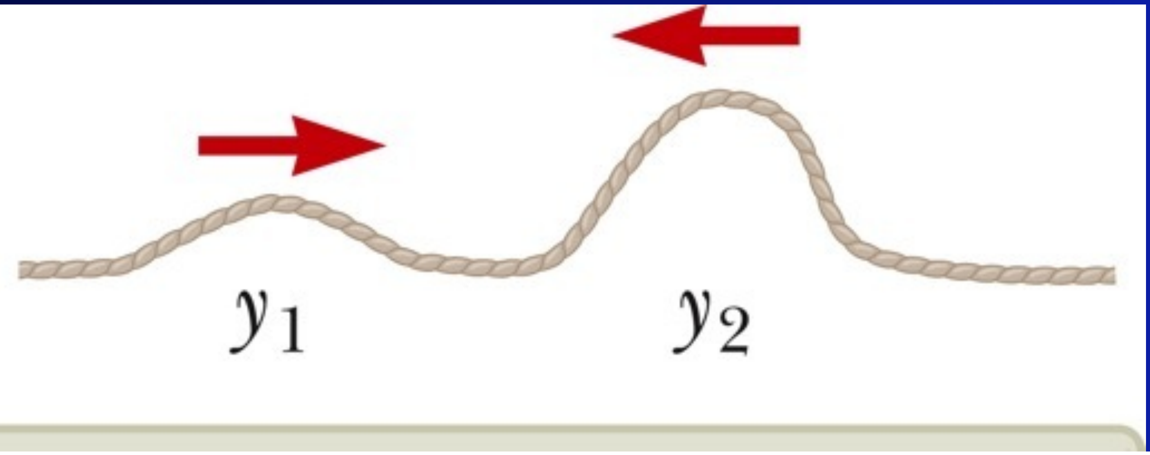
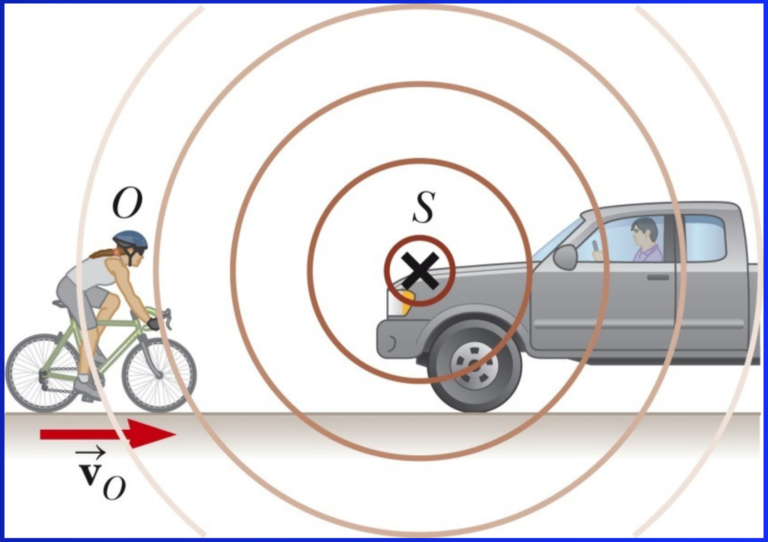
Physics 1C: Mechanical Waves and Interference

Thursday, 2 July 2015





Reminders

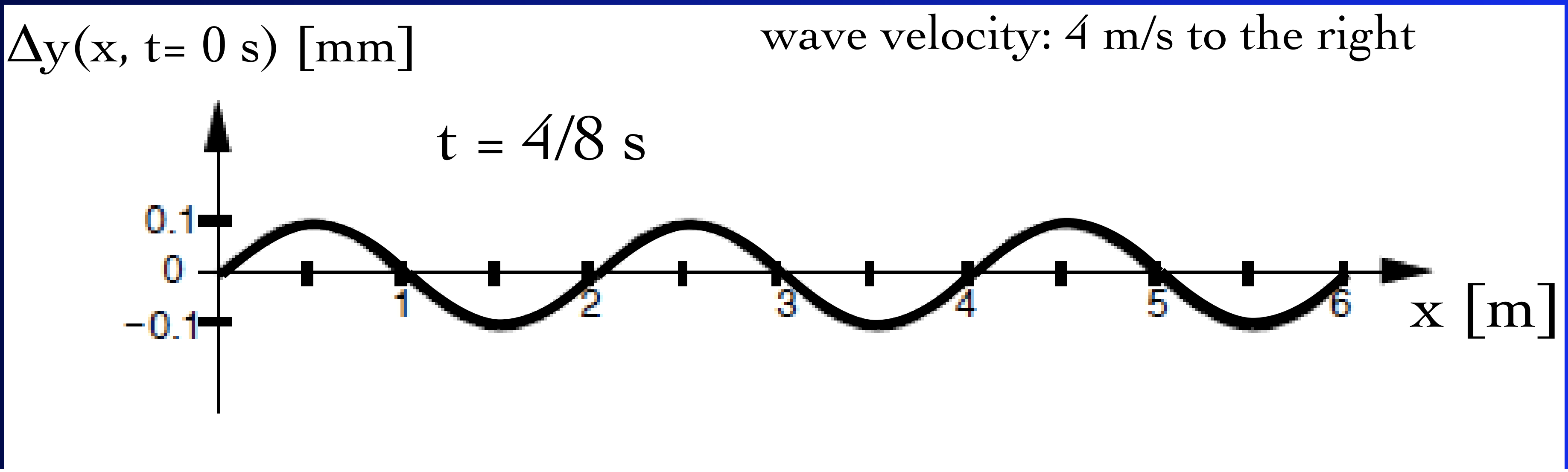
- homework (on WebAssign) due tomorrow
- first quiz will be on Monday (where you'll be given the equations you need; cheat sheets are not allowed)
- look at the Ted course website for the syllabus, lecture slides, announcements, and other info
- make sure that you've registered your clicker and bring it to class every day

Sinusoidal Waves

- How do we determine the period of a wave? It's the length of time needed to cycle through the wave, so $T = 1/2$ s

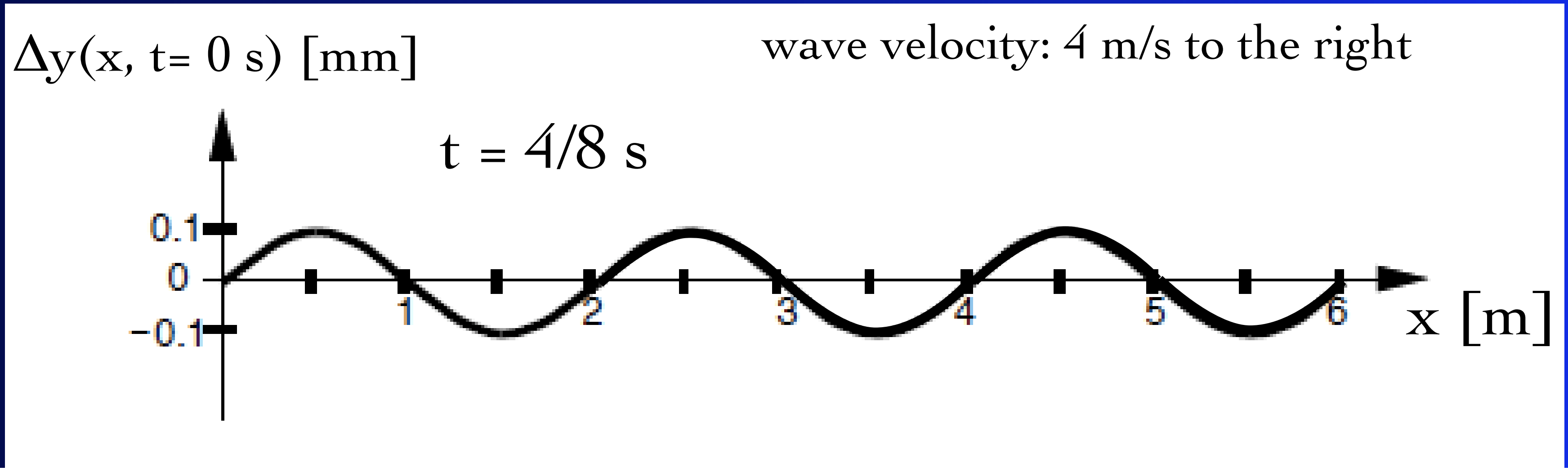
- How do we determine the frequency of the wave? $f = 1/T = 2$ Hz

