Pulse Propagation and Fast Transient Transport Model with Self Consistent Nonlinear Noise

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Outline

- *Motivation*
- *Simplest Model*
- *Results*
- *Conclusion*
- *Future Work*

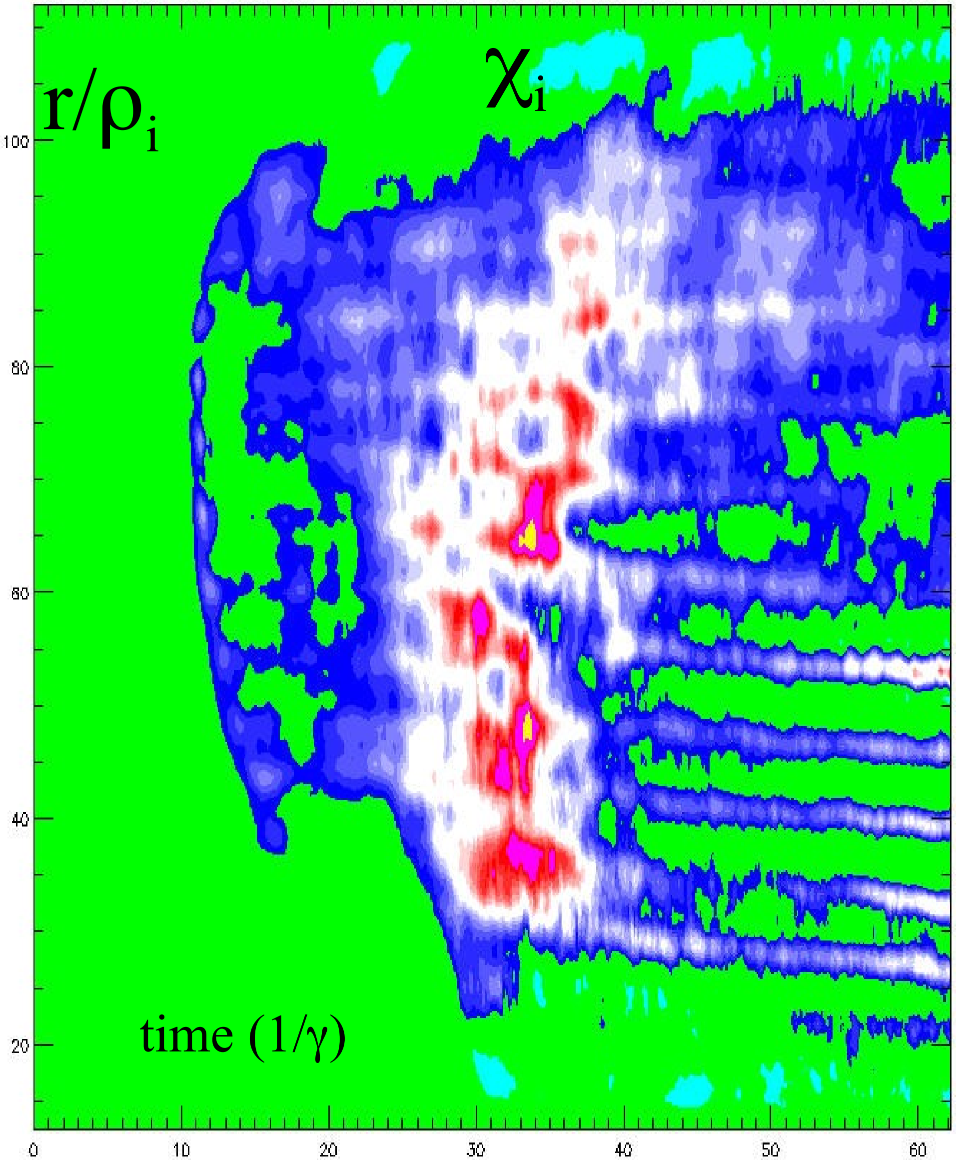
Motivation

- GB Scaling Breaking, *mesoscale* transport process, $L_{corr} < l < L_{sys}$
- *Non-locality* → Free energy delocalized from Source/Excitation
- Mechanisms { Avalanches (fluctuating gradient coupling) ; ✲Turbulence Spreading (*NL wave interactions*) }
- Observed both in experiments① and simulations (shown later)

