Physics 11 Homework II Solutions
Ch. 2 - Problems 1, 7, 18, 22, 30, 43, 59, 61, 62, 67.

Problem 1

(a) Recall that the average speed is defined as the total distance divided by the total displacement. Here, the total distance is given by

$$x = (80\frac{km}{h})(\frac{1}{2}h) + (100\frac{km}{h})(\frac{1}{5}h) + (40\frac{km}{h})(\frac{3}{4}h) = 90km.$$

Thus, the average speed is given by,

$$s = \frac{90km}{(\frac{1}{2}+\frac{1}{5}+\frac{3}{4}+\frac{1}{4})h} = 52.9\frac{km}{h}.$$

We could also have found this by multiplying each speed by the ratio of the time spent at that speed to the total time spent on the trip and then summing.

(b) From above, we found the total distance to be 90 km.

Problem 7

Recall that the average velocity during a time interval Δt is the displacement Δx divided by Δt. That is,

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}.$$

(a) The average velocity on the time interval 0 to 1 is

$$\bar{v} = \frac{(4-0)m}{(1-0)s} = 4\frac{m}{s}\mathbf{x}.$$

(b) The average velocity on the time interval 0 to 4 is

$$\bar{v} = \frac{((-2)-0)m}{(4-0)s} = -0.5\frac{m}{s}\mathbf{x}.$$

(c) The average velocity on the time interval 1 t o5 is

$$\bar{v} = \frac{(0-4)m}{(5-1)s} = -1\frac{m}{s}\mathbf{x}.$$

(d) The average velocity on the time interval 0 to 5 is

$$\bar{v} = \frac{(0-0)m}{(5-0)s} = 0\frac{m}{s}\mathbf{x}.$$

Problem 18

(a)

$$s_{25.2} = \frac{0.25mi}{25.2s}(\frac{3600s}{1h}) = 35.7\frac{mi}{h},$$
$$s_{24.0} = \frac{0.25mi}{24.0s}(\frac{3600s}{1h}) = 37.5\frac{mi}{h},$$
$$s_{23.8} = \frac{0.25mi}{23.8s}(\frac{3600s}{1h}) = 37.8\frac{mi}{h},$$
$$s_{23.0} = \frac{0.25mi}{23.0s}(\frac{3600s}{1h}) = 39.1\frac{mi}{h}.$$

(b) Using an instantaneous speed at the finish line of 39.1 $\frac{mi}{h}$, the average acceleration for the entire race is

$$\bar{a} = \frac{39.1\frac{mi}{h}}{(25.2+24.0+23.8+23.2)s}(\frac{3600s}{1h}) = 1460\frac{mi}{h^2}.$$

Problem 22

(a) Recall that the average acceleration during a time interval Δt is given by

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}.$$

Thus, the average accelerations for the given time intervals are,

$$\bar{a}_{0-5} = \frac{((-8)-(-8))\frac{m}{s}}{(5-0)s} = 0\frac{m}{s^2}\mathbf{v},$$
$$\bar{a}_{5-15} = \frac{(8-(-8))\frac{m}{s}}{(15-5)s} = 1.6\frac{m}{s^2}\mathbf{v},$$
$$\bar{a}_{0-20} = \frac{(8-(-8))\frac{m}{s}}{(20-0)s} = 0.8\frac{m}{s^2}\mathbf{v},$$

where **v** tells us that the acceleration is in the same direction as the velocity.

(b) The instantaneous acceleration at a given time can be found easily by determining the slope of the curve on the given velocity v. time plot at the time of interest.

For times t=2, 18s the instantaneous acceleration is obviously 0 $\frac{m}{s^2}\mathbf{v}$ as the curve is horizontal at those times. For t=10s, a is the same as $\bar{a}_{5-15}$ found above in part (a), namely, 1.6 $\frac{m}{s^2}\mathbf{v}$.

Problem 30

We are given that a truck accelerates from rest at 2 $\frac{m}{s^2}$ until it reaches a speed of 20 $\frac{m}{s}$. The truck then coasts for 20s at constant speed until it the brakes are applied, bringing the truck back to rest in 5s, in a uniform manner.

(a) So, we know that the truck coasts for 20s and brakes for 5s. Thus, in order to determine the total time, we need to figure out the time interval during which the truck accelerated from rest to 20 $\frac{m}{s}$. Recall, that for constant acceleration, we have:

$$v_f = v_0 + a * t.$$

With the truck initially at rest, we have

$$t = \frac{v_f}{a} = \frac{20\frac{m}{s}}{2\frac{m}{s^2}} = 10s.$$

Thus, the truck is in motion for t = (10 + 20 + 5)s = 35s.

(b) Well, we want the average velocity of the truck, which begins and ends its motion at rest. Thus, $\bar{v} = 0$ $\frac{m}{s}$.

