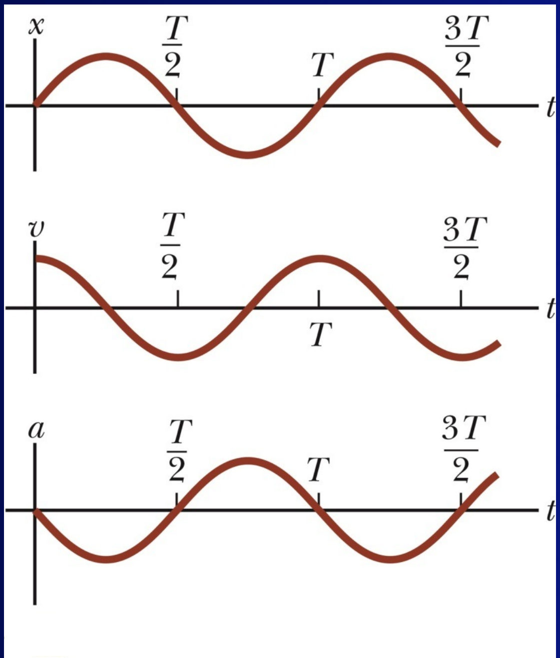
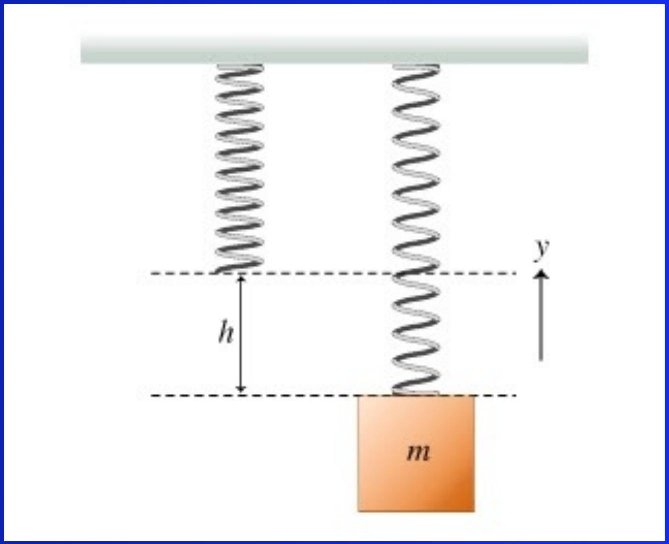
Physics 1C: Simple Harmonic Motion

Tuesday, 30 June 2015





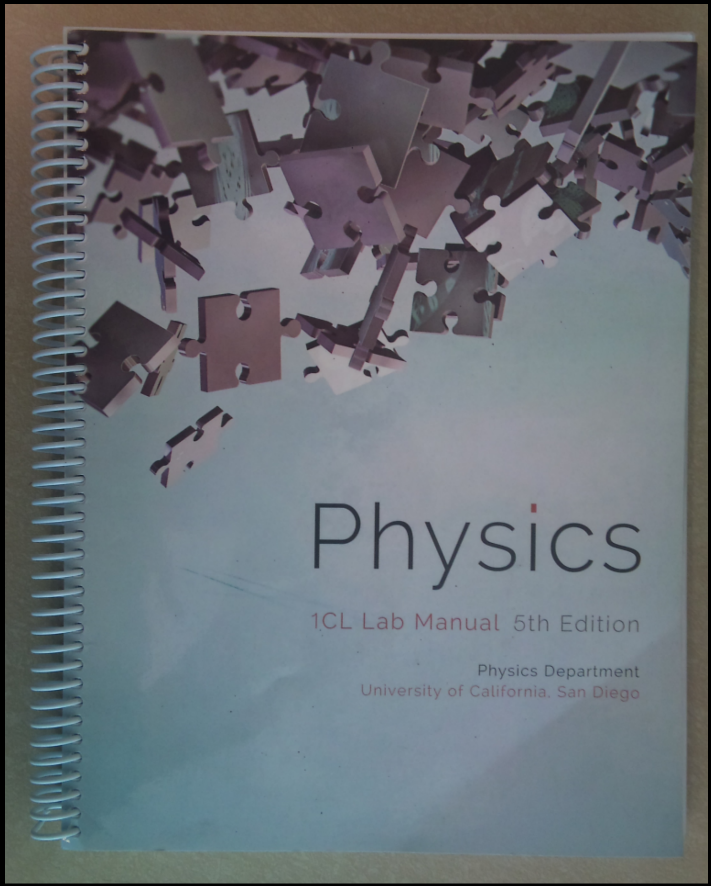
Physics 1CL

Lab TA Coordinators

- **Mike Eldridge (meldridge@ucsd.edu)**
- **Sebastian Diaz (s1diaz@ucsd.edu)**
- **Prarit Agarwal (pagarwal@ucsd.edu)**

Faculty Instructor

- **Ian Schanning**



Lab Manual (5th Edition) is available at the bookstore.

Syllabus/Calendar Link/Supplementary Materials on TED.

Labs start ***this*** week!

Reminders

- our Ted course website is now up! look there for the syllabus, lecture slides, announcements, and other info
- register your clicker on the Ted site if you haven't already
- self-enroll on `webassign.net` (class key: ucsd 1146 4985), and don't worry, "trial period" will be extended for course duration
- problems/questions for chapter 12 are similar to ones in book: Obj.Q. #1, 7, 11, 12; Conc.Q. #1 & 4; Problems #1, 3, 17, 20 & 33
- the first homework assignment (due tomorrow) won't be due for a grade, but Friday's will be due (at 5pm)
- the first quiz will be next Monday
- the first problem session is tomorrow after class
- the first lab is on Thursday

office hours

- Ramin Skibba

  rskibba@ucsd.edu, 429 SERF building

  office hours on Wednesdays at 1-3pm and by appointment

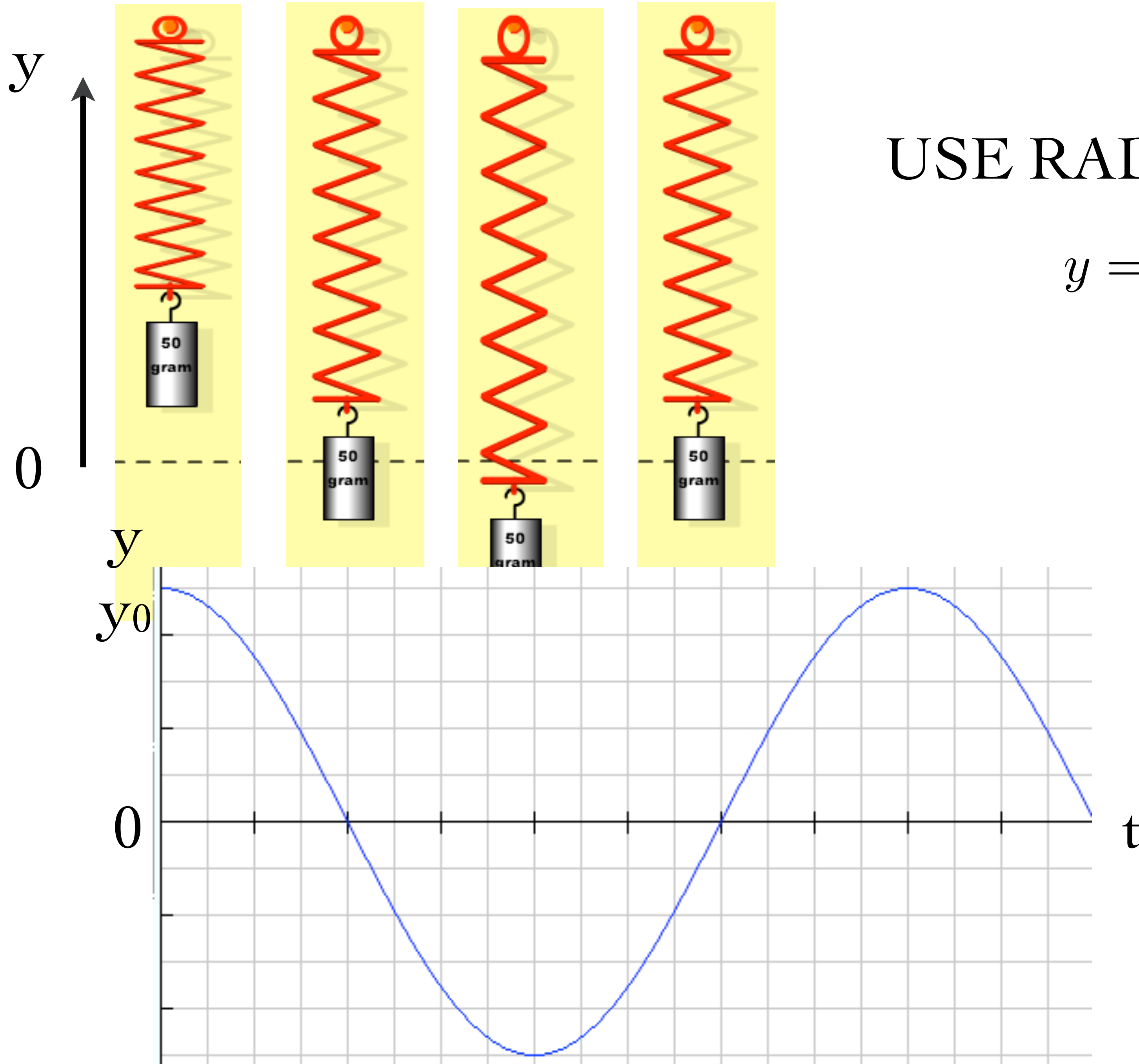
- TA: Raul Herrera, has office hours on Tuesdays at 11-12:50 in Mayer Hall 5426 and leads PBs on Wednesdays at 11-12:50 at Center Hall 222

  rherrera@physics.ucsd.edu

Oscillatory Motion

- Hooke's law: $F = -kx$
- Newton's second law: $F = ma$
- a point mass on a spring exhibits "simple harmonic motion"
- a simple pendulum at small angles approximately does too
- for a mass on a spring, the position as a function of time can be modeled as $x(t) = A \cos(\omega t+\phi)$, where A, ω, and ϕ are the amplitude, frequency, and phase, respectively
- Note how position, velocity, acceleration, and periods of masses on springs and pendula are similar to each other