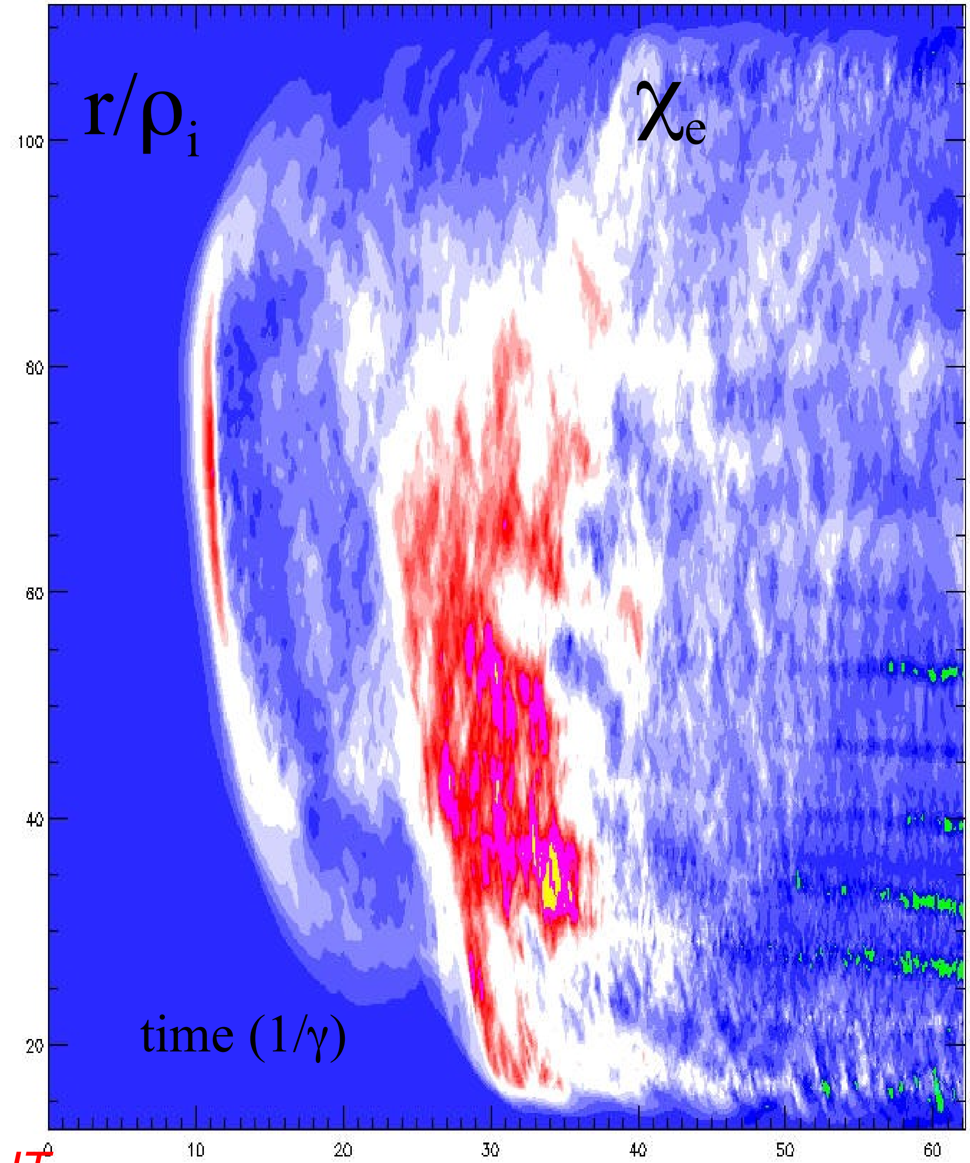
  Fast Propagation { Diffusive front (outwards) ; *Ballistic* front (inwards) }
- Recently, several theoretical models②③④ are *developed* for this theoretical challenge
- But, in general $K-\varepsilon$ models *noise* effects are ignored ***!?***
- Noise { Important (predicted by ②), enhance range of spreading ; Necessary(*self-consistent*), NL sink only (imperfect) }