- angular frequency $\omega = 2\pi f = 12.57$ rad/s



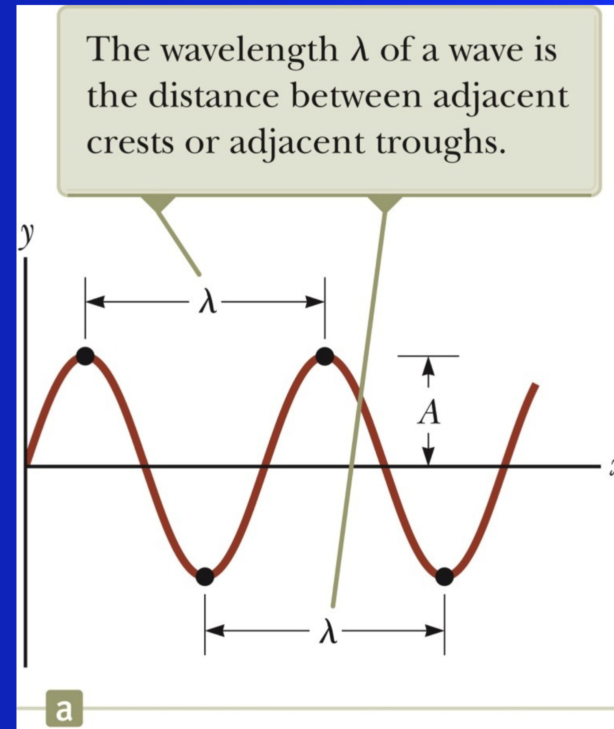
Sinusoidal Waves

- The period (T) is the amount of time necessary for the wave to travel by one wavelength (λ)
- speed=wavelength/period, which is equivalent to $v = \lambda f$
- v = (2 m) (2 Hz) = 4 m/s

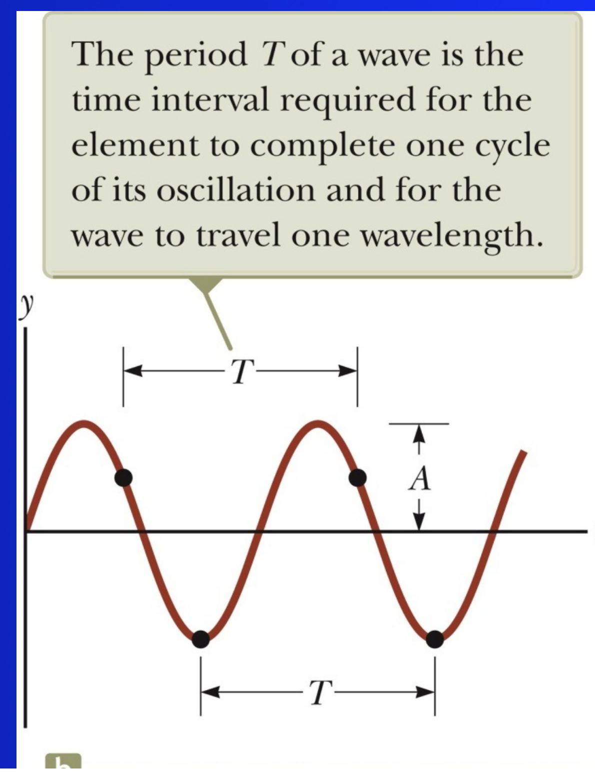


Traveling Waves

- **wavelength** λ (inversely related to *k*) and **amplitude** *A*



- **period** *T* and **frequency** (number of crests that pass a given point in a unit time interval: $f=1/T$)



generalized **wave function** (for a wave moving to the right):

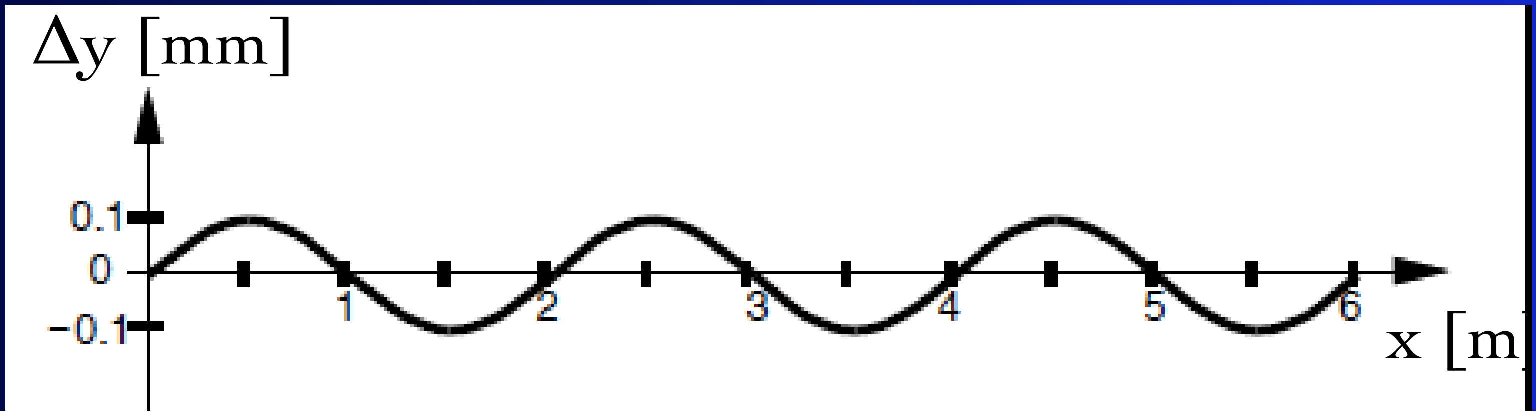
$$y = A\sin\,(kx - \omega t + \phi)$$

example: analyzing a traveling wave

The snapshot graph below (at t=0 s) shows a wave traveling to the right with speed 4.0 m/s. What is the first time (t>0) that a particle of the medium located at x=3m experiences zero velocity and negative acceleration?

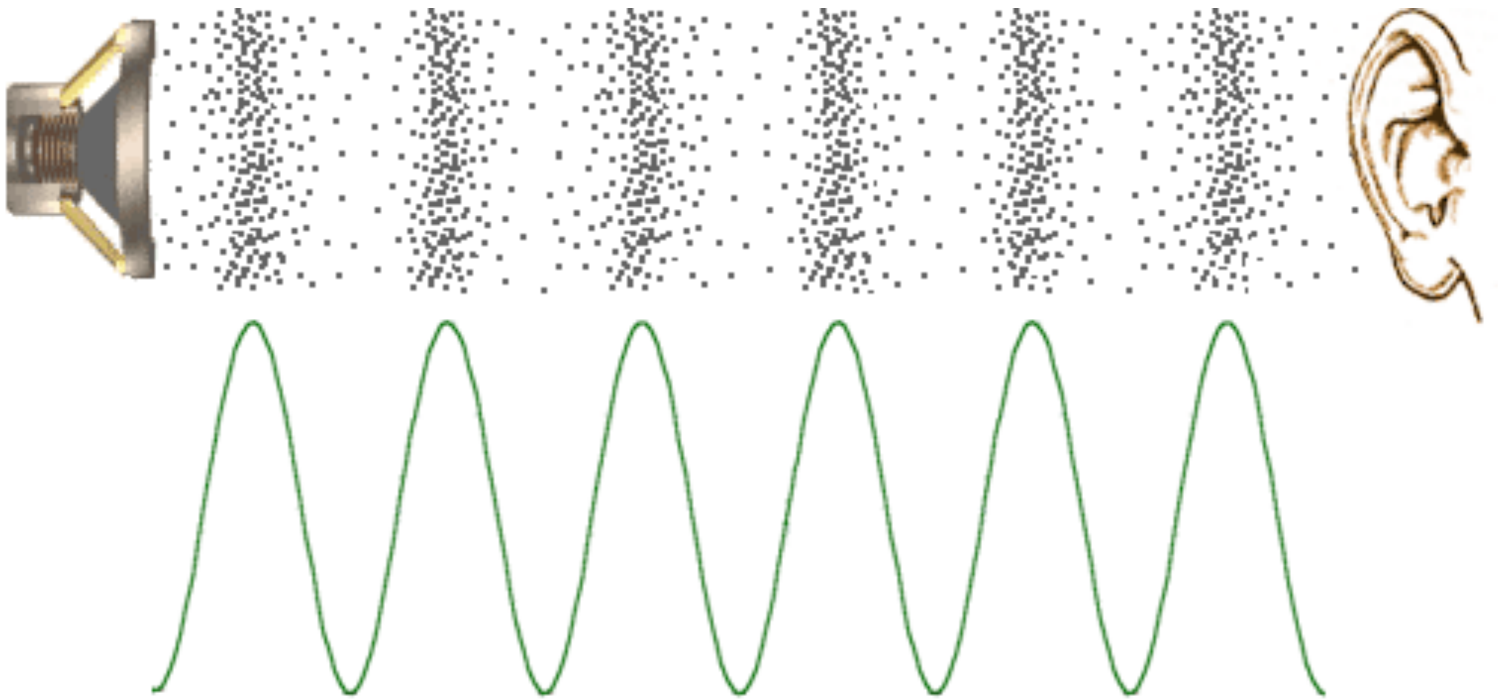
use the fact that $v=dy/dt$ and $a=dv/dt$ and