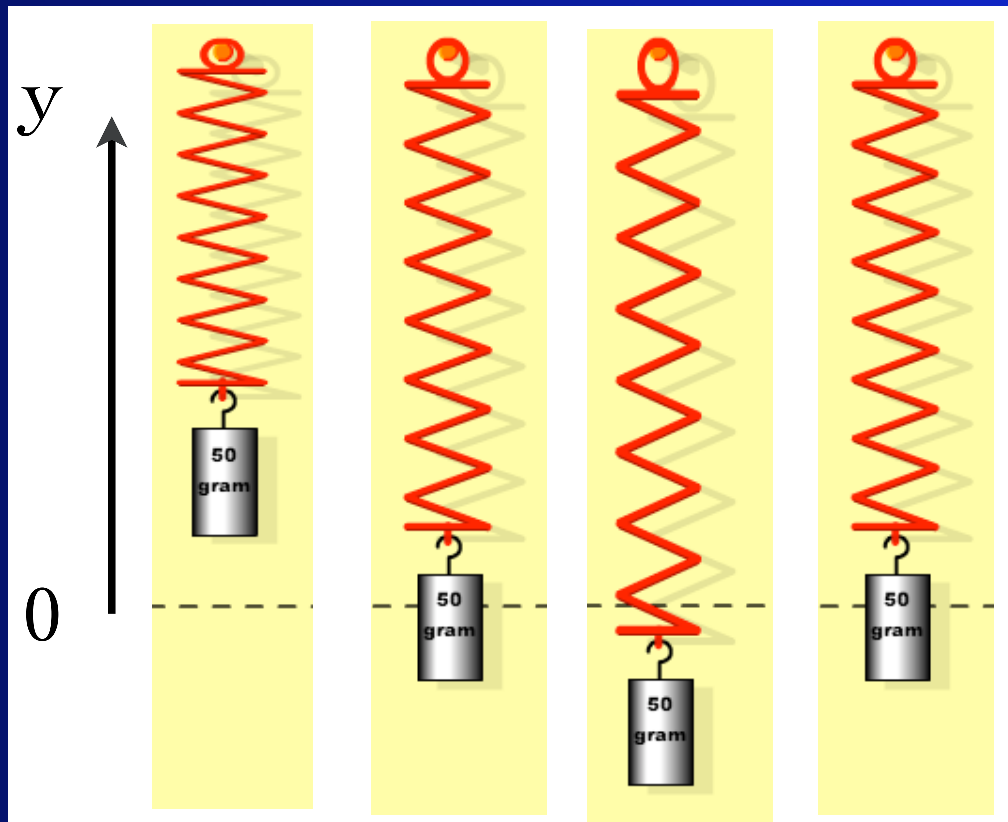
Mass's Position as a Function of Time



USE RADIANS!!

$$y = y_0 \cos(\omega t)$$

Mass on a Spring: vel. & accel.



v:	0	–	0	+
a:	–	0	+	0

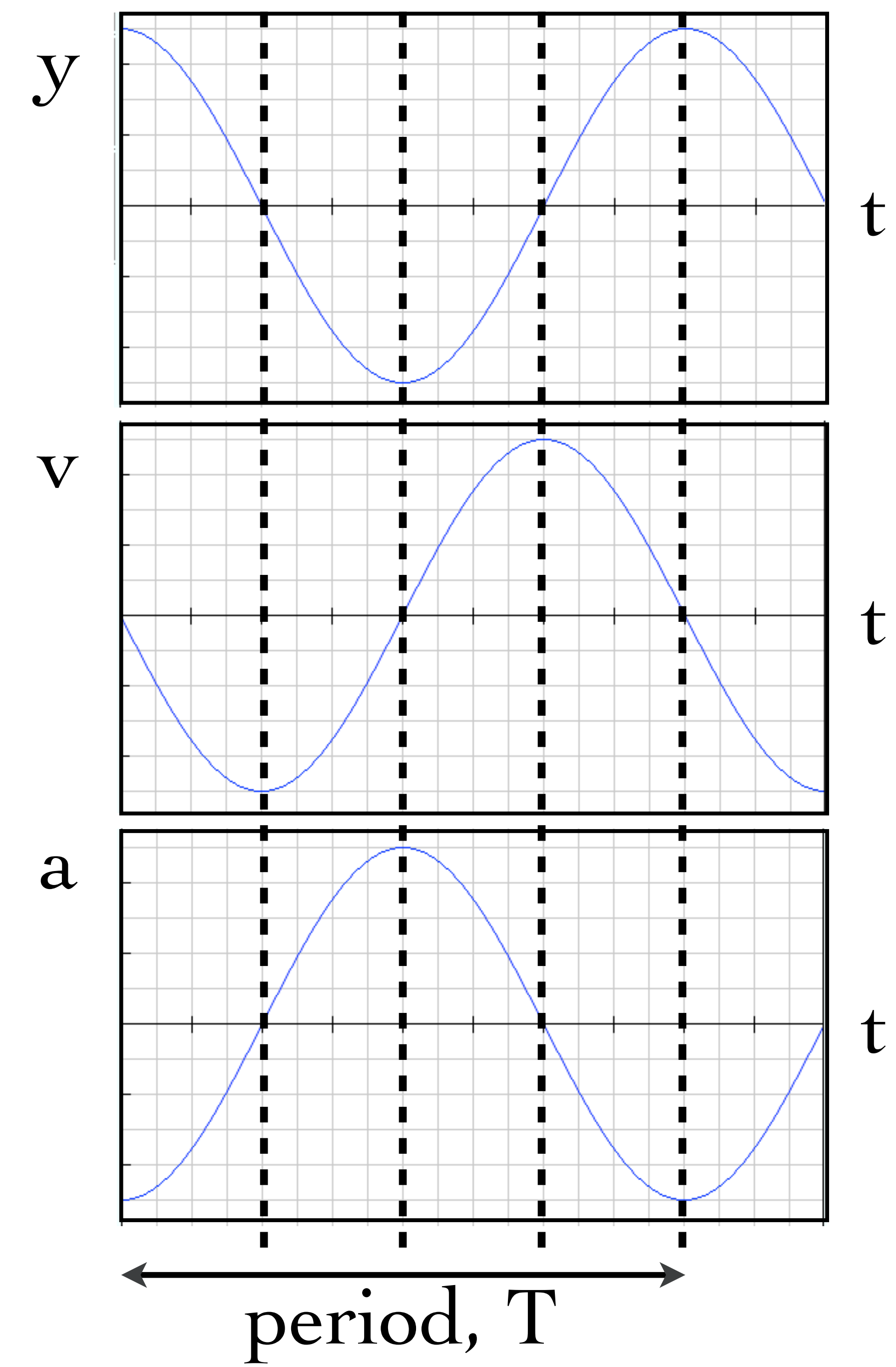
Review: position, velocity, acceleration

$y = y_0 \cos(\omega t)$

$v_y = -y_0 \omega \sin(\omega t)$

$a_y = -y_0 \omega^2 \cos(\omega t)$

and $\omega = 2\pi f$, $\omega = 2\pi / T$, $\omega = \sqrt{(k/m)}$



Simple Harmonic Motion (SHM)

position $$x = A\cos(\omega t + \phi_0)$$

velocity $$\frac{dx}{dt} = -A\omega\sin(\omega t + \phi_0)$$

acceleration $$\frac{d^2x}{dt^2} = -A\omega^2\cos(\omega t + \phi_0)$$

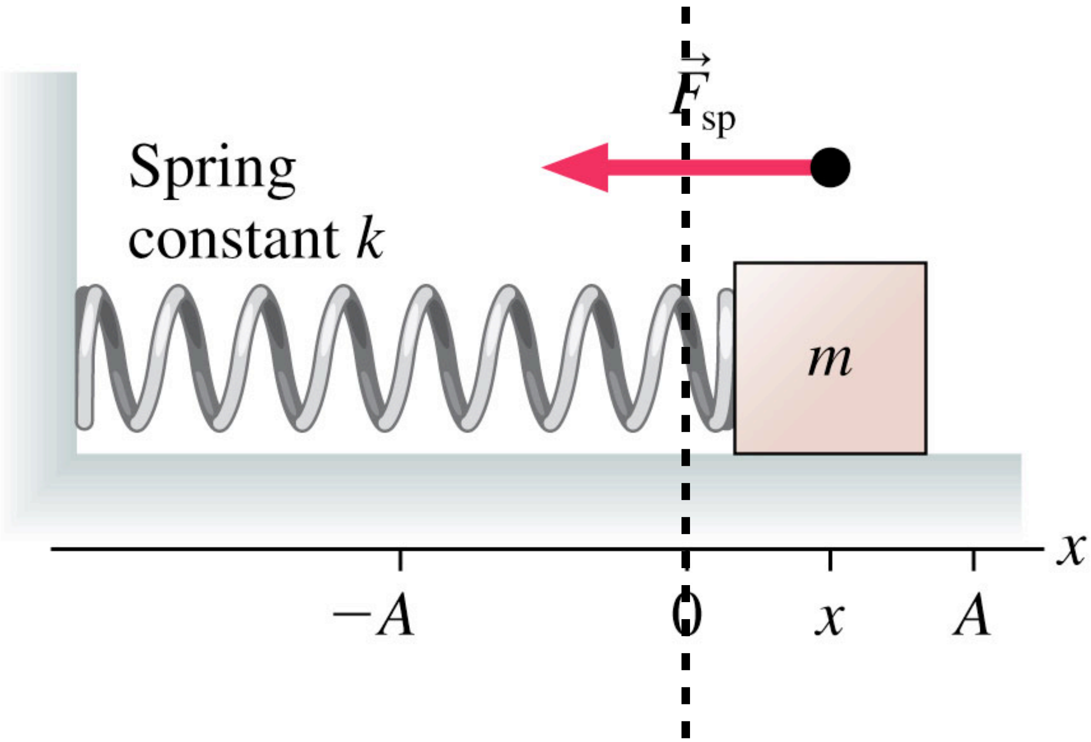
$$\frac{d^2x}{dt^2} = -\omega^2 x$$

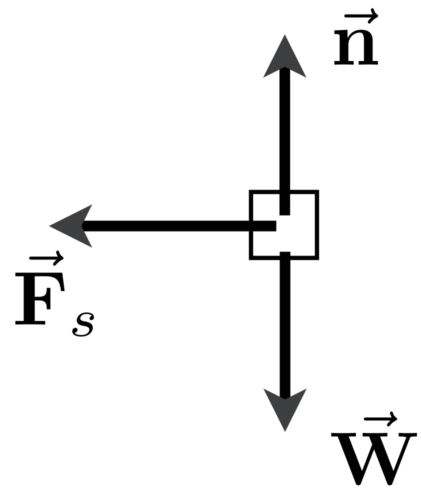
$$F_{\text{net}} = m\frac{d^2x}{dt^2} = -m\omega^2 x$$

"linear restoring force"