CTEM Nonlinear Bursting and Spreading By Z.Lin

Ion transport



Electron transport



BUT

Simplest Model-*Resistivity gradient driven turbulence*

Thermal Equation

$$\frac{\partial \tilde{T}}{\partial t}+\nabla\cdot(\tilde{V}\langle T\rangle)+\nabla\cdot(\tilde{V}\tilde{T})=\chi\nabla^2\tilde{T} \quad (1)$$

Electric Field Drift Velocity

$$\tilde{V}=\frac{\hat{z}\times\nabla\tilde{\varphi}}{B_T} \quad (2)$$

Ohm's Law with Resistivity Gradient

$$\nabla_{\parallel}\tilde{\varphi}=-\tilde{\eta}J_{\parallel}=-\left(\frac{d\eta}{dT}\tilde{T}\right)\frac{E_o}{\eta_o} \quad (3)$$

Linear theory,

$$\gamma=\frac{E_o}{B_T}\left(\frac{1}{\eta_o}\frac{d\eta_o}{dr}\right)\frac{k_\theta}{k_\parallel}-\chi_\parallel k_\parallel^2$$

Multiplies (1) by $\tilde{T}^*$ and making statistical average, yields

$$\frac{\partial\langle\tilde{T}_1^2\rangle}{\partial t}+\nabla\cdot\langle\tilde{V}\tilde{T}_1^*\langle T\rangle\rangle+\nabla\cdot\langle\tilde{V}\tilde{T}_2\tilde{T}_1^*\rangle=\chi\nabla^2\langle\tilde{T}_1^2\rangle$$

Triad Interaction Term

In-Coherent | **Coherent**

Noise ~ $\langle\tilde{V}^2\rangle\langle\tilde{T}_2^2\rangle$ ~ $\langle\tilde{V}^2\rangle\langle\tilde{T}_1^2\rangle$ ~ $\langle\tilde{T}_2^2\rangle\langle\tilde{T}_1^2\rangle$

Form ?

Simplest Model-Prediction

Intensity Continuity Equation

$$\frac{\partial \tilde{T}^2}{\partial t} + \nabla\cdot(\tilde{\vec{V}}\,\tilde{T}^2) = 0$$

Intensity Flux

Goal of Turbulence Spreading

Triad Interaction

- Coherent
- *In-Coherent*

NL effects

$$\nabla\cdot\vec{J}_{eff}$$

$$Term\left(T^2(r+\Delta_1),\, T^2(r+\Delta_2)\right)$$

Constraint in form of Intensity Flux

Inhomogeneous Noise with Radial Shifts

But, if theoretical model[3] like

unadjusted

$$\frac{\partial \varepsilon}{\partial t} + \upsilon_g \partial_x \varepsilon - \partial_x(D_o \varepsilon^{\alpha} \partial_x \varepsilon) = \gamma_l \varepsilon - \gamma_{nl} \varepsilon^{\alpha+1}$$

Results-Spectral Structure

coherent — **In-coherent**

$$\frac{\partial \langle \tilde{T}^2_{\vec{k}} \rangle}{\partial t} + \nabla \cdot \vec{J} = (\gamma^l_{\vec{k}} + \chi \nabla^2) \langle \tilde{T}^2_{\vec{k}} \rangle + \nabla \cdot \vec{J}_{noise}$$

$$-\nabla \cdot (\vec{\vec{D}}(\tilde{T}^2_{k''}) \cdot \nabla^* \langle \tilde{T}^2_k \rangle)$$