$y = A \sin(kx - \omega t) = A \sin((2\pi/\lambda)x - (2\pi v/\lambda)t)\ldots$



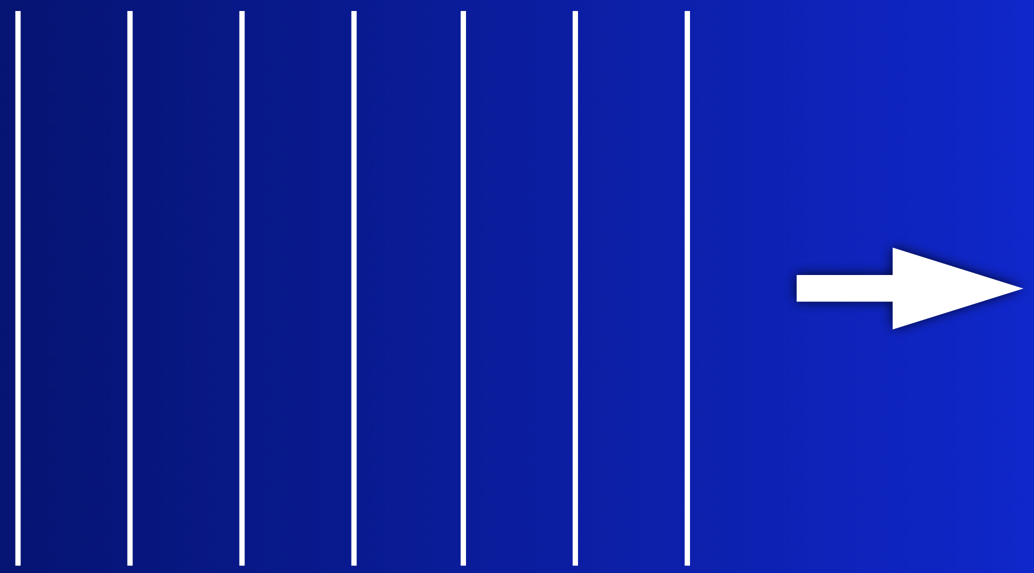
Sound Waves

- Longitudinal Wave traveling through the air
- speed of sound at room temperature ~ 340 m/s
- audio frequency range: 20-20,000 Hz



Reflection & Transmission, Wavefronts

- Wavefronts: a way to draw a "snapshot graph," but in multiple dimensions
- lines represent the peaks of the waves
- as the wave propagates, the wavefronts move

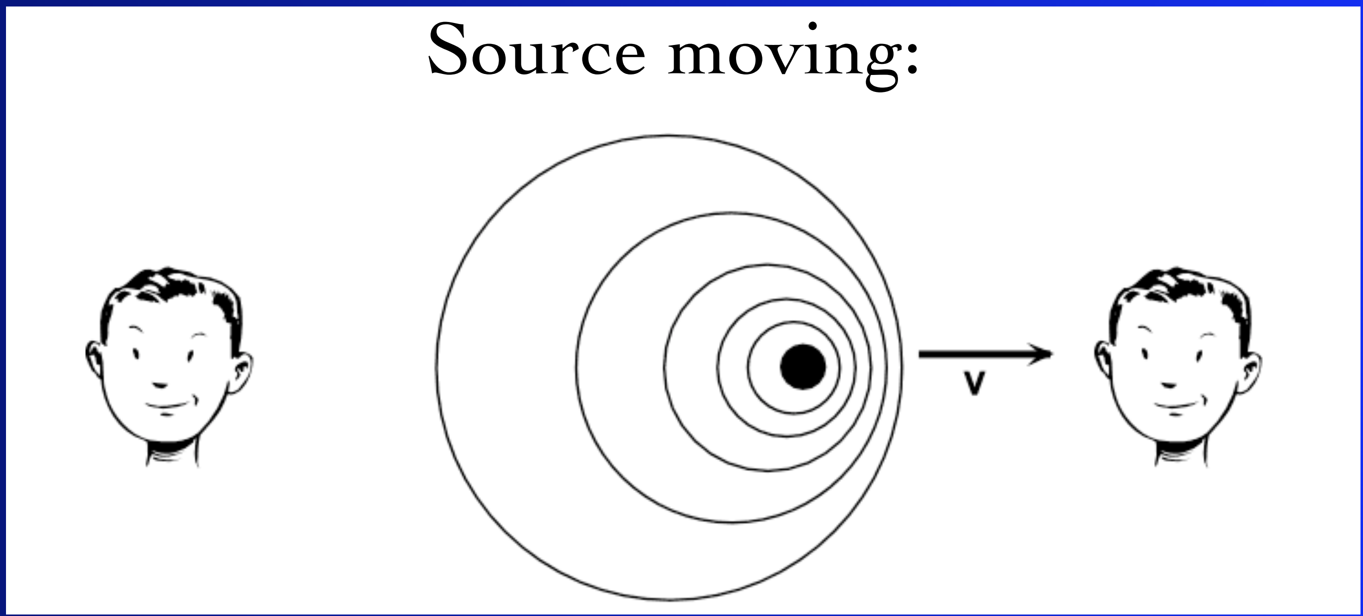


Doppler Effect

- Observer moves toward Source: observes crests to occur *more* often ⇒ *higher* pitch (higher frequency *f*)

- Observer moves away from Source: observes crests to occur *less* often ⇒ *lower* pitch

the surface of each circle is the wavefront:



describing the Doppler effect

If either the speed changes (because we're moving) or the wavelength changes (because the source is moving), then the frequency we hear, f', will change, because of $f=v/\lambda$. (We are the observer, O.)

Always remember: if the source and observer are moving toward each other, then $f'>f$. If the source and observer are moving away from each other, then $f'<f$.

$$f' = f\left(\frac{v+v_O}{v-v_S}\right)$$

deriving the Doppler effect

- if the observer moves toward the source (which isn't moving), then the observed frequency is: $f' = v'/\lambda = (v+v_O)/\lambda = f(v+v_O)/v$
- if the source moves toward the observer (which isn't moving), then the observed frequency is also shifted: $$f' = \frac{v}{\lambda'} = \left(\frac{v}{v - v_S}\right) f$$
- the general expression for Doppler effect when either or both source & observer are moving:

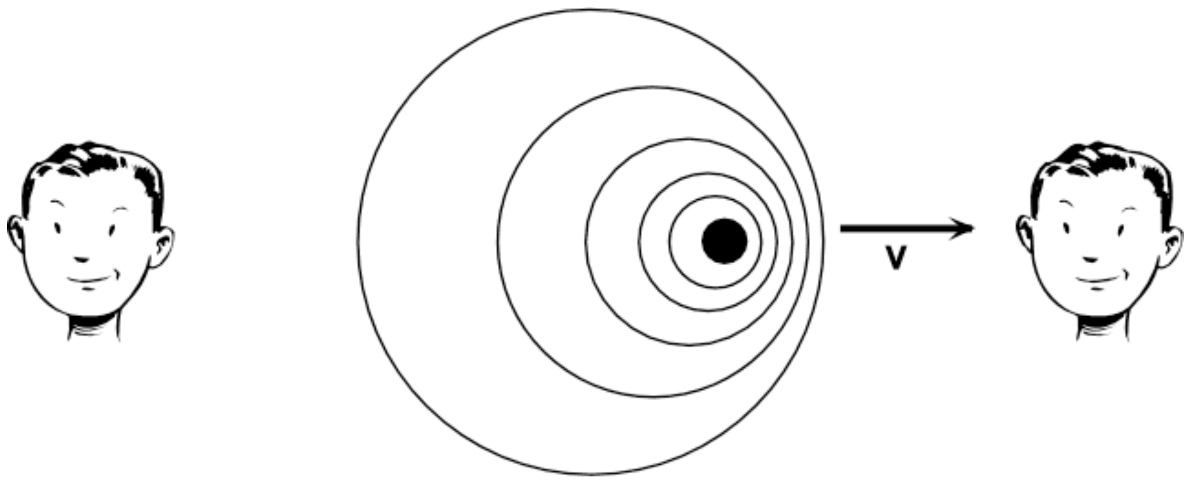
$$f' = f\left(\frac{v + v_O}{v - v_S}\right)$$

Doppler Effect

- velocities are positive if observer/source are moving ***toward*** each other
- velocities are negative if observer/source are moving ***away*** from each other
- velocities in either direction affect the perceived frequency

$$f' = f\left(\frac{v + v_O}{v - v_S}\right)$$

Source moving:



historical note: relativistic Doppler effect

- 50 years after Christian Doppler's death, Einstein calculated the relativistic version red/blue-shifted light sources in 1905:

$$f' = f\left(\frac{v + v_O}{v - v_S}\right) \quad \Longrightarrow \quad f' = f\left(\frac{1 -/+ (v/c)}{1 +/- (v/c)}\right)$$

Doppler Effect: Example

An ambulance siren is blaring at you with 800 Hz sound. The ambulance is heading east at 50 mph, and we know that the speed of sound is 340 m/s.

The ambulance passes and you start up again, so you're heading east too but at 35 mph behind it. Now what frequency of sound do you hear?

A. greater than 800 Hz
B. equal to 800 Hz
C. less than 800 Hz
D. I'm totally lost

example: applying the Doppler effect

$$f' = f\left(\frac{v + v_O}{v - v_S}\right)$$

An ambulance siren is blaring at you with 800 Hz sound. The ambulance is heading east at 50 mph, and we know that the speed of sound is 340 m/s.

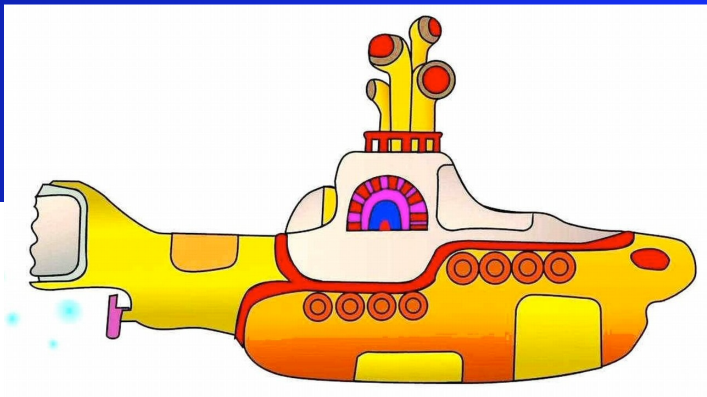
A. You stop as the ambulance approaches you on the road. What frequency of sound do you hear?

