Homework VII

Ch. 5 - Problems 4, 8, 9, 18, 24, 30, 33, 41, 51, 61, 75.

Problem 4

The component of force parallel to the motion of the block is given by F = 35cos(25). Thus, the work done is given by

$$W = \mathbf{F} \ \cdot \ \mathbf{r} = 35cos(25) \ N \ \cdot \ 50 \ m = 1586 \ J.$$

Problem 8

(a) The work done by the applied force is simply

$$W = \mathbf{F} \cdot \mathbf{r} = 16cos(25)N \cdot 2.20m = 31.9 \ J.$$

(b) The normal force acts in a direction perpendicular to that of motion. Thus, the work done is 0 J, as follows naturally from the definition of the scalar product.

(c) Again the force of gravity acts perpendicular to the direction of motion and thus the work done is 0 J.

(d) The work done by the net force acting on the block is just the work done by the net force acting parallel to the motion of the block. Here, it is just the work done in part (a), 31.9 J.

Problem 9

(a) It follows from the Work-Energy theorem that the work done is here contained in the kinetic energy of the car. Thus,

$$\begin{aligned} \frac{1}{2}mv^2 &= W, \\ v &= \sqrt{\frac{2W}{m}}, \\ &= 2 \ \frac{m}{s}. \end{aligned}$$

(b) The horizontal force follows from the definition of work,

$$F = \frac{W}{r} = \frac{5000 \ J}{25.0 \ m} = 200 \ N.$$

Problem 18

The sled has an inital velocity of 2 $\frac{m}{s}$ and comes to a rest. It has a mass of 10 kg and a coefficient of kinetic friction of 0.1. Thus, there exists a normal force of magnitude 98 N and a force due to friction of f = 9.8 N. By the work-energy theorem, we have

$$\begin{aligned}
W &= \Delta E, \\
\mathbf{F} \cdot \mathbf{r} &= \frac{1}{2} m v_0^2, \\
r &= \frac{m v_0^2}{2 x \mu_k \ x \ m \ x \ g}, \\
&= 2.04 \ m.
\end{aligned}$$

Problem 24

We can solve this problem using the work-energy theorem. Defining the zero of the gravitational potential to be at the top of the circle, we have only kinetic energy at the top of the circle given by

$$K_0 = \frac{1}{2} m v_0^2 = 28.125 \ J.$$

At the bottom we have a final kinetic energy given by $\frac{1}{2} m v_f^2$ and a final potential energy -2mgr. Then, from the work energy theorem it follows that

$$\begin{aligned}
W &= E_f - E_i, \\
&= K_f + U_f - K_i, \\
\mathbf{F} \cdot \mathbf{r} &= \frac{1}{2} m v_f^2 - 2mgr - \frac{1}{2} m v_0^2, \\
v_f &= 26.5 \ \frac{m}{s}.
\end{aligned}$$

Problem 30

(a) Since the track is frictionless, we can simply apply conservation of energy. Thus, at point B we have

$$
\begin{aligned}
K_i + U_i &= K_f + U_f, \\
(0\ J) + (5.00\ kg)(9.8\ \frac{m}{s^2})(5.00m) &= \frac{1}{2}mv_B^2 + (5.00\ kg)(9.8\ \frac{m}{s^2})(3.20m), \\
v_B &= 5.94\ \frac{m}{s}
\end{aligned}
$$

and at point C we have

$$
\begin{aligned}
K_i + U_i &= K_f + U_f, \\
(0\ J) + (5.00\ kg)(9.8\ \frac{m}{s^2})(5.00m) &= \frac{1}{2}mv_C^2 + (5.00\ kg)(9.8\ \frac{m}{s^2})(2.00m), \\
v_C &= 7.67\ \frac{m}{s}.
\end{aligned}
$$

(b) The net work done by the force of gravity in moving in moving the bead from point A to point C is given by

$$
W = \mathbf{F}\ \cdot\ \mathbf{r} = (5.00\ kg)(9.8\ \frac{m}{s^2})(3.00\ m) = 147\ J.
$$

Problem 33

This is just a problem in conservation of energy. Let's define the zero of potential energy to be at the equilibrium position of the spring. Then, when the spring is compressed, we have have

$$
E_c = K_c + U_c = (0\ J) - (20.0e - 3\ kg)(9.8\ \frac{m}{s^2})x + \frac{1}{2}kx^2,
$$

where the first nonzero term is the gravitational potential energy and the second is the potential energy due to the compressed spring.