Simple Harmonic Motion

$$F_{\text{net}} = -m\omega^2 x$$





equilibrium

$$F_{\text{net}} = -kx$$

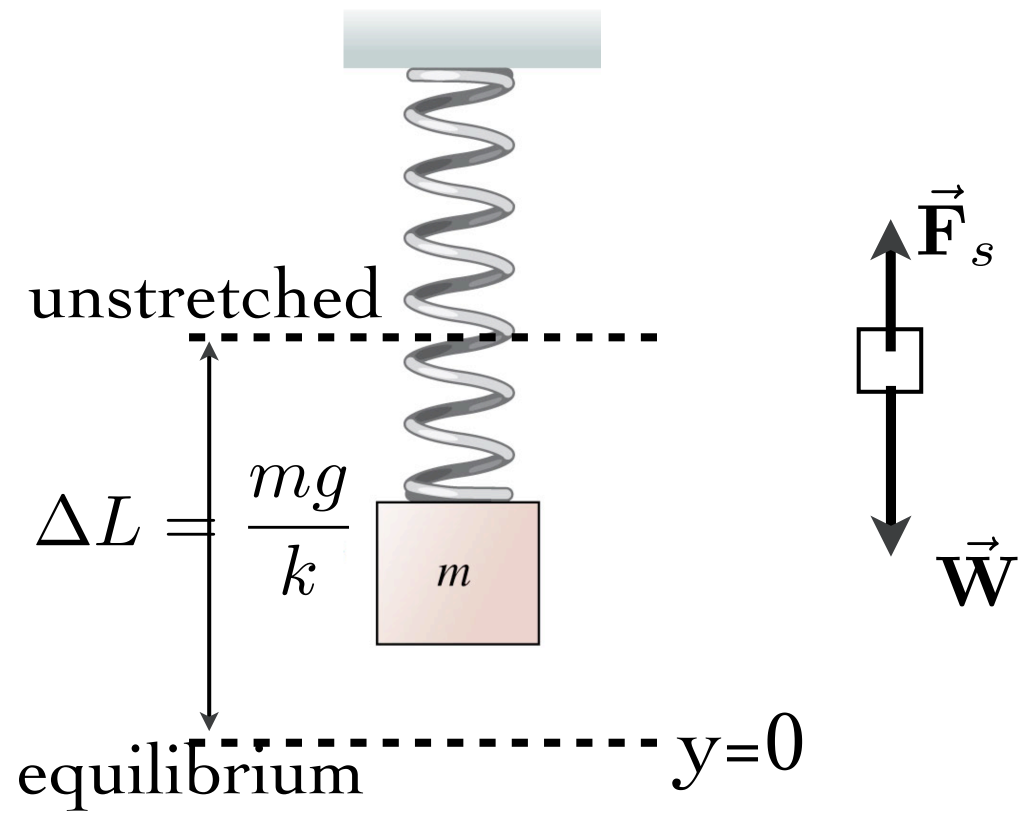
$$\omega = \sqrt{\frac{k}{m}} \qquad T = 2\pi\sqrt{\frac{m}{k}}$$

Simple Harmonic Motion

$F_{\text{net}} = -mg + k(\Delta L - y)$

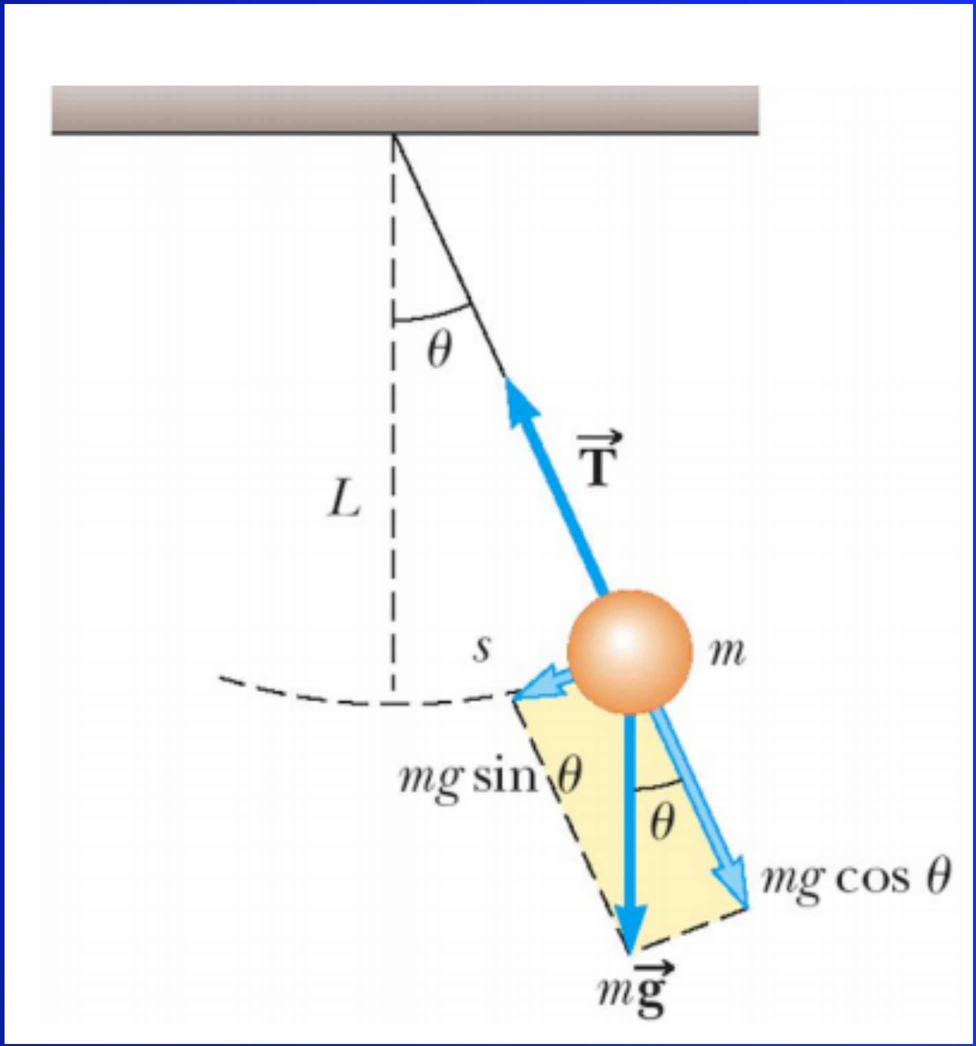
$F_{\text{net}} = -ky + (k\Delta L - mg)$

so $F_{\text{net}} = -ky$ is the "linear restoring force"

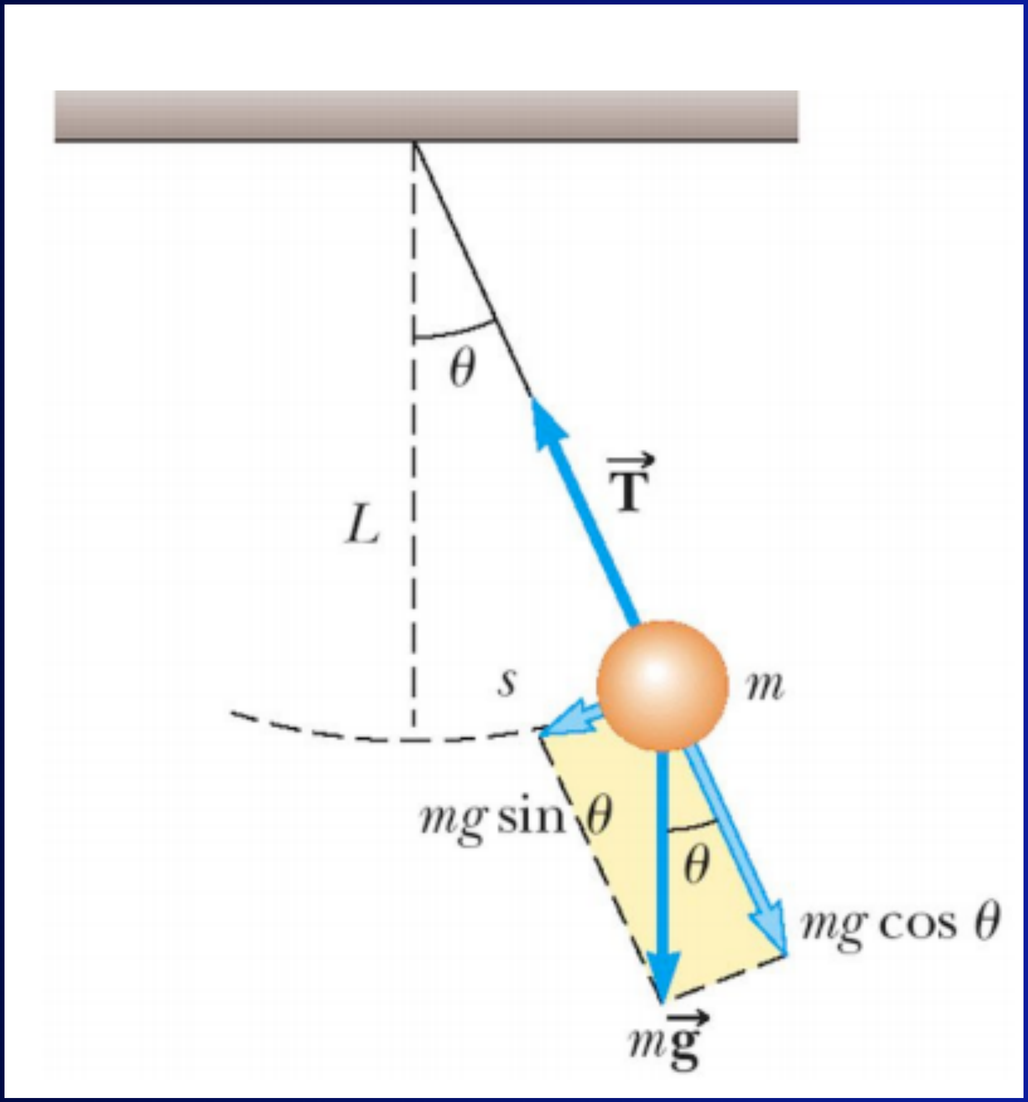


Simple Harmonic Motion: Pendulum

- with a pendulum, we have simple harmonic motion for small angles
- small angle approx applies when $\theta < 10°$ (or 0.17 rad), when $\sin(\theta) \approx \theta$
- $F_{net} = ma = -m\omega^2 x$
- so the tangential force on the pendulum is $F_{net} = -mg\sin(\theta)$



Simple Harmonic Motion: Pendulum



- $F_{net,t} = -mg\sin(\theta)$
- so $F_{net,t} \approx -m(g/L)s$
- angular frequency: $\omega=\sqrt{(g/L)}$ and $T=2\pi/\omega$

Simple Harmonic Motion: Pendulum

$$y(t) = A\cos(\omega t + \phi_0)$$

$$\text{phase} = \omega t + \phi_0$$

What is the phase when the position of the mass is at its minimum value?