B. The ambulance passes and you start up again, so you're heading east too but at 35 mph behind it. Now what frequency of sound do you hear?

another example, but under water

A submarine (sub A) travels through water at a speed of 8.00 m/s, emitting a sonar wave at a frequency of 1400 Hz. A second submarine (sub B) is traveling directly toward it at 9.00 m/s. Assuming the speed of sound in water is 1533 m/s, what is the detected frequency?

Fortunately, the subs barely miss each other and pass. What frequency is detected by an observer riding on sub B as the subs recede from each other?



$$f' = \left(\frac{v + v_O}{v - v_S}\right) f$$

$$f' = \left[\frac{1\,533 \text{ m/s} + (-9.00 \text{ m/s})}{1\,533 \text{ m/s} - (-8.00 \text{ m/s})}\right](1\,400 \text{ Hz}) = 1\,385 \text{ Hz}$$

Chapter 14: Superposition of Waves

Waves are very different from particles (except in quantum mechanics):

- An "ideal" particle is of zero size
- An "ideal" wave is of infinite length
- Two or more waves may combine at a single point in the same medium
- We can combine particles to form extended objects, but the particles must be at different locations

Superposition of Waves

Superposition Principle:

- If two or more traveling waves are moving through a medium, the resultant value of the wave function at any point is the algebraic sum of the values of the wave functions of the individual waves

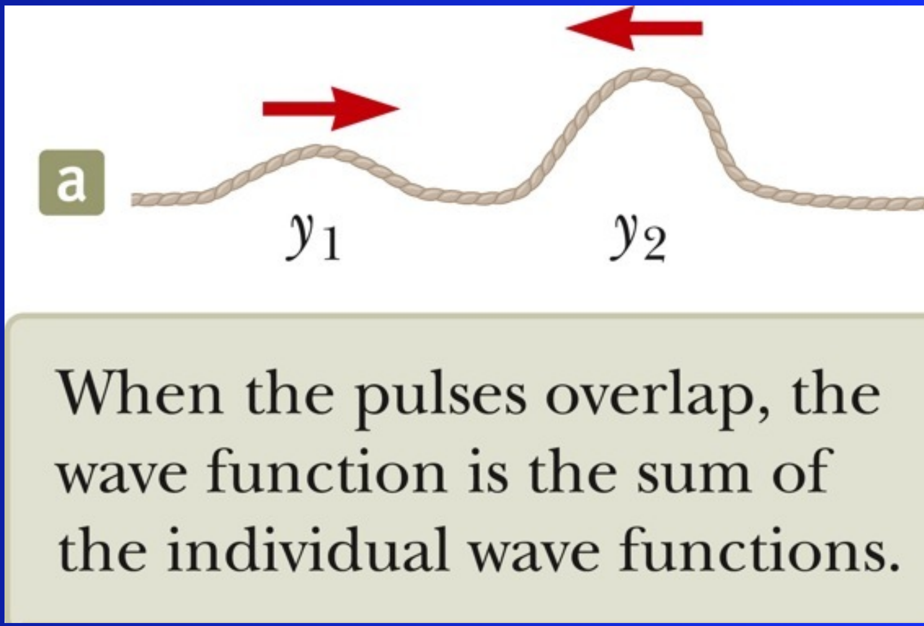
Waves that obey the superposition principle are **linear waves**

- Linear waves generally have amplitudes A much smaller than their wavelengths λ

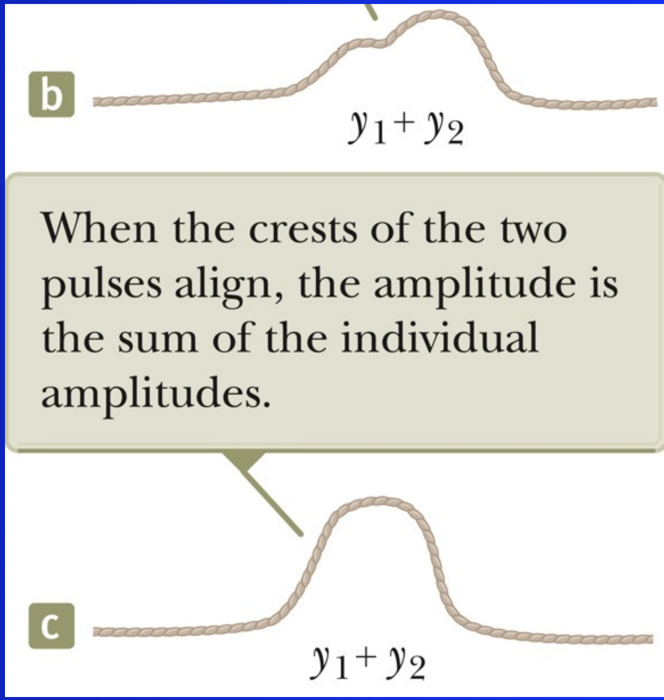
Superposition of Waves

- when there are two moving waves at the same location, add their displacements

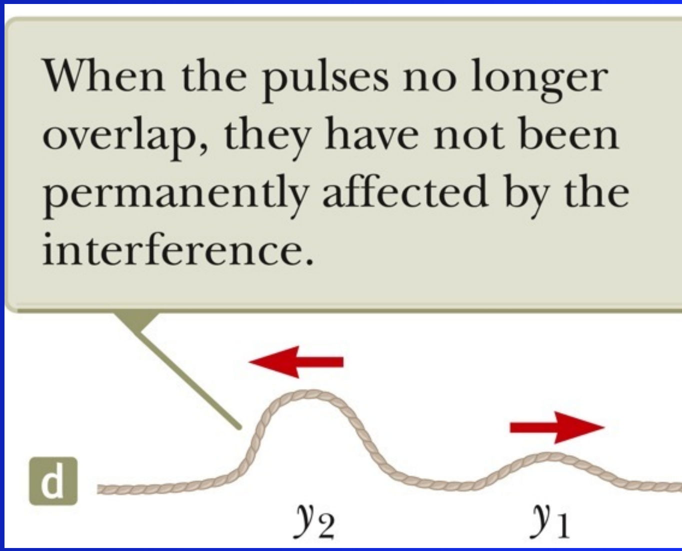
- in this example, the wave pulses have the same speed but different shapes
  - the displacement of the elements is *positive* for both



Superposition of Waves



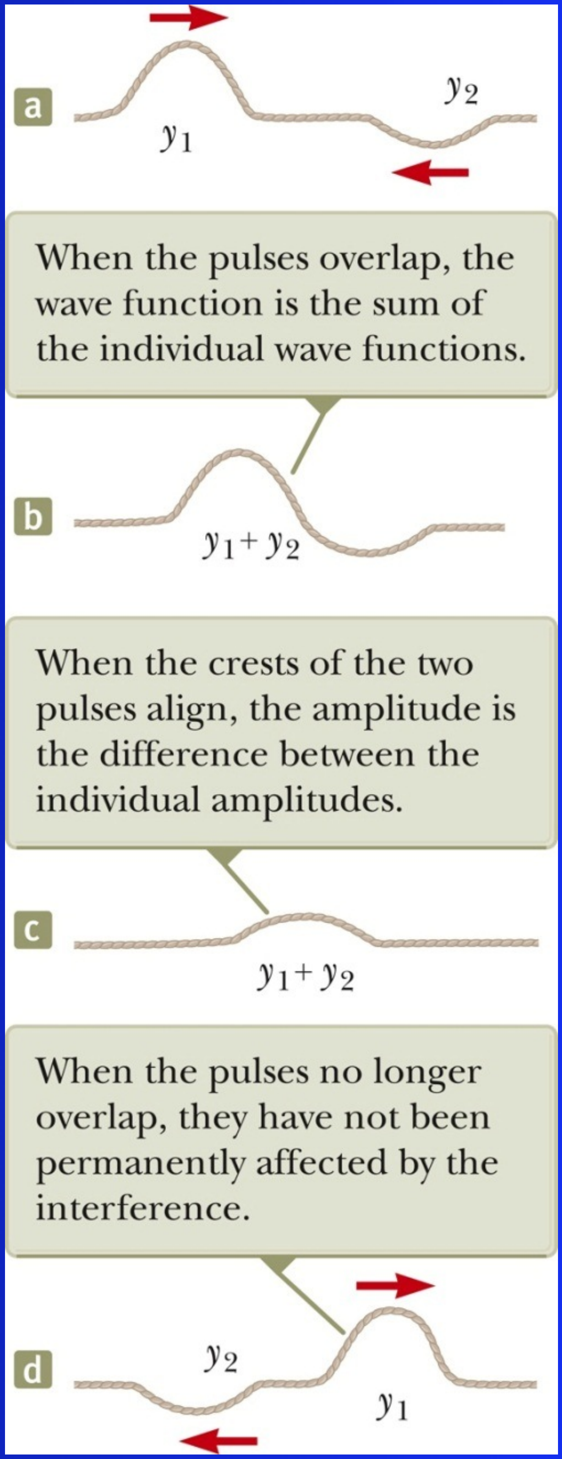
- When the waves start to overlap, the resultant wave function is $y_1 + y_2$
- When crest meets crest, the resultant wave has a *larger amplitude* than the original waves
- The two pulses separate and continue moving in their original directions, while the pulses' shapes remain unchanged
- The combination of separate waves in the same region to produce a resultant wave is called ***interference***