NL Diffusion

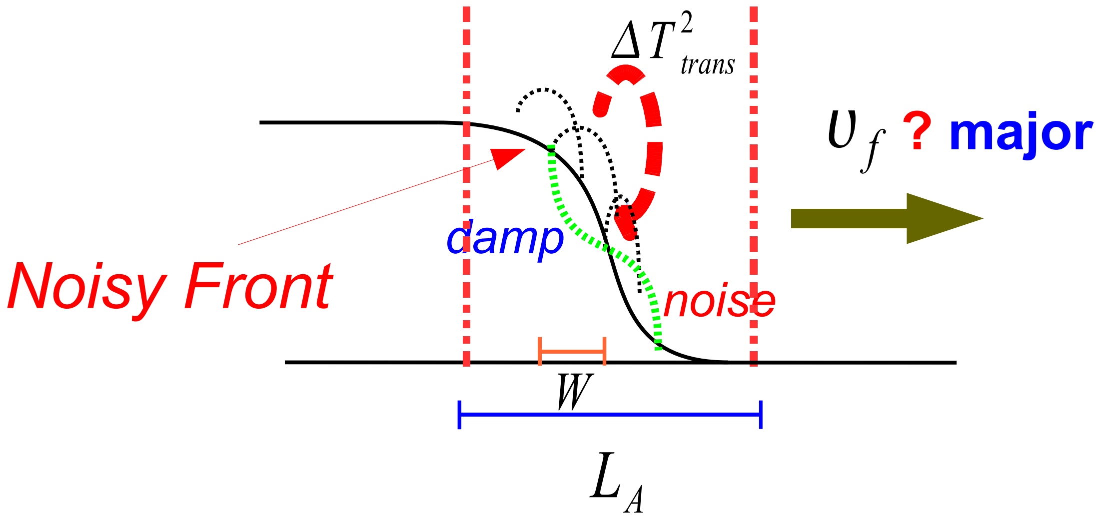
$$\nabla \cdot (\vec{V}(\tilde{T}^2_{k'}) \langle \tilde{T}^2_k \rangle)$$

NL Damp

$$\nabla \cdot (\vec{V}(\tilde{T}^2_{k''}) \langle \tilde{T}^2_{k'} \rangle)$$

Inhomogeneous noise

Self-consistent system



Gradient Length

Results-Spectrial Equation

Rewriting Spectral Eqn. (in *Two Radial Scales*)

NL Diffusion

$$\partial_t(A^2 \tilde{f}_k^2) - [\partial_r(\alpha A^2 \tilde{f}_k^2 |\partial_r A^2|) + \partial_r(\alpha A^4 \tilde{f}_k^2 x / w^2)] = (\gamma_k^l + \chi \nabla^2) A^2 \tilde{f}_k^2$$

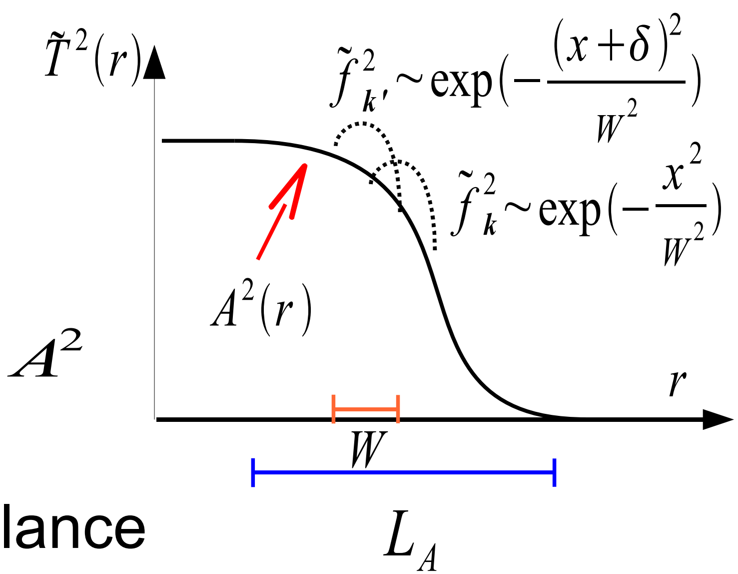
$$- \partial_r(\alpha |\partial_r A^2| A^2 \tilde{f}_k^2) + \partial_r(\alpha |\partial_r A^2| A^2 \tilde{f}_{k'}^2)$$

NL Damping Noise

cancel

during $\sum_k \rightarrow \int |m| dm \int \frac{q'}{q^2} dx$

because $\int \exp(-\frac{x^2}{W^2}) dx = \int \exp(-\frac{(x+\delta)^2}{W^2}) dx$



Energy *conserved* in NL Processes

$$\partial_t A^2 - \partial_r(\alpha A^2 \partial_r A^2) = (\gamma_l + \chi \nabla^2) A^2$$

$\int dr$

0 — Steady

0 — (Boundaries)

Linear Balance

Results-Contrast with Prediction & Fisher Eqn.