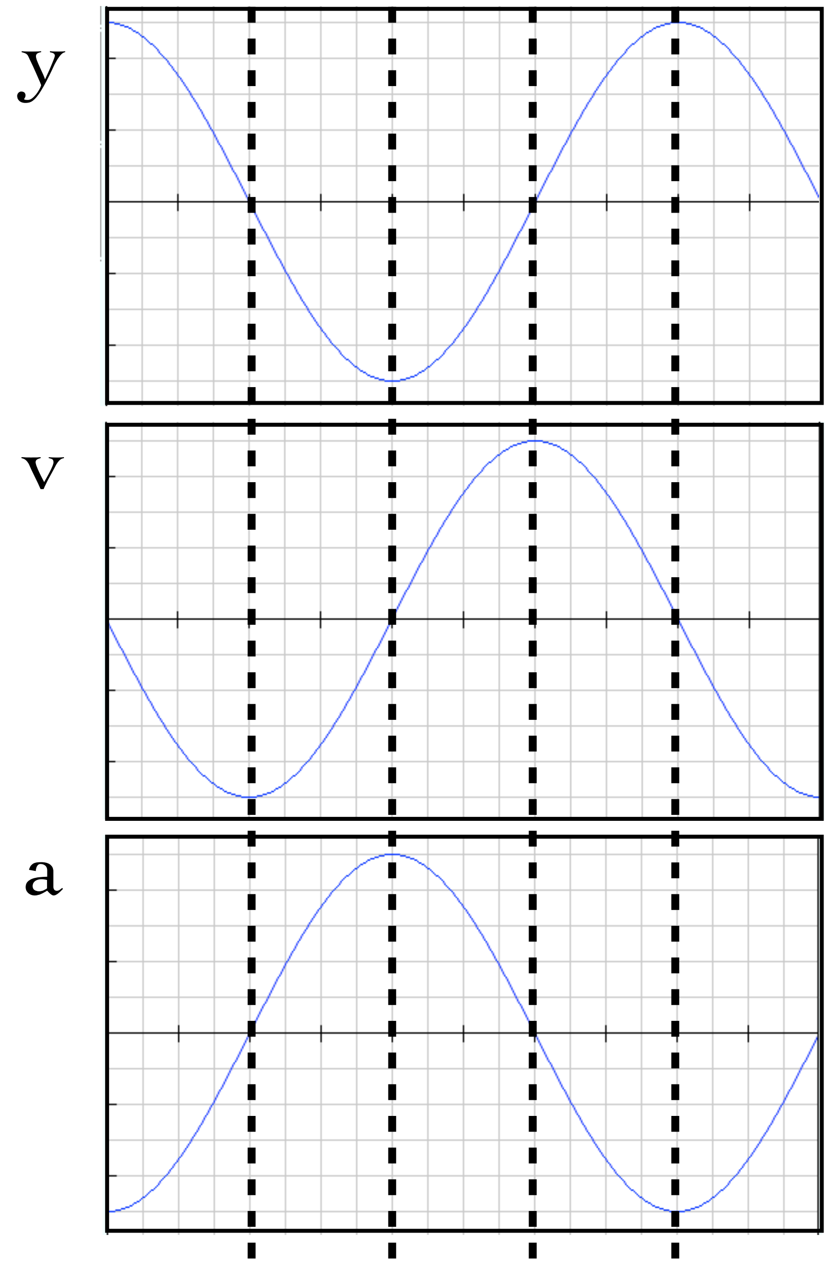
A. 0

B. $\pi/2$

C. π

D. $3\pi/2$

E. more than one of these



Simple Harmonic Motion: Pendulum

$$y(t) = A\cos(\omega t + \phi_0)$$

$$\text{phase} = \omega t + \phi_0$$

What is the phase when the position of the mass is at its equilibrium (y=0) value?

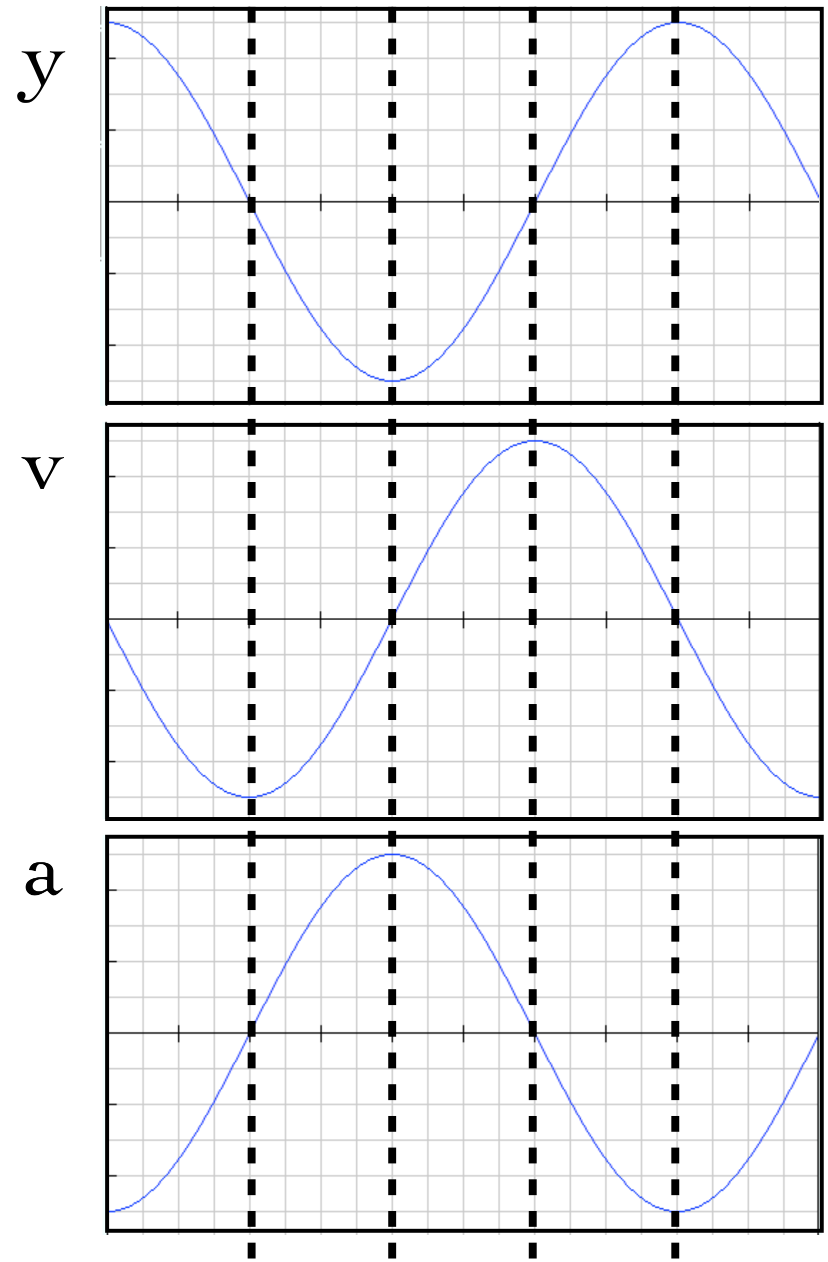
A. 0

B. $\pi/2$

C. π

D. $3\pi/2$

E. more than one of these



Simple Harmonic Motion: Pendulum

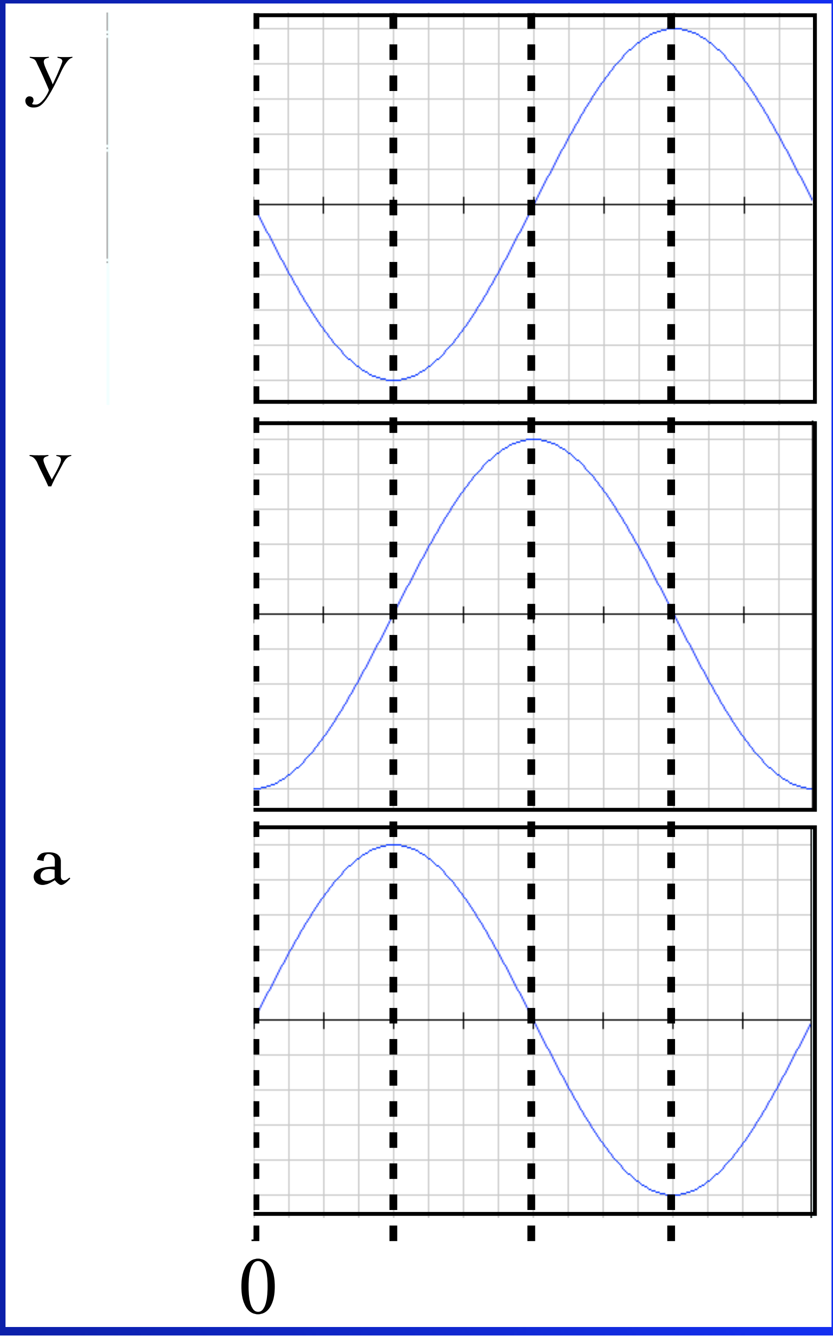
$$y(t) = A\cos(\omega t + \phi_0)$$

$$\text{phase} = \omega t + \phi_0$$

The ***phase constant*** is ϕ_0

It tells us the initial phase— where to start the graph at $t=0$

for example, when $\phi_0 = \pi/2$



SHM: spring example

A mass on a spring oscillates with period 0.80 s and amplitude 10 cm. At t=0 s, it is 5.0 cm to the left of equilibrium and moving to the left. What are the position and velocity of the mass at time t=2.0 s?