Superposition of Waves

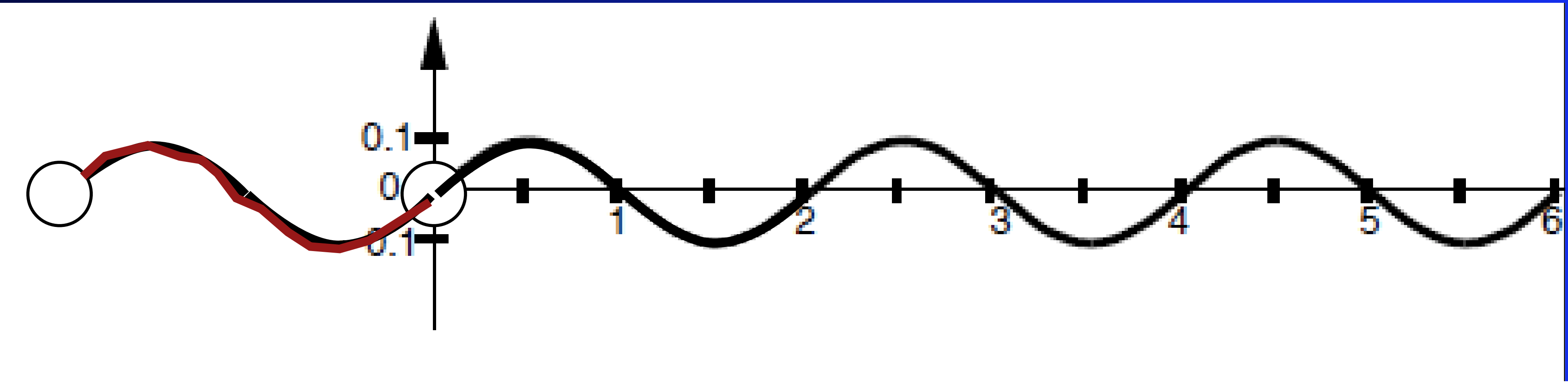
- *Constructive interference* occurs when the displacements caused by the two pulses are in the *same* direction
  - amplitude of resultant pulse is greater than either individual pulse

- *Destructive interference* occurs when the displacements caused by the two pulses are in *opposite* directions
  - amplitude of resultant pulse is less than either individual pulse

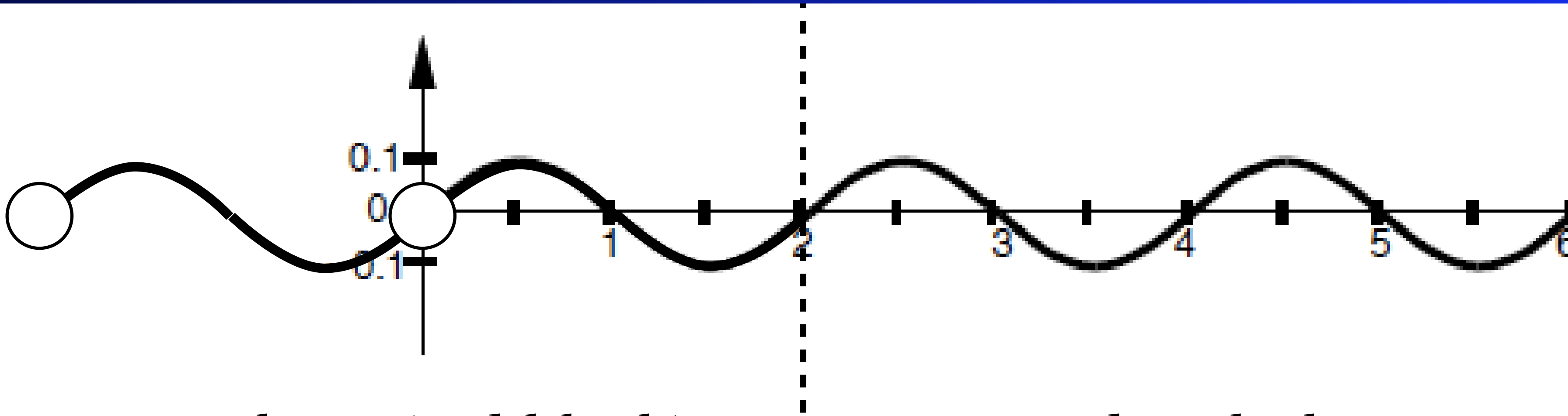


What happens if…

- two speakers emitting sound with the same frequency and in phase with each other
- does the superposition of these waves make an overall stronger or weaker wave?



What happens if...

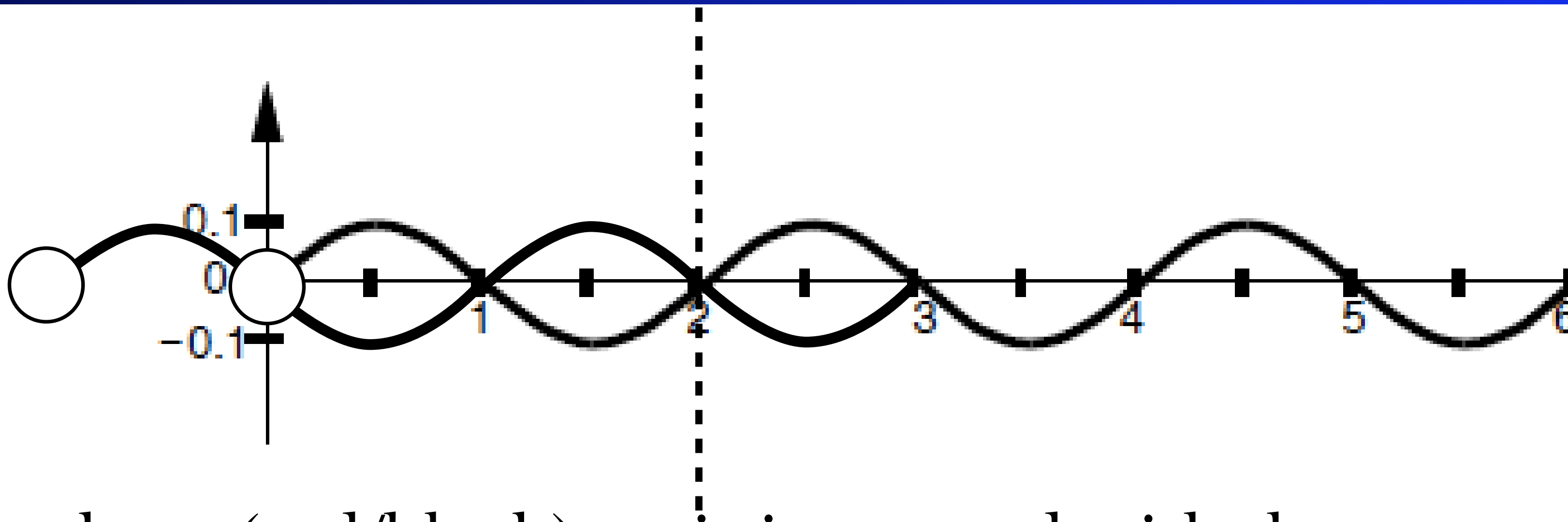


two speakers (red/black) emitting sound with the same frequency and in phase with each other

as the waves travel, the two always add to make a larger wave:
constructive interference

$$\Delta r = \lambda, 2\lambda, \ldots$$

What happens if…



two speakers (red/black) emitting sound with the same frequency and in phase with each other