At the top of the projectile's path, we have

$$
E_t = K_t + U_t = (0\ J) + (20.0e - 3\ kg)(9.8\ \frac{m}{s^2})(20.0\ m - x) + (0\ J),
$$

where the only contribution to the energy is from the gravitational potential.

Finally, the energy at the equilibrium position when the spring is released and the projectile fired is given by

$$
E_e = K_e + U_e = \frac{1}{2}(20.0e - 3\ kg)v^2,
$$

where the only contribution to the energy is from the kinetic energy of the fired projectile.

(b) Equating E_e and E_t and solving for v yields

$$v = \sqrt{\frac{2\ x\ (20.0e-3\ kg)(9.8\ \frac{m}{s^2})(20.0\ m)}{(20.0e-3\ kg)}} = 19.8\ \frac{m}{s}.$$

(a) Equating E_e and E_t, substituting for v and x and solving for the spring constant yields, k = 548 $\frac{N}{m}$.

Problem 41

This problem is most easily solved using Newton's second law. The total force acting parallel to the plane is the just the component of the gravitational force acting in the that direction less the average friction force. Thus, the acceleration of the car is given by

$$a = \frac{F}{m} = \frac{mgsin\theta - f}{m} = 1.45\ \frac{m}{s^2}.$$

Then, the final velocity is simply

$$v_f = \sqrt{2ad} = 3.80\ \frac{m}{s}.$$

Problem 51

The average acceleration of the train is given by

$$a = \frac{v_f - v_0}{t} = \frac{0.620\ \frac{m}{s}}{21.0\ ms} = 29.5\ \frac{m}{s^2}.$$

Thus, the average force acting on the train is simply F = ma = 25.8 N. The average velocity of the train is 0.310 $\frac{m}{s}$. Then it follows that the average power delivered to the train during its acceleration is just the average force times the average velocity or

$$P = \bar{F}\bar{v} = (29.5\ N)(0.310\ \frac{m}{s}) = 8.01\ W.$$

Problem 61

(a) The speed of the man when he hits the pavement (independent of his mass) is simply given by

$$v = \sqrt{2gh} = \sqrt{2(9.8\ \frac{m}{s^2})(1.00\ m)} = 4.43\ \frac{m}{s}.$$

(b) The average acceleration is given by

$$\bar{a} = \frac{v^2}{2d} = \frac{(4.43\ \frac{m}{s})^2}{2(0.5e-3\ m} = 1960\ \frac{m}{s^2}.$$

Thus, the average force is just given by

$$\bar{F} = m\bar{a} = 1.5e5\ N.$$

Problem 75

(a) Using conservation of energy, it can be discerned from position A that the total energy of the system is given by

$$E = \frac{1}{2}kx_1^2 - mgx_1 = 101\ J.$$

(b) Again, using conservation of energy, we find x_2 to be

$$\begin{aligned} E &= mgx_2, \\ x_2 &= \frac{E}{mg}, \\ &= 41.0\ cm. \end{aligned}$$

(c) Once again, from conservation of energy it follows that

$$\begin{aligned} E &= \frac{1}{2}mv^2, \\ v &= \sqrt{\frac{2E}{m}}, \\ &= 2.84\ \frac{m}{s}. \end{aligned}$$

(d) If we want to maximize the kinetic energy, we want to maximize the velocity. Alternately, we can look at the problem as though we want to minimize the potential energy. A moment's consideration convinces us that the minimum of the potential energy occurs when the spring is compressed. (To see this, just plot E as a function of x, where

$$
\begin{aligned}
E(x) &= \frac{1}{2}kx^2 + mgx, x \leq 0 \\
&= mgx, x > 0.
\end{aligned}
$$

In order to plot this, we must choose values for the constants, but since they are only scale factors we are free to choose whatever values we want, k=m=g=1 being a convenient choice.)

Anyway, to minimize E(x), we simply caluclate $\frac{dE}{dx}$ and set it equal to zero. (In order to ensure that the value x$_m$ found from the previous equation is truly a minimum, we must also check that the second derivate evaluated at x$_m$ is greater than zero.) Anyway, here we have

$$
\begin{aligned}
\frac{dE}{dx} &= kx_m + mg = 0, \\
x_m &= \frac{-mg}{k}, \\
&= 9.8\ mm, \\
\frac{d^2E}{dx^2}|_{x_m} &= k > 0.
\end{aligned}
$$

Thus, we find that the potential energy is an extremum (the kinetic energy is also extremum) for x = -9.8 mm and, since the second derivative evaluated at this point is greater than zero, is a minimum.

Solutions to the remaining short answer questions can be found in the class notes on General Relativity and Black Holes as posted on the course website.