remember, we know that $x(t) = A \cos(\omega t+\phi_0)$,
$\omega=2\pi f$ and $f=1/T$

[Hint: start by finding the angular frequency, then the phase constant…]

SHM: spring example

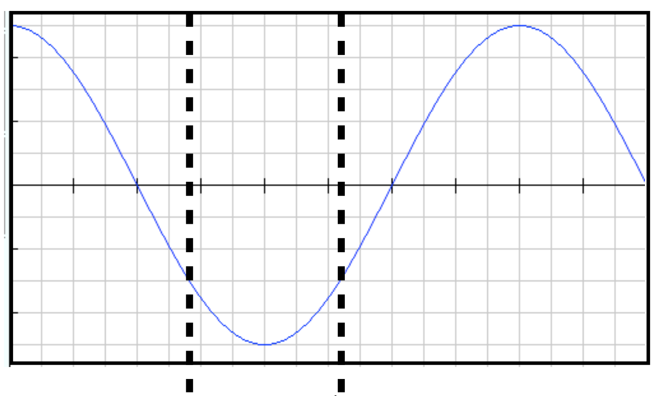
A mass on a spring oscillates with period 0.80 s and amplitude 10 cm. At t = 0 s, it is 5.0 cm to the left of equilibrium and moving to the left. What are the position and velocity of the mass at time t = 2.0 s?

$$x = A\cos(\omega t + \phi_0) \qquad t = 0 \qquad x = -5.0 \text{ cm}$$

$$A = 10 \text{ cm} \qquad \omega = \frac{2\pi}{T} = 7.854 \text{ rad/s}$$

$$-5.0 \text{ cm} = (10 \text{ cm})\cos\phi_0$$

$$\phi_0 = \frac{2\pi}{3}, \frac{4\pi}{3}$$

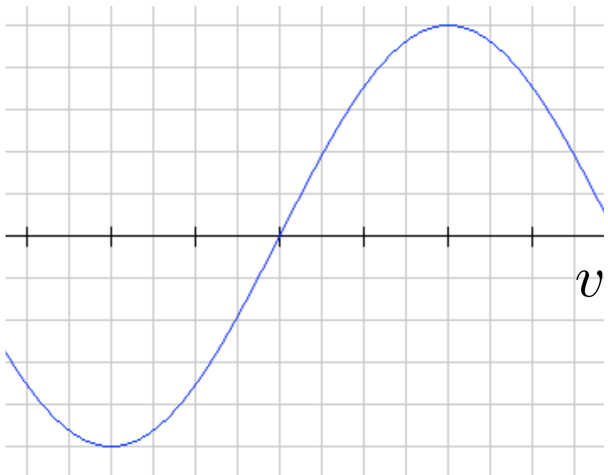


$$\frac{2\pi}{3}, \quad \frac{4\pi}{3}$$

$$x = (10 \text{ cm})\cos\left((7.854 \text{ rad/s})t + \frac{2\pi}{3}\right)$$

SHM: spring example

A mass on a spring oscillates with period 0.80 s and amplitude 10 cm. At t = 0 s, it is 5.0 cm to the left of equilibrium and moving to the left. What are the position and velocity of the mass at time t = 2.0 s?



$$x(t) = (10\ \mathrm{cm})\cos\left((7.854\ \mathrm{rad/s})t + \frac{2\pi}{3}\right)$$

$$v(t) = -(78.54\ \mathrm{cm/s})\sin\left((7.854\ \mathrm{rad/s})t + \frac{2\pi}{3}\right)$$

$$x(2\ \mathrm{s}) = 5.0\ \mathrm{cm}$$

$$v(2\ \mathrm{s}) = 68\ \mathrm{cm/s}$$

INTERMISSION

Conservation of Energy

- assuming an isolated spring-mass system is moving on a frictionless surface, then the ***total energy is constant***
- kinetic energy:

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi)$$

- elastic potential energy:

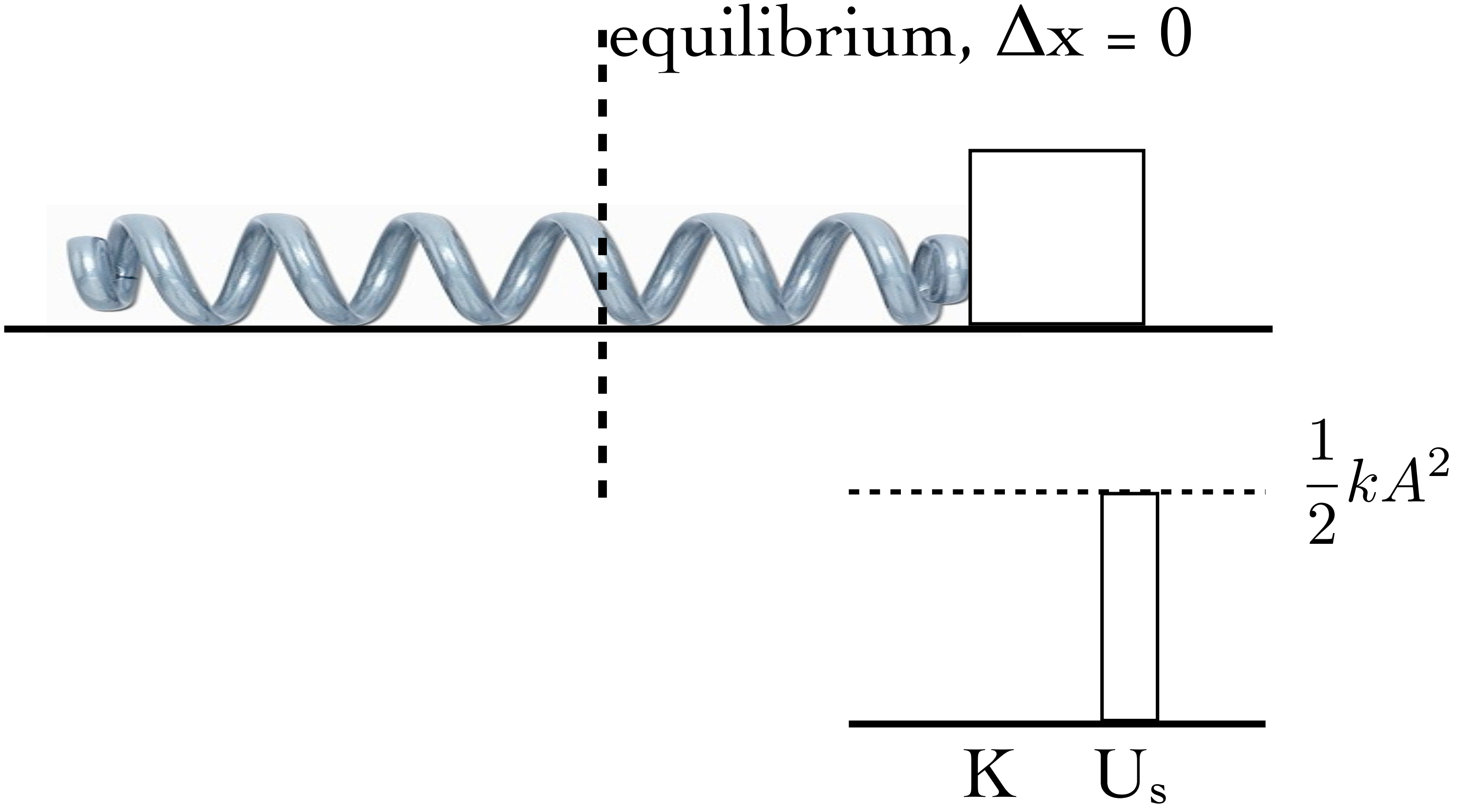
$$U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \cos^2(\omega t + \phi)$$