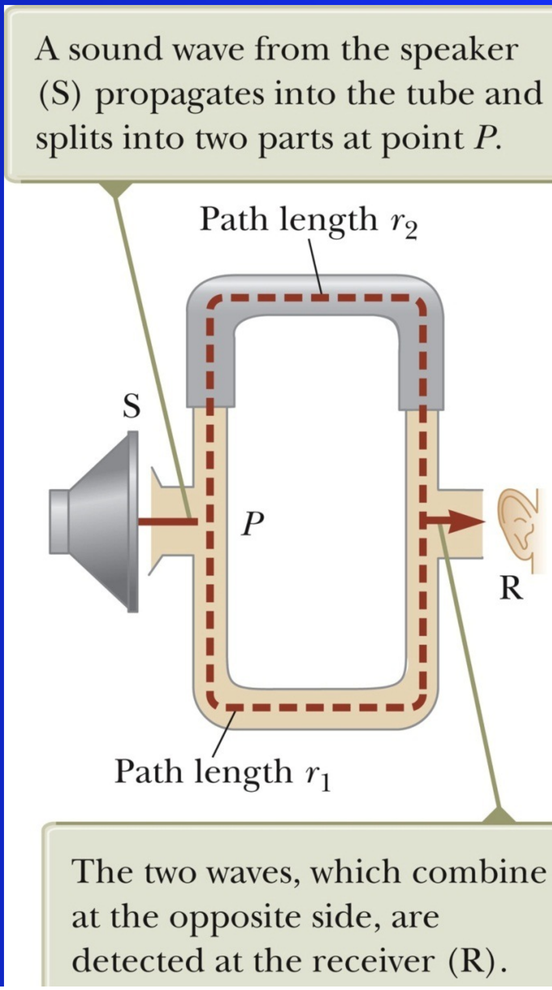
as the waves travel, the two always subtract to make a smaller wave:

destructive interference

$$\Delta r = \frac{1}{2}\lambda, \frac{3}{2}\lambda, \ldots$$

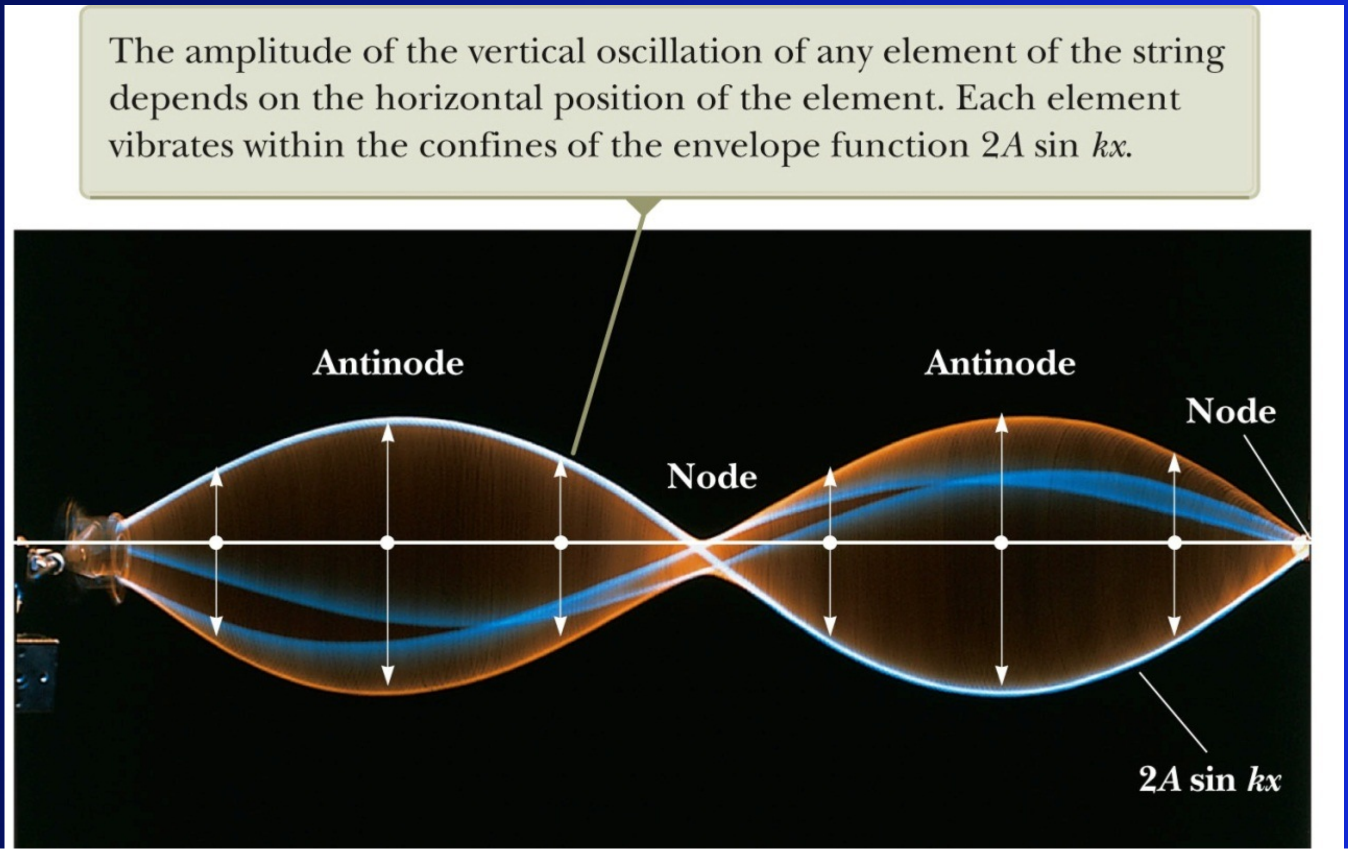
Constructive/Destructive Interference

- $\Delta r = |r_1 - r_2|$ is the difference in the distance from a location to each source of waves
- *constructive* interference: $\Delta r = |r_2 - r_1| = n\lambda$, where n=0, 1, 2, 3…
- *destructive* interference: $\Delta r = |r_2 - r_1| = (n+1/2)\lambda$, where n=0, 1, 2, 3…
- a half wavelength is all the difference
- for example, interference of sound waves



Standing Waves

- When we have two traveling waves going in opposite directions, the resulting oscillation pattern is a *standing wave*



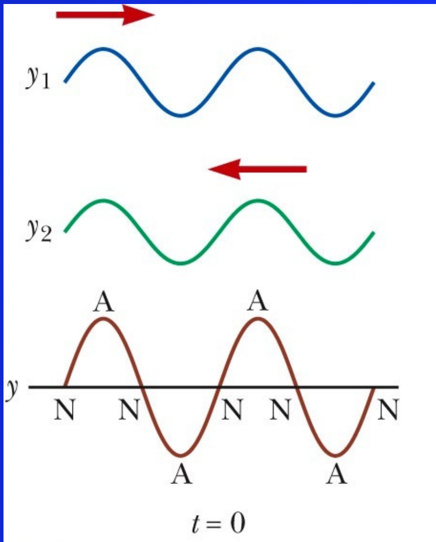
Standing Waves

- When we have two traveling waves going in opposite directions, the resulting oscillation pattern is a ***standing wave***
- for example, suppose you have these two traveling waves moving with $+v$ and $-v$

$$y_1 = A\sin(kx - \omega t) \qquad y_2 = A\sin(kx + \omega t)$$

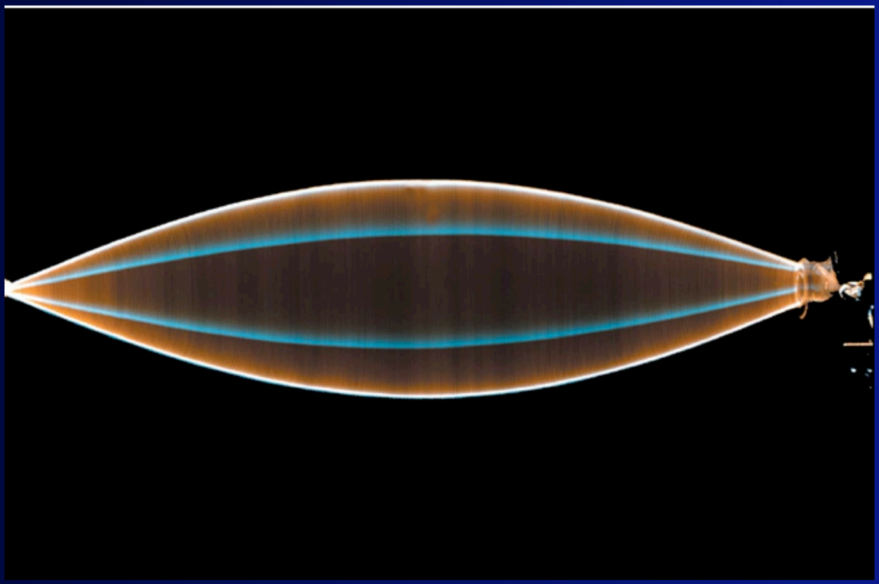
- then with the superposition principle, $y = y_1 + y_2 =$