Constraint in form of Intensity Flux

Spectral Equation

$$\frac{\partial \langle \tilde{T}_k^2 \rangle}{\partial t} + \nabla \cdot \vec{J} = (\gamma_k^l + \chi \nabla^2) \langle \tilde{T}_k^2 \rangle + \nabla \cdot \vec{J}_{noise}$$

$$-\nabla \cdot (\vec{\vec{D}}(\tilde{T}_{k''}^2) \cdot \nabla^* \langle \tilde{T}_k^2 \rangle) \qquad \nabla \cdot (\vec{V}(\tilde{T}_{k'}^2) \langle \tilde{T}_k^2 \rangle) \qquad \nabla \cdot (\vec{V}(\tilde{T}_{k''}^2) \langle \tilde{T}_{k'}^2 \rangle)$$

NL Diffusion *NL Damp*

$$\sim \langle \tilde{T}_{k''}^2 \rangle \langle \tilde{T}_{k'}^2 \rangle$$

Compare

Fisher Equation

$$\frac{\partial u}{\partial t} - D \frac{\partial^2 u}{\partial x^2} = \gamma_l u - \gamma_{nl} u^2$$

$$Term(T^2(r+\Delta_1), T^2(r+\Delta_2))$$

Linear *But* $\neq \nabla \cdot \vec{J}$

Inhomogeneous Noise with Radial Shifts

Conclusion

- Nonlinear *noise* is usually *neglected* in turbulence spreading models (i.e. $K-\epsilon$);
- Noise from nonlinear beats can deliver power to leading edge of spreading front ⟶ *impact* on spreading (*ballistic* **!?**);
- *Interactions* of noise with leading edge of front are *restricted* by mode resonance structure and finite spectral width;
- Noise is *Self-Consistent* with NL Damping effects;
- All NL effects are constraint in forms of $\nabla\cdot\vec{J}$ and noise is term with little radial shifts; *Purely* local NL damp effects are unphysical ~~$-\gamma_{nl}\varepsilon^2$~~;
- "Front" ⟶ Noisy front.

Future work

- Solve equation and determine nonlinear noise effects on front propagation speed;
- Smooth leading edge ⟶ noisy leading edge ⟶ effects on front speed;
- Consider more realistic models, more physics (esp. noise feedback on $\langle T \rangle$ avalanche trigger **?**)

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Prof. Diamond's Great and Unwearied Help;