total energy $E = K + U = (1/2)kA^2 =$ <u>constant</u>

Conservation of Energy

block released from rest

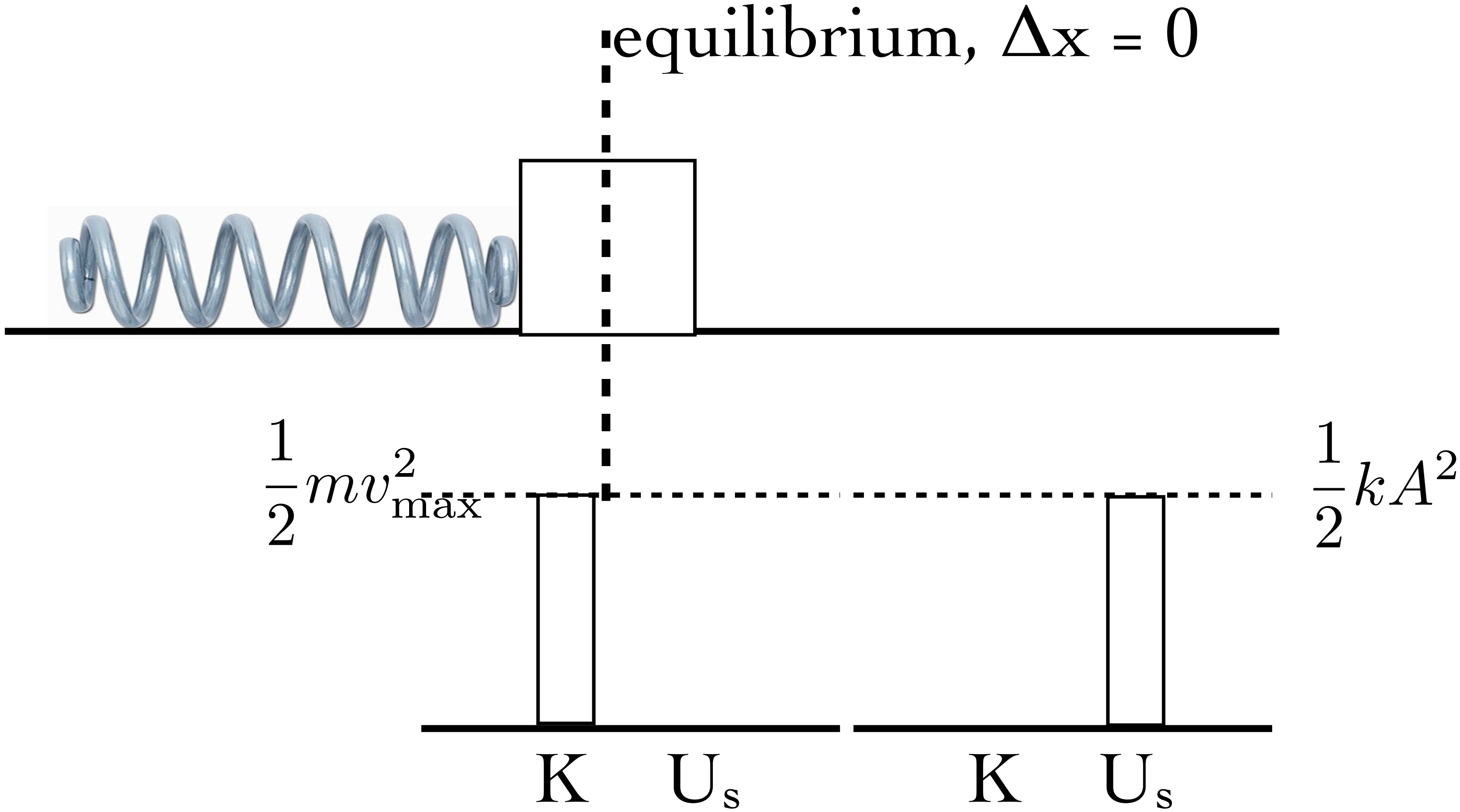
A = Amplitude



Conservation of Energy

block released from rest

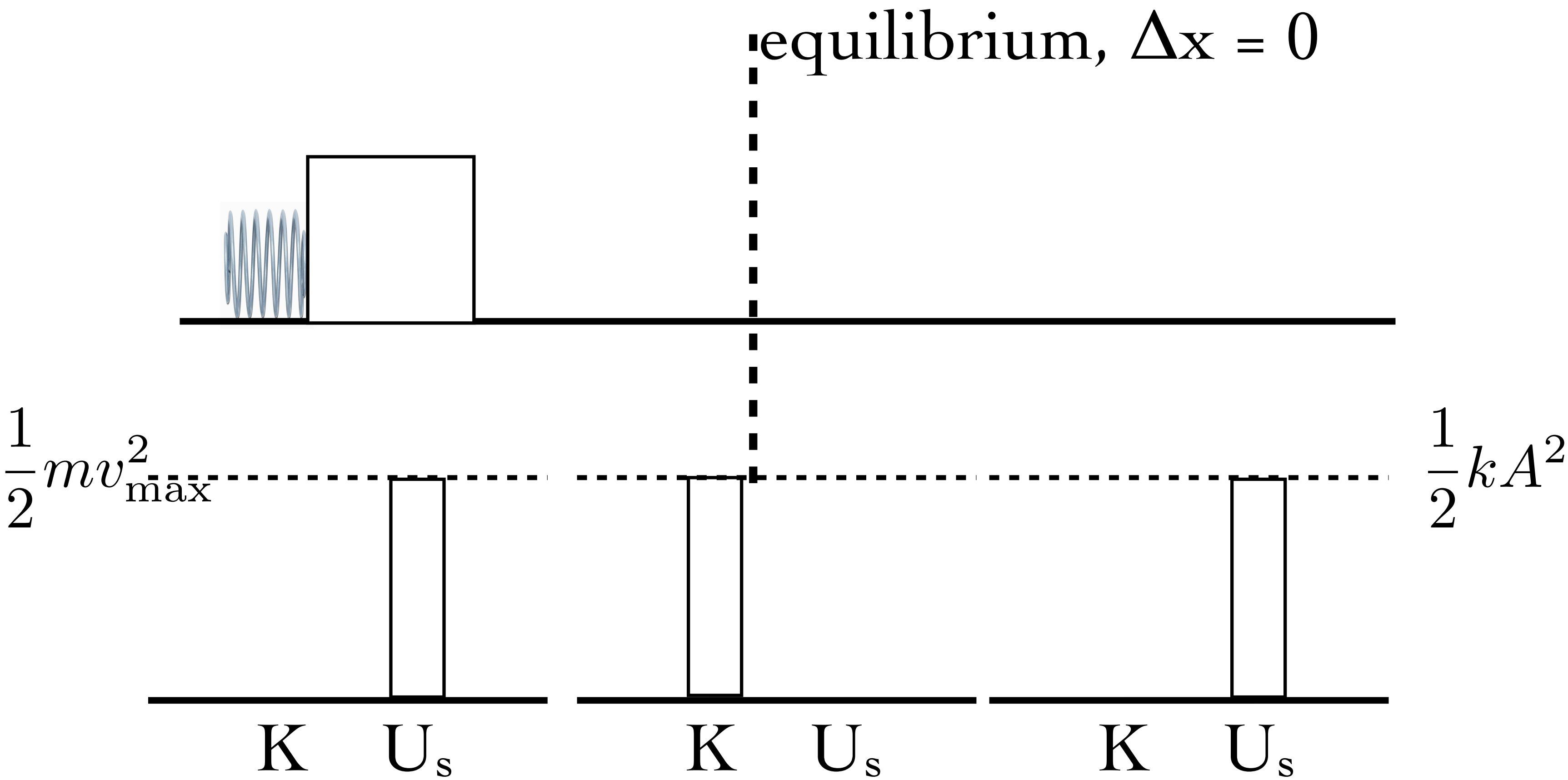
A = Amplitude



Conservation of Energy

block released from rest

A = Amplitude

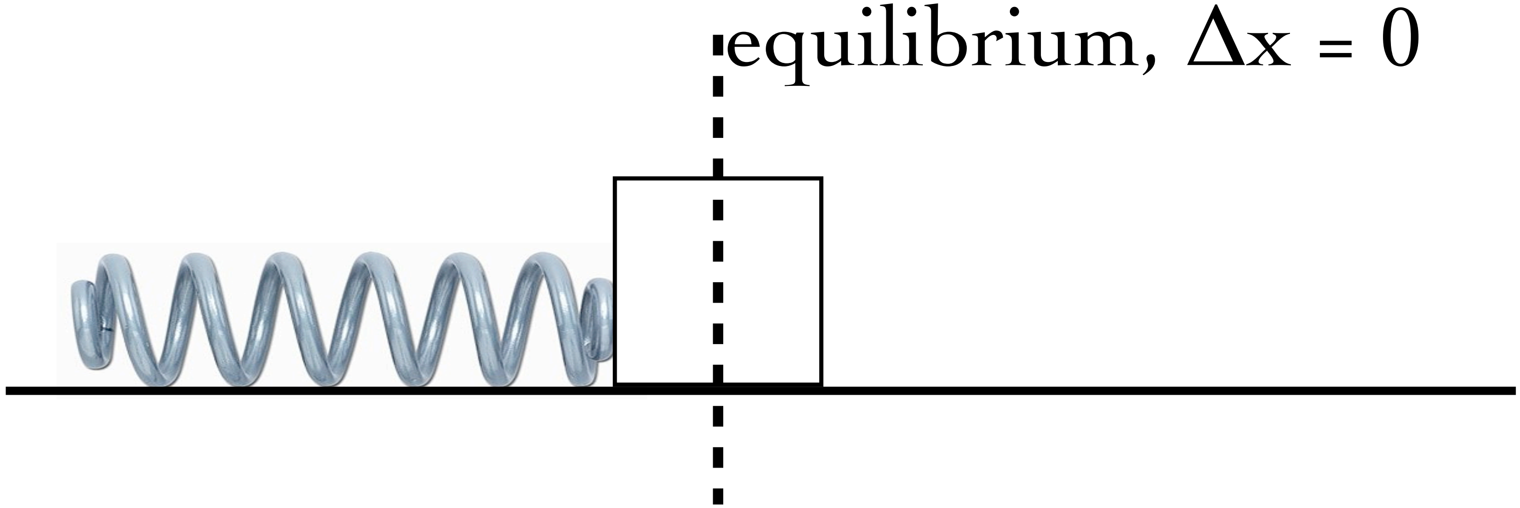


Conservation of Energy

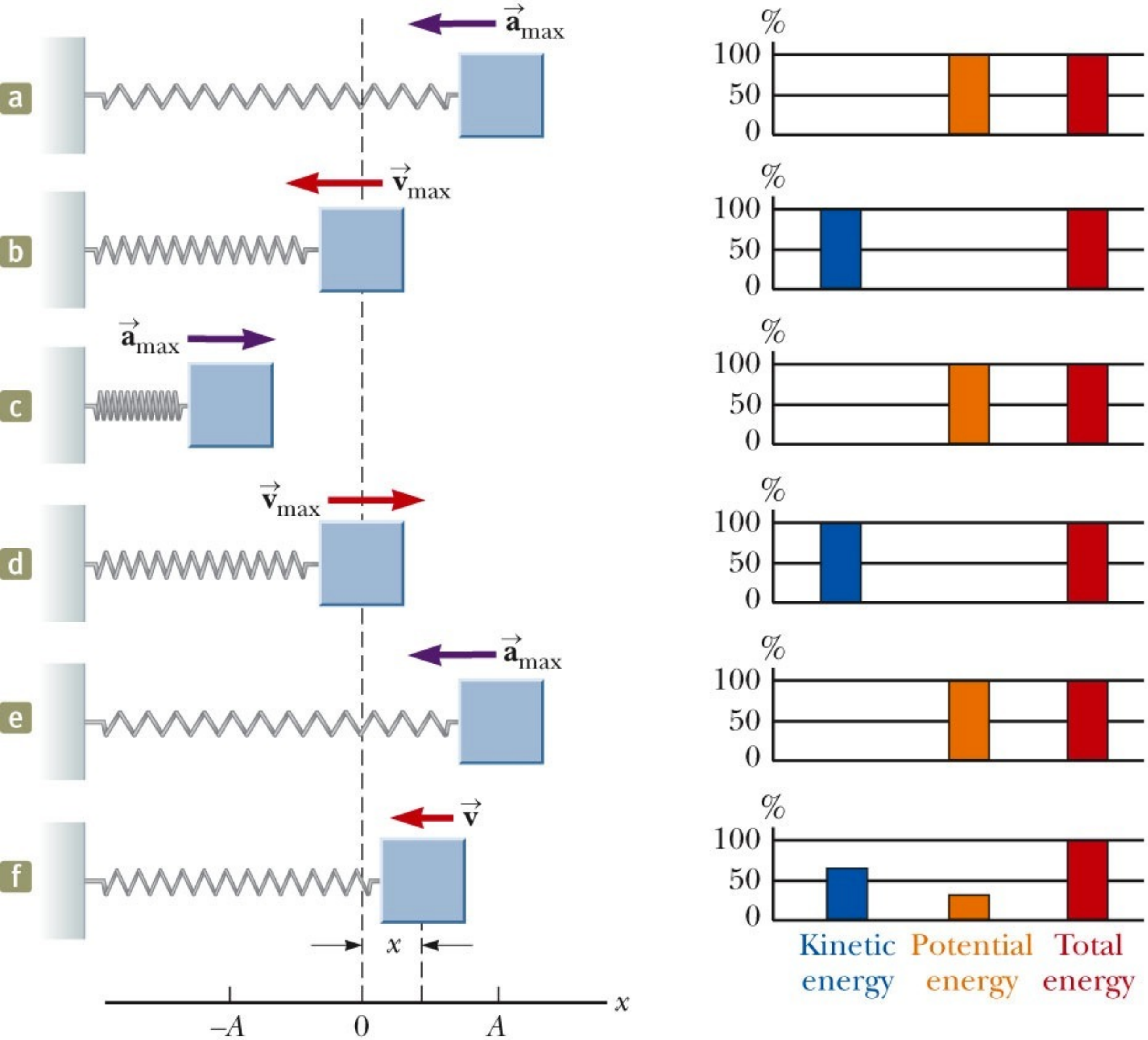
- kinetic and potential energy change with time (and are different than each other)
- but the total energy remains constant:

$E_{\mathbf{total}}$ **= K+U = (1/2)$m(v_{\mathbf{max}})^2$ = (1/2)kA^2**

$$K = \frac{1}{2}mv^2 \qquad U_s = \frac{1}{2}k(\Delta x)^2$$



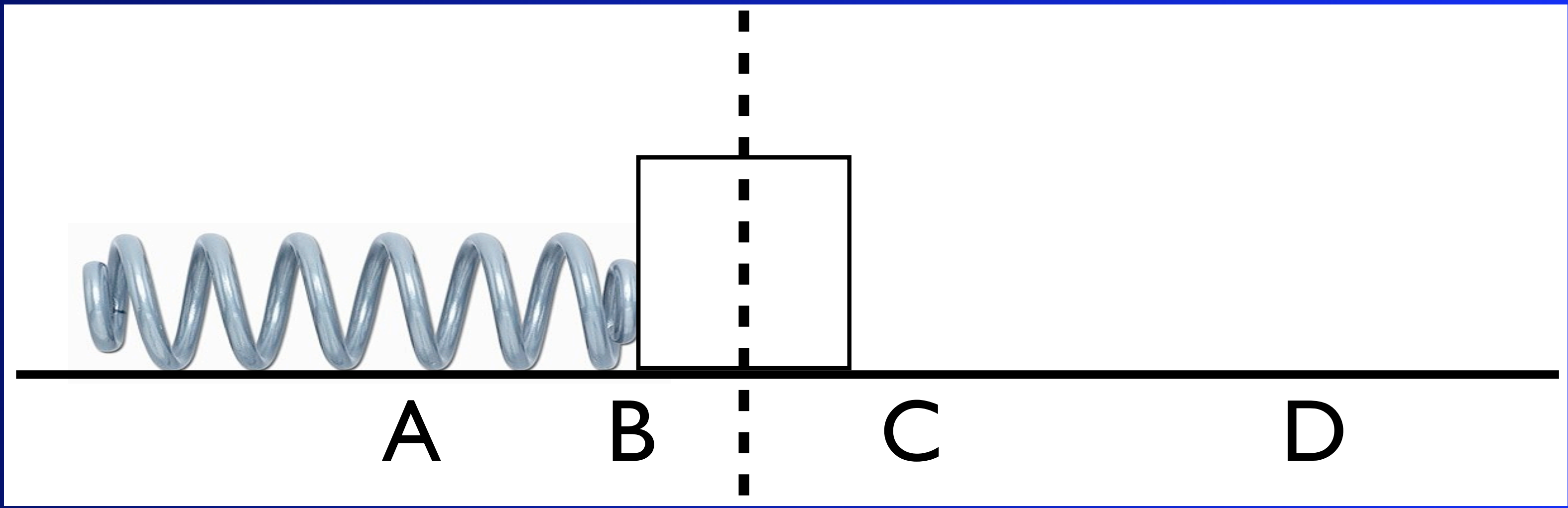
Conservation of Energy



t	x	v	a	K	U
0	A	0	$-\omega^2 A$	0	$\frac{1}{2}kA^2$
$\frac{T}{4}$	0	$-\omega A$	0	$\frac{1}{2}kA^2$	0
$\frac{T}{2}$	$-A$	0	$\omega^2 A$	0	$\frac{1}{2}kA^2$
$\frac{3T}{4}$	0	ωA	0	$\frac{1}{2}kA^2$	0
T	A	0	$-\omega^2 A$	0	$\frac{1}{2}kA^2$
t	x	v	$-\omega^2 x$	$\frac{1}{2}mv^2$	$\frac{1}{2}kx^2$

Conservation of Energy: Example

A mass affixed to the end of a horizontal spring oscillates with amplitude A. When the mass is located at which position below (A, B, C, or D) will the potential energy of the mass-spring system be largest?



A. $x = -0.6A$

B. $x = -0.2A$

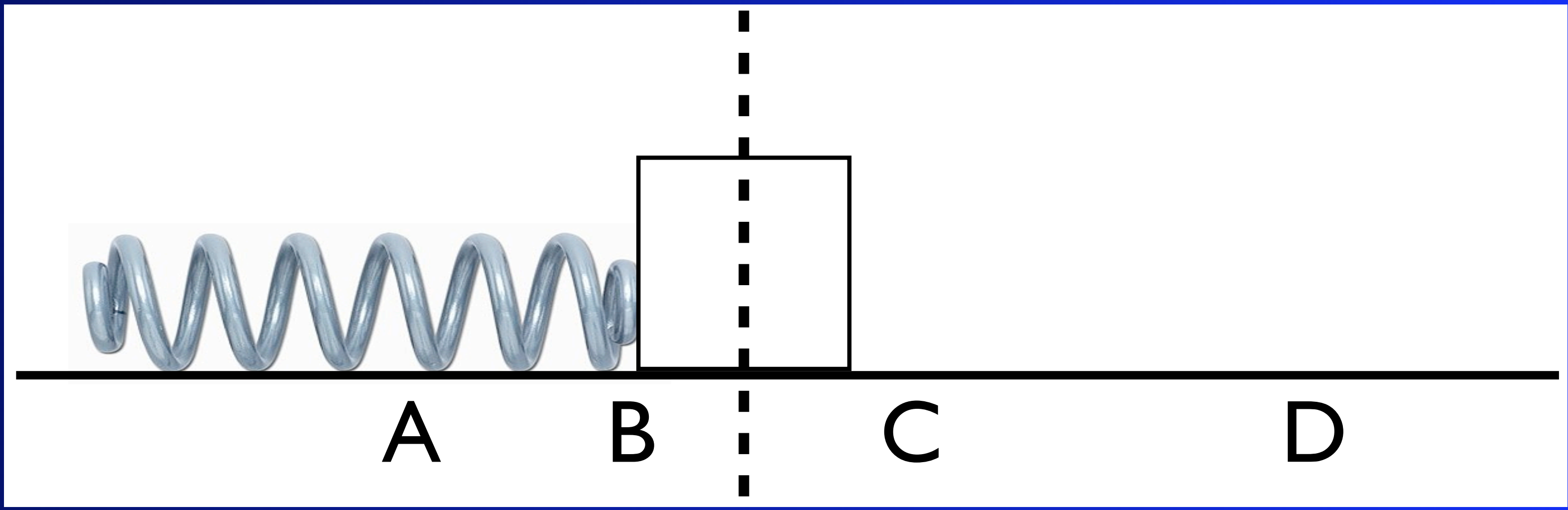
C. $x = +0.3A$

D. $x = +0.8A$

E. All four configurations have the same potential energy

Conservation of Energy: Example

A mass affixed to the end of a horizontal spring oscillates with amplitude A. At which location will the mass have the largest speed? [Hint: think conservation of energy.]