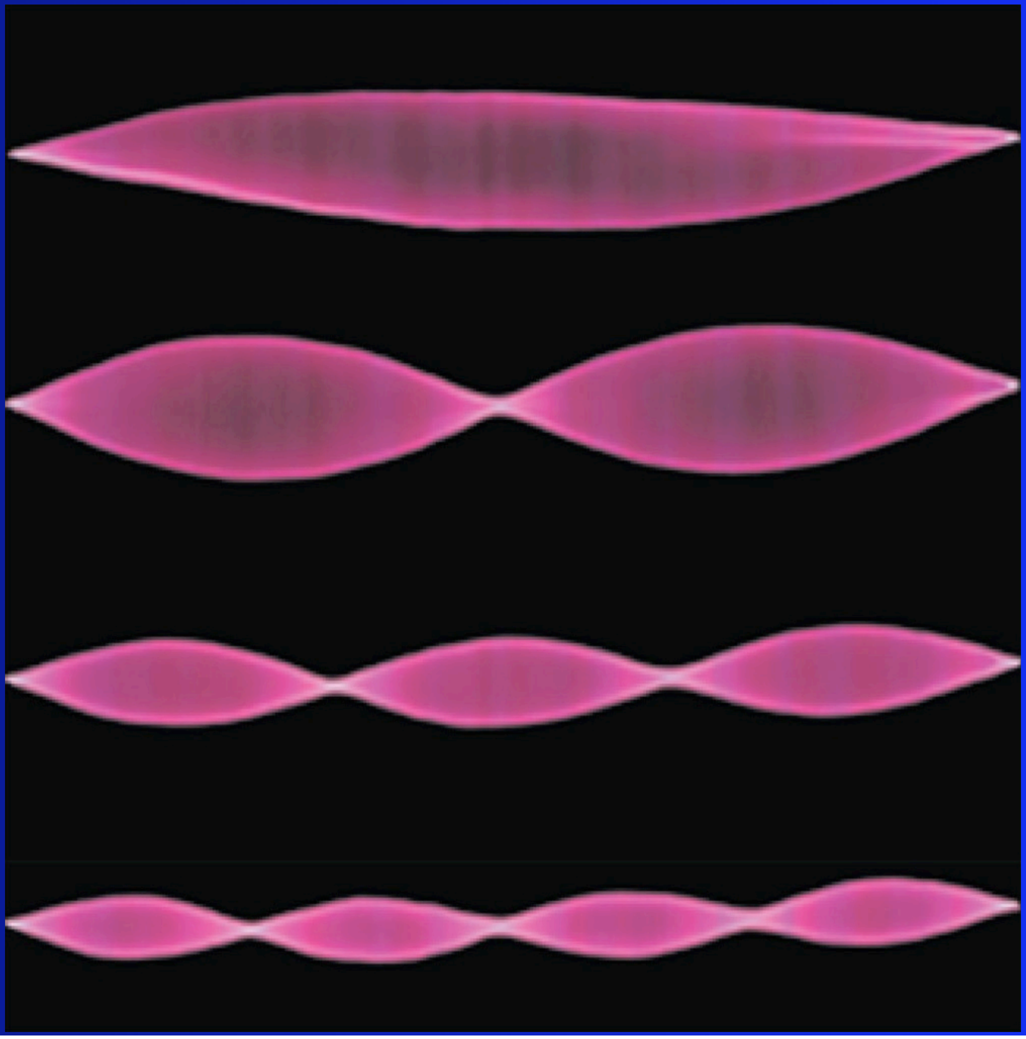
$$y = (2A\sin kx)\cos\omega t$$



Standing Waves

- a wave with fixed points that do not vibrate (***nodes***), which are at zero amplitude, and points going between maximum amplitudes (***antinodes***)





- note that consecutive (anti)nodes are a half wavelength ($\lambda/2$) apart from each other

Wave Interference: Example

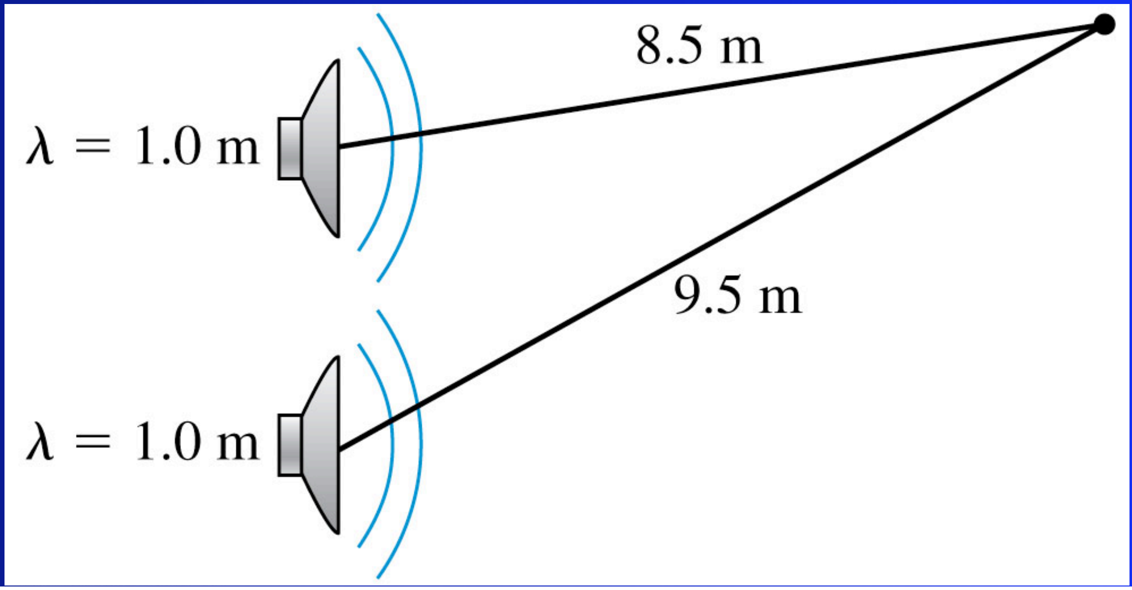
Two speakers emit sound waves in phase with each other with wavelength 1.0 m. The distance from a given location to each speaker is shown in the figure. A listener at the location on the right will experience:

A. constructive interference

B. destructive interference

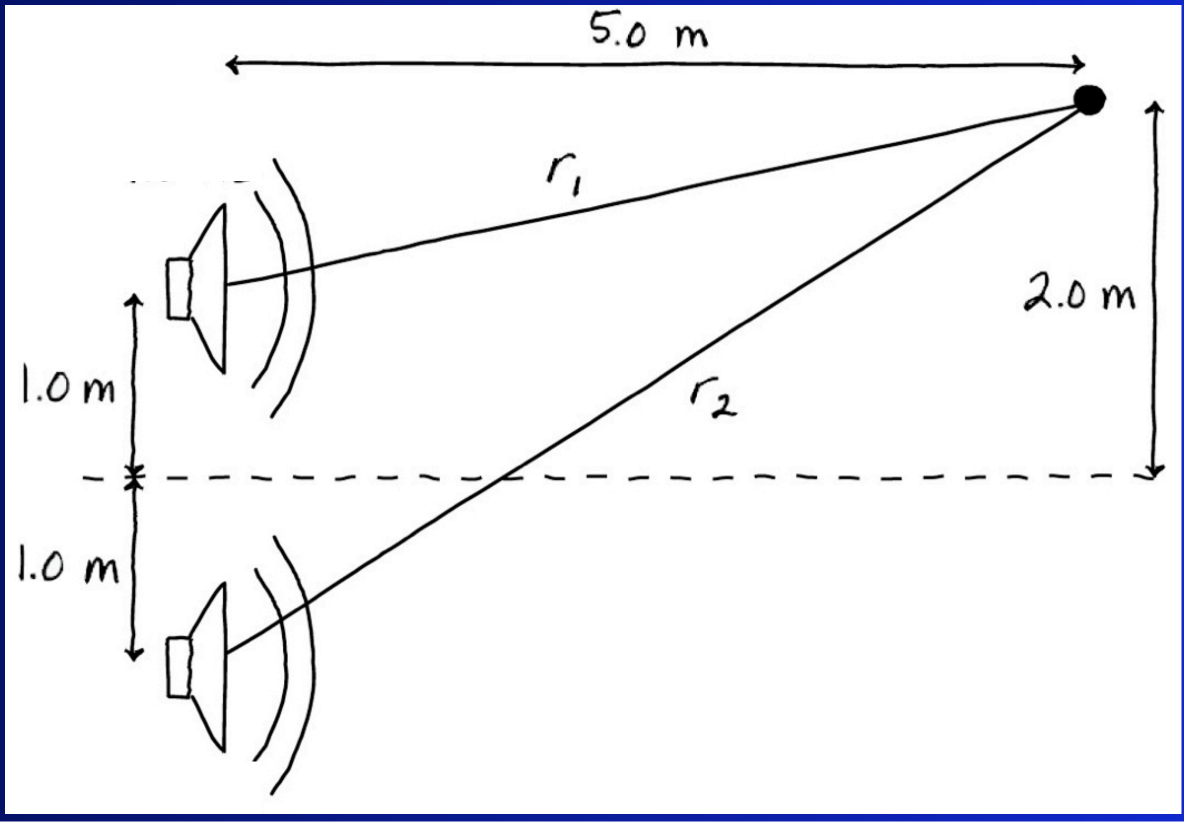
C. something in between the two

D. huh?



example of wave interference

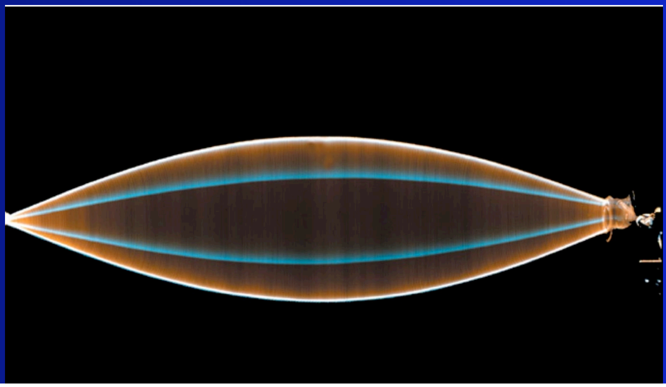
You move about in a room with two speakers and notice that when you stand at the location shown, there is a minimum in the intensity of sound. What is the longest wavelength for which this could occur? what about the second longest? [Hint: calculate Δr.]