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References

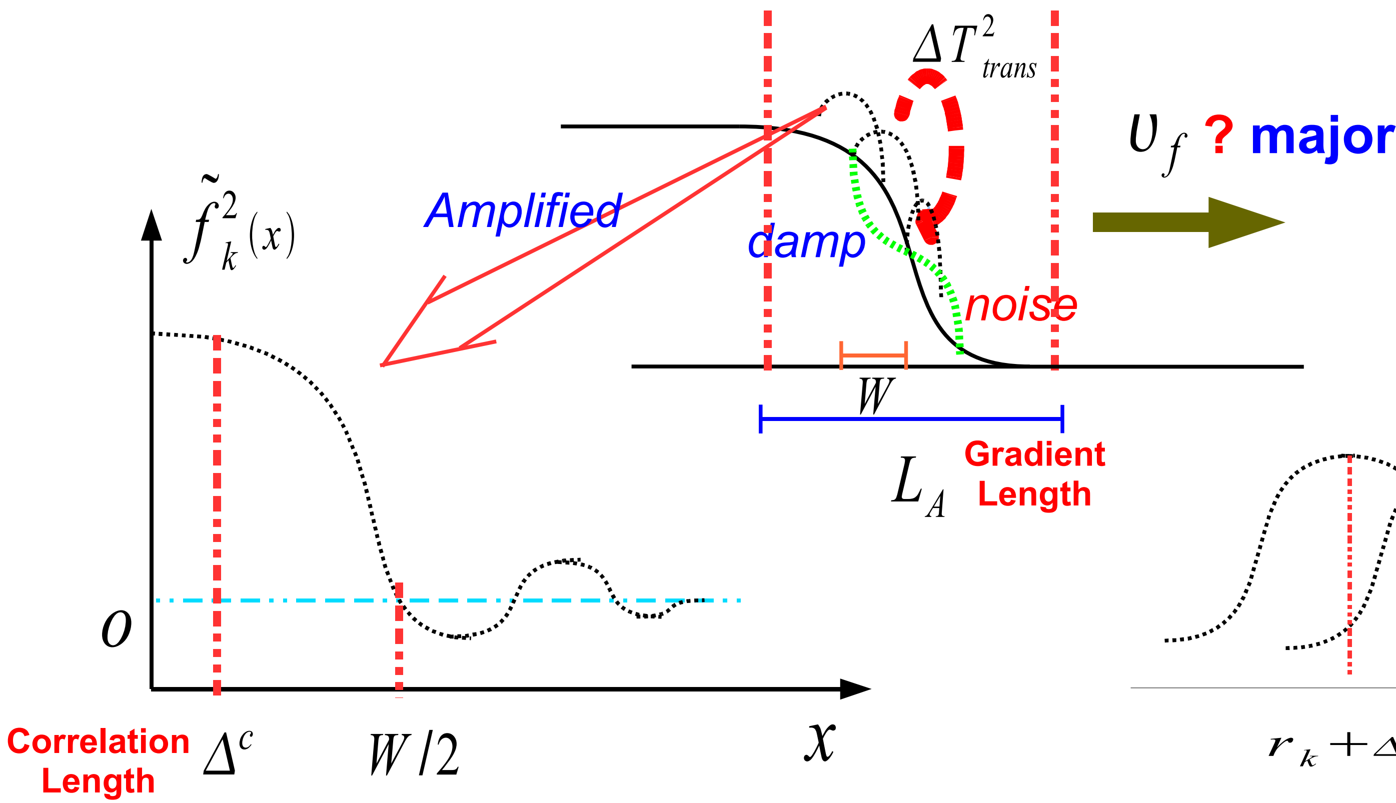
① P. Mantica, D. Van Eester, X. Garbet, *et al*. , Phys. Rev. Lett. **96**, 095002 (2006).

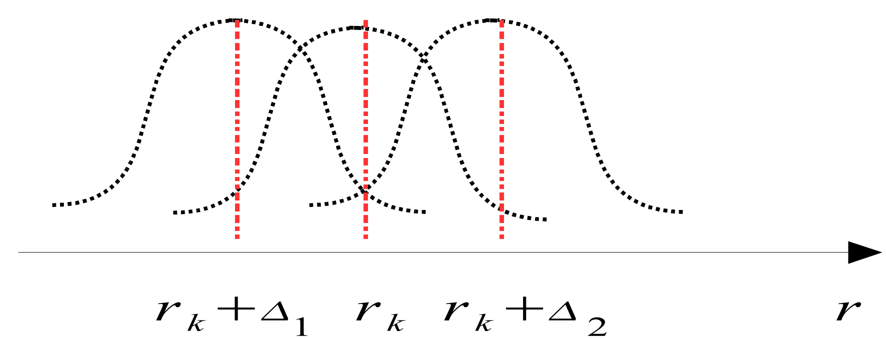
② T. S. Hahm, P. H. Diamond, Z. Lin, *et al.*, Plasma Phys. Control Fusion **46**, A323 (2004).

③ Ö. D. Gürcan, P. H. Diamond and T. S. Hahm, Phys. Plasmas **14**, 032303 (2005).

④ X. Garbet, Y. Sarazin, F. Imbeaux, *et al.*, Phys. Plasma **14**, 122305 (2007).