A. $x = -0.6A$

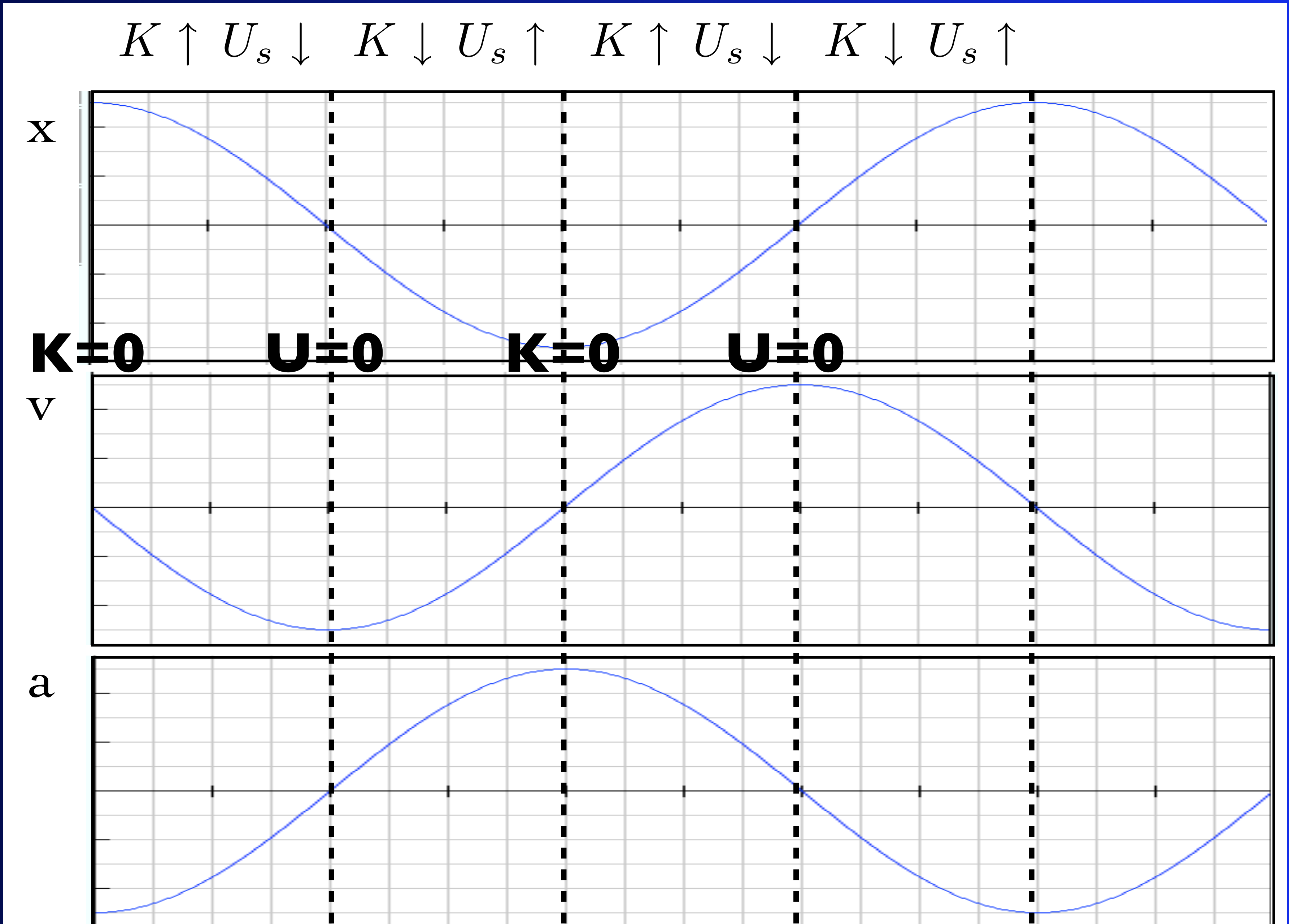
B. $x = -0.2A$

C. $x = +0.3A$

D. $x = +0.8A$

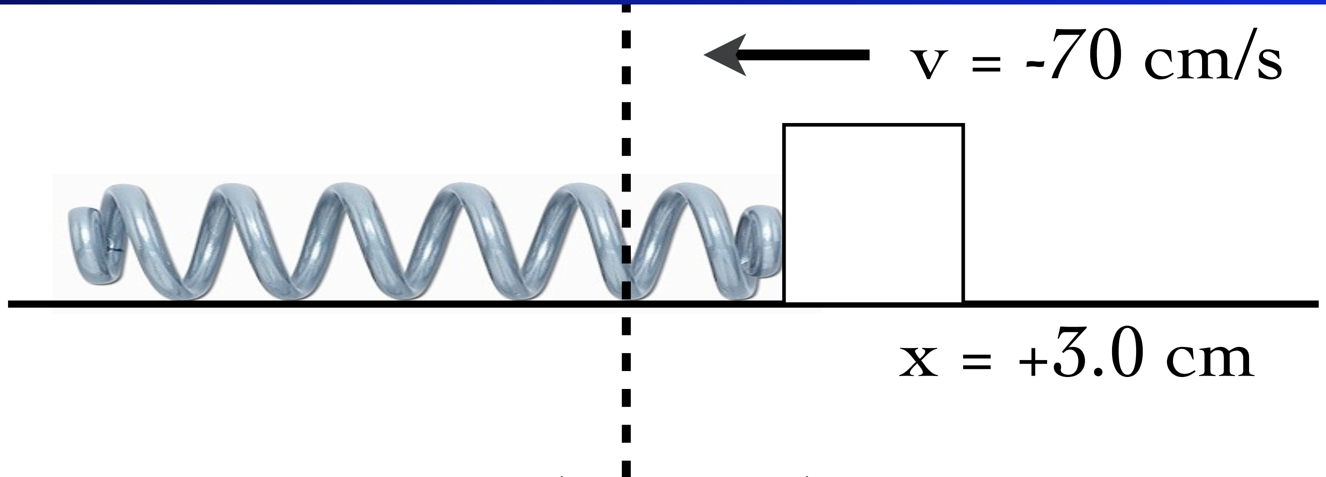
E. The speed is the same at all four points

Conservation of Energy



Conservation of Energy: An Example

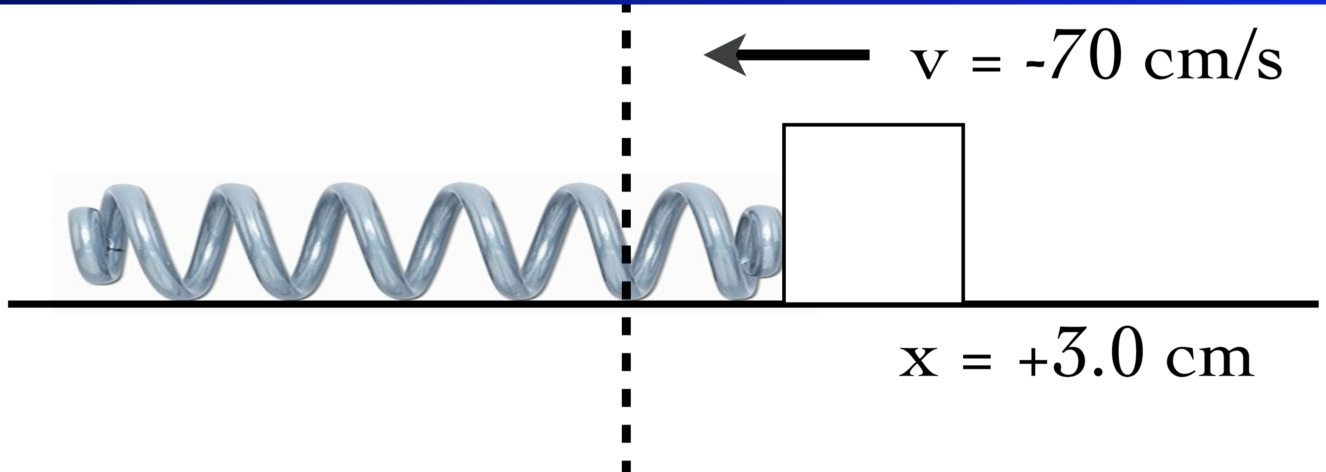
A 2.0 kg block is connected to a 150 N/m spring. At time t=0 s, the block is located at x=+3.0 cm from equilibrium and has velocity v=–70 cm/s. What is the amplitude of the block's oscillations?



$$E_{\text{total}} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

Conservation of Energy: An Example

A 2.0 kg block is connected to a 150 N/m spring. At time t=0 s, the block is located at x=+3.0 cm from equilibrium and has velocity v=–70 cm/s. What is the amplitude of the block's oscillations?



$$E_{\text{total}} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$E_{\text{total}} = \frac{1}{2}(2.0\text{ kg})(-0.70\text{ m/s})^2 + \frac{1}{2}(150\text{ N/m})(0.03\text{ m})^2 = 0.5575\text{ J}$$

$$E_{\text{total}} = \frac{1}{2}kA^2$$

$$A = 8.6\text{ cm}$$

Conservation of Energy: An Example

A 2.0 kg block is connected to a 150 N/m spring. At time t=0 s, the block is located at x=3.0 cm from equilibrium and has velocity v=–70 cm/s.

What is the speed of the block when it is located at x = –2.0 cm?

a. 0.528 m/s
b. 0.726 m/s
c. 1.027 m/s
d. need more information

Conservation of Energy: An Example

A 2.0 kg block is connected to a 150 N/m spring. At time t = 0 s, the block is located at x = +3.0 cm from equilibrium and has velocity v = -70 cm/s.

a. What is the speed of the block when it is located at x = -2.0 cm?

$$E_{\text{total}} = \frac{1}{2}(2.0\ \text{kg})(-0.70\ \text{m/s})^2 + \frac{1}{2}(150\ \text{N/m})(0.03\ \text{m})^2 = 0.5575\ \text{J}$$

$$E_{\text{total}} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$0.5575\ \text{J} = \frac{1}{2}(2.0\ \text{kg})v^2 + \frac{1}{2}(150\ \text{N/m})(-0.02\ \text{m})^2$$

$$v = 0.726\ \text{m/s}$$

Conservation of Energy: An Example

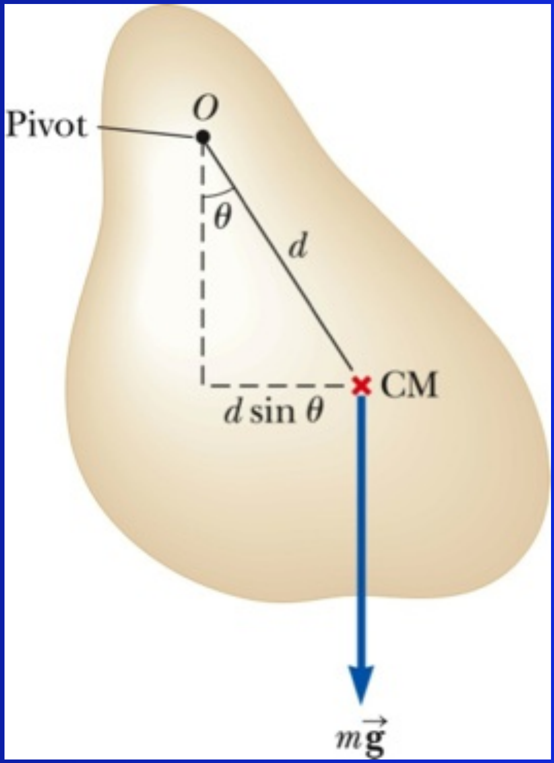
A 2.0 kg block is connected to a 150 N/m spring. At time t=0 s, the block is located at x=3.0 cm from equilibrium and has velocity v=–70 cm/s.

a. What is the speed of the block when it is located at x = –2.0 cm?
b. Use energy considerations to determine the maximum speed of the block.
c. What is the angular frequency and period of oscillation?
d. What is the phase constant, ϕ_0?

(Remember that we calculated E_{total}=0.56 J and A=8.6cm)

Physical Pendulum

- The gravitational force provides a torque about an axis through *O*
  - The magnitude of the torque is $mgd \sin \theta$
  - I is the moment of inertia about the axis through *O*



$$\omega = \sqrt{\frac{mgd}{I}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mgd}}$$

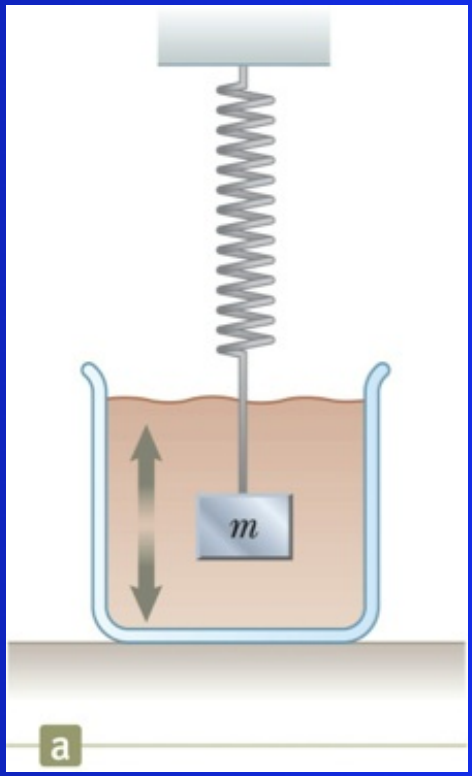
Note: if $I=md^2$ for a flat rigid object, then the physical pendulum behaves as a simple pendulum.

Forced and Damped Oscillations

In the real world, on object moving through a medium experiences a *resistive force* that *damps* oscillations.

This affects the angular frequency and makes the amplitude, A, decrease with time. The energy, $(1/2)kA^2$, also decreases.

But *forced oscillations* can compensate for the loss in energy.



For Wednesday:

1. review chapter 12 and work on chapter 12 homework problems and questions on WebAssign
2. if there are problems you're not sure about, ask about them at tomorrow's problem session or at office hours
3. make sure you understand: position, velocity, acceleration; frequency and period; force and energy
4. start reading the next chapter on mechanical waves, at least sections 13.1 and 13.2