Standing Waves

- standing waves are the superposition of two opposite directed waves

- for example, waves on a string fixed at both ends, like a guitar or violin string
- such a string as natural patterns or modes of oscillation
- the waves have similar relationships as before, such as $v = f\lambda$ and $v = \sqrt{(T/\mu)}$



example: standing wave frequency

We have a string instrument and we pluck a string, which is fixed at both ends. Which of the following actions would create a higher pitch (and therefore a higher frequency)?

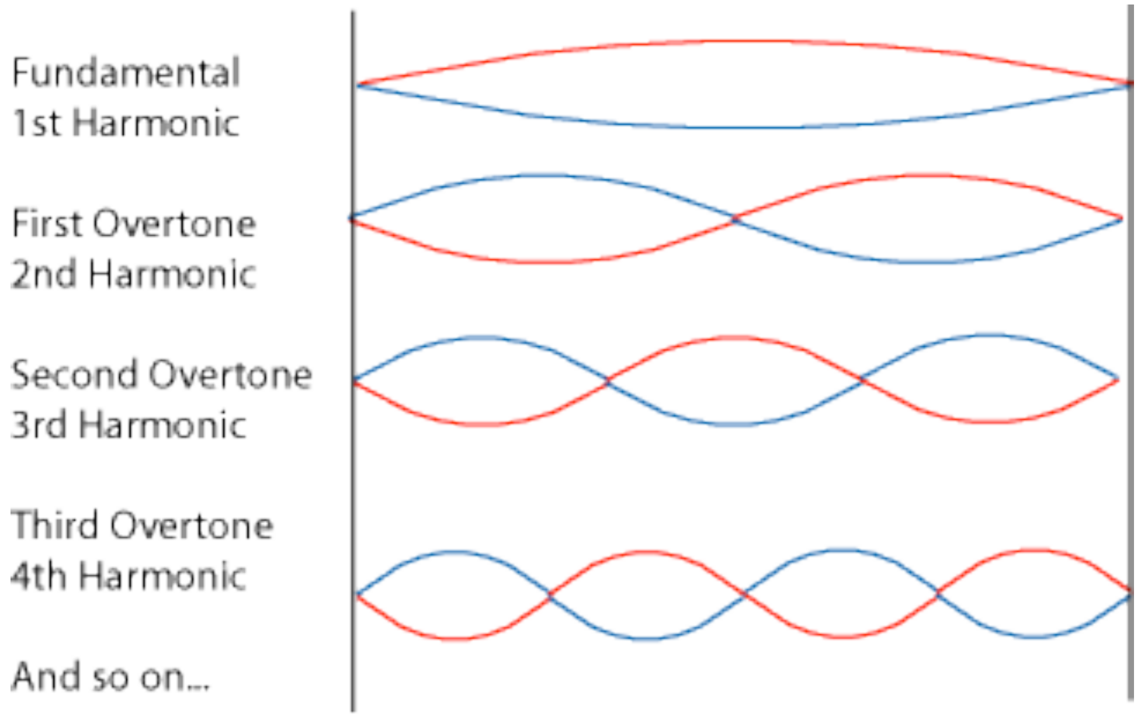
I. Increase the tension in the string
II. Shorten the string

A. I only
B. II only
C. I and II
D. neither of these
E. not sure

$$f = \frac{v}{(2L/n)}$$

$$v = \sqrt{\frac{T}{\mu}}$$

Waves under Boundary Conditions: Harmonics



$$\lambda_1 = 2L$$

$$\lambda_2 = L$$

$$\lambda_3 = \frac{2L}{3}$$

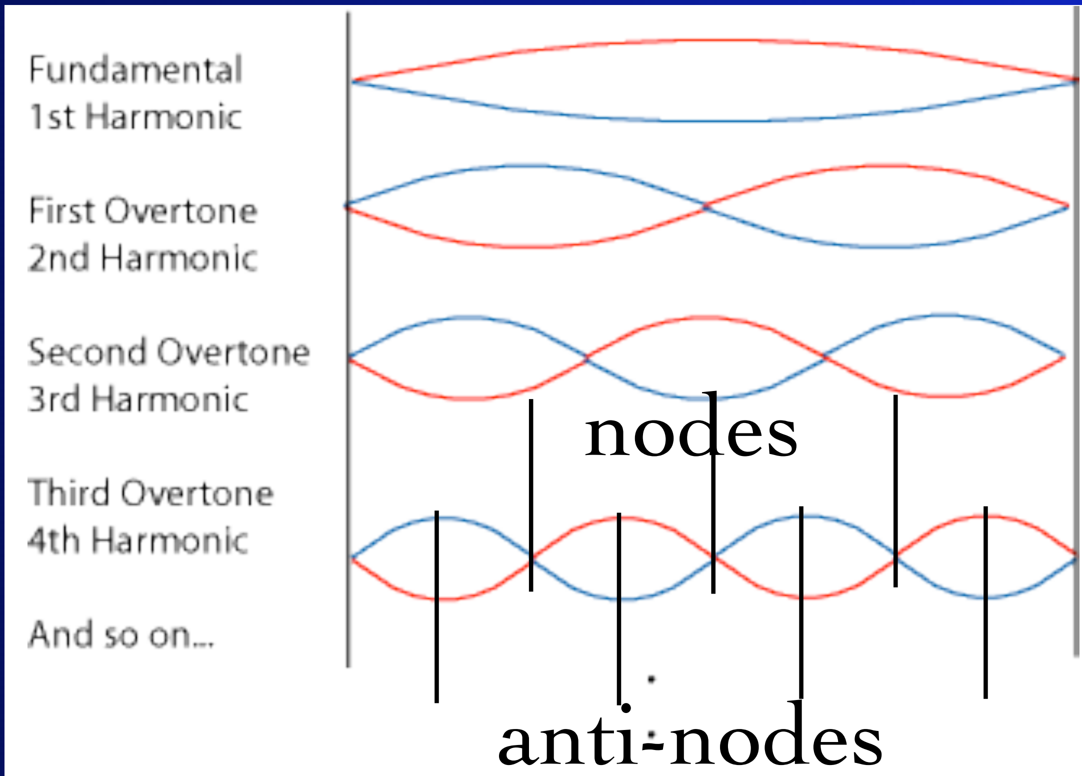
$$\lambda_4 = \frac{L}{2}$$

$$\vdots$$

$$\lambda_n = \frac{2L}{n}$$

- **fundamental frequency** f_1 is frequency of first harmonic, and $f_n = n\, f_1$

Nodes & Anti-nodes



$$\lambda_1 = 2L$$

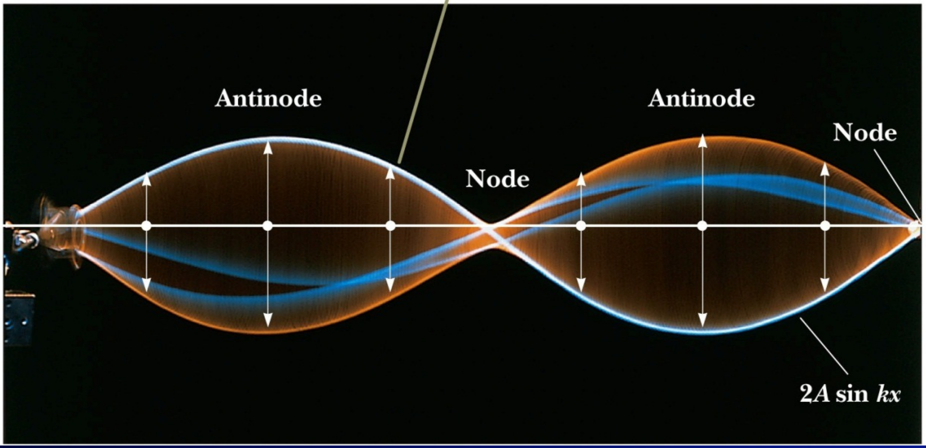
$$\lambda_2 = L$$

$$\lambda_3 = \frac{2L}{3}$$

$$\lambda_4 = \frac{L}{2}$$

$$\lambda_n = \frac{2L}{n}$$

The amplitude of the vertical oscillation of any element of the string depends on the horizontal position of the element. Each element vibrates within the confines of the envelope function $2A \sin kx$.



- note that only distinct modes are allowed (since *n* must be a whole number). later we'll call this ***quantization***…

Example: C note on a piano

- The middle C string on a piano has a fundamental frequency of 262 Hz. Calculate the frequencies of the next two harmonics of the string:

$$f_2 = 2f_1 = 524\ \text{Hz}$$
$$f_3 = 3f_1 = 786\ \text{Hz}$$

- The string for the first A above the middle C has a fundamental frequency of 440 Hz. If both strings have the same linear mass density μ and length *L*, what's the ratio of tensions in the two strings?

$$f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

$$\frac{f_{1\mathrm{A}}}{f_{1\mathrm{C}}} = \sqrt{\frac{T_\mathrm{A}}{T_\mathrm{C}}} \quad \rightarrow \quad \frac{T_\mathrm{A}}{T_\mathrm{C}} = \left(\frac{f_{1\mathrm{A}}}{f_{1\mathrm{C}}}\right)^2 = \left(\frac{440}{262}\right)^2 = 2.82$$

for Monday:

- remember to do the Chapter 13 homework problems and reading quiz on WebAssign by tomorrow at 5pm
- have a good weekend and good luck on Monday's quiz
- …and good luck to the US women's soccer team in the World Cup final on Sunday!