Self-consistent system





Scales

Triad modes interaction

Results-Renormalization

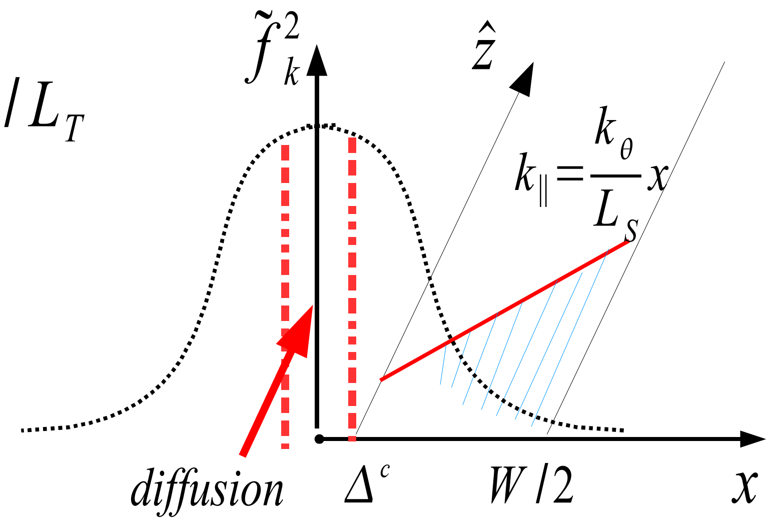
NL Diffusion Conduction Growth

$$\partial_r\left(D_k(\tilde T_k^2)\,\partial_r \tilde T_k^2\right) \qquad \chi_\parallel k_\parallel \tilde T_k^2 \qquad \left(\frac{c_t E_o L_s}{B_T x}\right)\tilde T_k^2 / L_T$$

Radial Profile Steady State

$$D_k \sim x^4 \chi_\parallel k'^2_\parallel \qquad x \sim \Delta^c$$

$$D_k \sim (\Delta^c)^4 \chi_\parallel k'^2_\parallel$$



Compare and adding $\sum_{k'} \langle \tilde T_{k''}^2 \rangle / \langle T \rangle^2$

$$D_k \simeq (\Delta^c)^4 \chi_\parallel k'^2_\parallel \frac{A^2}{\langle T \rangle^2} \sum_{k'} \tilde f_{k''}^2 = \alpha A^2$$

1

Free energy is first Diffused before being Conducted near Resonance Surface

⑤ *L.Garcia, P. H. Diamond, B. A. Carreras, and J. D. Callen, PFL,* ***28****,2147,(1985);*

Results-Spectral Structure

$$\frac{\partial \langle \tilde{T}^2_{\vec{k}} \rangle}{\partial t} + \nabla \cdot \vec{J} = (\gamma^l_{\vec{k}} + \chi \nabla^2) \langle \tilde{T}^2_{\vec{k}} \rangle + \nabla \cdot \vec{J}_{noise}$$

Where

$$\vec{\vec{D}}(\tilde{T}^2_{\vec{k}''}) = \frac{1}{2} \sum_{\vec{k}'} \Theta_{\vec{k},\vec{k}',\vec{k}''} \vec{V}^{eff}_{\vec{k}'} \vec{V}^{eff}_{-\vec{k}''} \langle \tilde{T}^2_{\vec{k}''} \rangle / \langle T \rangle^2$$

$$\vec{V}(\tilde{T}^2_{\vec{k}'}) = -\frac{1}{2} \sum_{\vec{k}'} \Theta_{\vec{k},\vec{k}',\vec{k}''} \vec{V}^{eff}_{\vec{k}'} \vec{V}^{eff}_{\vec{k}} \cdot \nabla^* \langle \tilde{T}^2_{\vec{k}'} \rangle / \langle T \rangle^2$$

damp

Cancel ?!

$$\vec{\vec{D}}(\tilde{T}^2_{\vec{k}''}) = \frac{1}{2} \sum_{\vec{k}'} \Theta_{\vec{k},\vec{k}',\vec{k}''} \vec{V}^{eff}_{\vec{k}'} \vec{V}^{eff}_{-\vec{k}''} \langle \tilde{T}^2_{\vec{k}''} \rangle / \langle T \rangle^2$$

$$\vec{J}_{noise} = \frac{1}{2} \sum_{\vec{k}'} \Theta_{\vec{k},\vec{k}',\vec{k}''} \vec{V}^{eff}_{\vec{k}'} \vec{V}^{eff}_{\vec{k}'} \cdot \nabla^* \langle \tilde{T}^2_{\vec{k}''} \rangle / \langle T \rangle^2 \langle \tilde{T}^2_{\vec{k}'} \rangle$$

noise

$$\vec{V}^{eff}_{\vec{k}} = \left| \frac{C_t E_o}{k_{\parallel} B_T} \right| \vec{k} \times \hat{z}$$

$$\gamma^l_k = \vec{V}^{eff}_{\vec{k}} \cdot |\nabla \langle T \rangle / \langle T \rangle|$$