Problem 43

A ball is thrown vertically with a speed of 25.0 $\frac{m}{s}$.

(a) Recall the equation,

$$v_f^2 = v_0^2 + 2 * a * y.$$

Also, let's define up as the positive direction, making the constant acceleration due to gravity, g, act in the negative direction. So, in determining the height to which the ball rose, we can make use of the fact that at the peak of its motion, its speed is 0 $\frac{m}{s}$. Thus, we have

$$y = \frac{v_0^2}{-2 * a} = \frac{(25.0\frac{m}{s})^2}{-2 * (-9.8\frac{m}{s^2})} = 31.9m.$$

(b) Using the same equation as in Problem 30, part (a), we find

$$t = \frac{v_0}{-a} = \frac{25.0\frac{m}{s}}{-(-9.8\frac{m}{s^2})} = 2.55s.$$

(c) Recall the equation,

$$y_f - y_i = v_0 * t + \frac{1}{2} * a * t^2.$$

Now, recall from part (a) that at the top of its path, the ball had speed $0\ \frac{m}{s}$. Thus, the ball begins the downward path at rest, so $v_0 = 0\ \frac{m}{s}$, we have

$$t = \sqrt{\frac{2 * y}{a}} = \sqrt{\frac{2 * (-31.9m)}{-9.8\frac{m}{s^2}}} = 2.55s.$$

(d) Using the same equation as part (b), we have:

$$v_f = v_0 + a * t = (9.8\frac{m}{s^2})(2.55s) = -25.0\frac{m}{s}\mathbf{y}.$$

Hmm...interesting that the time down is the same as the time down. Also interesting is that the speed with which the ball is released is the same as that with which it returns to the starting point. Do you think this is a coincidence? The interesting *coincidences* we find here and in other problems will be covered in depth during Wednesday's discussion.

Problem 59

Two students are on a balcony 19.6m above the street from which one student throws a ball downward and the other upward at the same instant with a speed of 14.7 $\frac{m}{s}$.

Define up as the positive direction, so gravity acts in the negative direction.

(a) For the ball thrown downward, we have:

$$\begin{aligned} h_i &= 19.6m, \\ h_f &= 0m, \\ v_0 &= -14.7\frac{m}{s}\mathbf{y}, \\ a &= -9.8\frac{m}{s^2}\mathbf{y}. \end{aligned}$$

Thus, using the equation,

$$y_f - y_i = v_0 * t + \frac{1}{2} * a * t^2.$$

we have (units temporarily omitted)

$$\begin{aligned} -19.6 = -14.7 * t - 4.9 * t^2, or \\ 4.9t^2 + 14.7t - 19.6 = 0, \end{aligned}$$

which has solutions t = -4, 1s. Now, clearly, a negative time is meaningless. Although it comes out as a solution to the problem, our intuition tells us that it is an artifact of the mathematics and not a physical solution. Thus, we keep the solution t = 1s.

Similarly, for the ball thrown upward, we have:

$$\begin{aligned} h_i &= 19.6m, \\ h_f &= 0m, \\ v_0 &= 14.7\frac{m}{s}\mathbf{y}, \\ a &= -9.8\frac{m}{s^2}\mathbf{y}. \end{aligned}$$

Thus, using the equation,

$$y_f - y_i = v_0 * t + \frac{1}{2} * a * t^2.$$

we have (units temporarily omitted)

$$\begin{aligned} -19.6 = 14.7 * t - 4.9 * t^2, or \\ 4.9t^2 - 14.7t - 19.6 = 0, \end{aligned}$$

which has solutions t = -1, 4s. Again, the negative solution is a mathematical artifact. We want to keep t = 4s. So, the difference in the two balls' time in the air is t = (4-1)s = 3s.

(b) For the ball thrown downward we have,

$$\begin{aligned} v_f &= v_0 + a * t, \\ &= (-14.7\frac{m}{s} + (-9.8\frac{m}{s^2})(1s))\mathbf{y}, \\ &= -24.5\frac{m}{s}\mathbf{y}. \end{aligned}$$

Similarly, for the ball thrown upward we have,

$$\begin{aligned} v_f &= v_0 + a * t, \\ &= (14.7\frac{m}{s} + (-9.8\frac{m}{s^2})(4s))\mathbf{y}, \\ &= -24.5\frac{m}{s}\mathbf{y}. \end{aligned}$$

Still think it's a coincidence?
(c) Recall once again that

$$y_f = y_i + v_0 * t + \frac{1}{2} * a * t^2.$$

For the ball thrown downward, we have

$$y_{\downarrow}(0.8) = 19.6m + (-14.7\frac{m}{s})(0.8s) + (-9.8\frac{m}{s^2})(0.8s)^2 = 4.7m.$$

And for the ball thrown upward, we have

$$y_{\uparrow}(0.8) = 19.6m + (14.7\frac{m}{s})(0.8s) + (-9.8\frac{m}{s^2})(0.8s)^2 = 28.2m.$$

Thus, 0.8s after they are thrown, the two balls are $y_{\downarrow}$ - $y_{\uparrow}$ = (23.5 − 4.7)m = 23.5m apart.

Problem 61

Let's define the following quantities:

$$\begin{aligned} a_k &= 4.9\frac{m}{s^2}, \\ a_s &= 3.5\frac{m}{s^2}, \\ t_s &= t_k + 1. \end{aligned}$$

(a) Since both drivers start from rest, we may write (units omitted)

$$\begin{aligned} x_k &= \frac{1}{2}a_k t_k^2, \\ x_s &= \frac{1}{2}a_s t_s^2 = \frac{1}{2}a_s(t_k + 1)^2. \end{aligned}$$

In order to determine the time it takes Kathy to overtake Stan, we equate the two and solve for t_k.

$$\begin{aligned} x_k &= x_s, \\ a_k t_k^2 &= a_s(t_k+1)^2, \\ 0 &= (a_k - a_s)t_k^2 - 2a_s t_k - a_s. \end{aligned}$$

which has solutions

$$\begin{aligned} t_k &= \frac{2a_s \pm \sqrt{4a_s^2 + 4a_s(a_k - a_s)}}{2(a_s - a_s)}, \\ &= \frac{a_s \pm \sqrt{a_s^2 + a_s(a_k - a_s)}}{a_k - a_s}, \\ &= \frac{a_s \pm \sqrt{a_k a_s}}{a_k - a_s}, \\ &= \frac{3.5 \pm \sqrt{17.15}}{4.9 - 3.5}, \\ &= -0.46s, 5.46s \end{aligned}$$

So, we find $t_k = 5.46$s.
(b) Kathy travels $x_k = \frac{1}{2}(4.9)(5.46)^2 = 73$m.
(c) Since both drivers are initially at rest, we have

$$\begin{aligned} v_k &= a_k t_k = 26.7\frac{m}{s}, \\ v_s &= a_s t_s = a_s(t_k + 1) = 22.6\frac{m}{s}. \end{aligned}$$

Problem 62

For consistency, let's define up as positive, so gravity as in the negative direction.
(a) For the first stone, we have

$$\begin{aligned} h_f &= 50m, \\ h_i &= 0m, \\ v_i &= -2.00\frac{m}{s}\mathbf{y}, \\ a &= -9.8\frac{m}{s^2}\mathbf{y}. \end{aligned}$$

Since the stones hit the water at the same time, we just need the $timeofflight$ of the first stone. This can be found using the equation,

$$y_f = y_i + v_0 * t + \frac{1}{2} * a * t^2.$$

Thus, we have (units omitted)

$$0 = 50 + (-2)t - \frac{1}{2}(-9.8)t^2,$$

which has solutions t = -3.40s, 2.99s.

So, the stones hit the water 2.99s after the release of the first stone.

(b) Now, we are told that the second stone is released 1s after the first. Thus, $t_2 = t_1$ - 1. Using the same equation as above, but here with reference to the second stone, we find (again, units omitted)

$$0 = 50 + v_2(1.99) + \frac{1}{2}(-9.8)(1.99)^2,$$

which yields an initially velocity for the second stone of v = - 15.4 $\frac{m}{s}\mathbf{y}$.

(c) The velocity of each stone at the instant it hits the water can be found using

$$v_f = v_0 + at.$$

Thus, for the first stone we have (units omitted)

$$v_f = v_0 + at_1 = -2 + (-9.8)(2.99) = -31.3\frac{m}{s}\mathbf{y},$$

while for the second stone we have

$$v_f = v_0 + at_1 = -15.4 + (-9.8)(1.99) = -34.9\frac{m}{s}\mathbf{y},$$

Note the similarities of this problem to the previous one.

Problem 67

So, we have

$$\begin{aligned} v_h &= 10\frac{m}{s}\mathbf{x}, \\ a_h &= 0\frac{m}{s^2}\mathbf{x}, \\ h_i &= 3m, \\ h_f &= 0m. \end{aligned}$$

(a) Now, in order for the man to land on the horse, he must jump at the instant in time when the interval it takes him to fall is equal to the interval it takes the horse to reach the tree. Now, since the man starts from rest, the time it takes him to fall 3.00m is

$$t = \sqrt{\frac{2h_i}{g}} = 0.782s.$$

Thus, he must jump when the horse is a distance x = v_ht = 7.82m from the base of the tree.

(b) This solution was found above. The man spends t = 0.782s in the air.