Nonlinear excitation and damping of Zonal Flows using a renormalized polarization response

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GPS-TTBP February 2008

Abstract

The nonlinear interaction of drift-wave turbulence and zonal flows is considered using an analogy with dressed test-particles in a stable plasma. The incoherent mode coupling potentials from the drift waves are treated as a source of noise driving the zonal flows. The coherent mode coupling potentials are included in a renormalized nonlinear polarization response to this noise source, analogous to the shielding of test-particles. The nonlinear damping of zonal flows and the conditions for a steady turbulent state are determined from the nonlinear polarizability. This calculation attempts to systematically address the effects of fluctuations and turbulence on the otherwise 'neoclassical' zonal flow polarization response. Thus it offers the possibility of identifying new nonlinear, kinetic 'channels' for the coupling of zonal flow energy to dissipation. The implications for zonal flow saturation will be discussed.

This work was supported by DoE Grant No. DE-FG02-04ER54738.

Basic Equations

The nonadiabatic part $g(\vec{x}, \mu, v, t)$ of the ion d.f., defined by $f = -(e/T_i)F_i\phi + g$, is a solution of the drift-kinetic equation (for $k_\perp \rho_i \ll 1$):

$$\frac{\partial g}{\partial t} + (v_\| \hat{b} + \vec{v}_d) \cdot \nabla g - \frac{e}{T_i} F_i \frac{\partial \phi}{\partial t} - \frac{1}{B} \hat{b} \times \nabla F_i \cdot \nabla \phi = -\frac{1}{B} \hat{b} \times \nabla \phi \cdot \nabla g \quad (1)$$

where $v_\| = \left(v^2 - 2\mu B\right)^{1/2}$, and $\vec{v}_d$ is the grad-B drift.

The potential is determined by quasineutrality:

$$n_{i1} = -\frac{n_0 e}{T_i} \left(\phi - \rho_i^2 \nabla_\perp^2 \phi\right) + \int d^3v \; g = n_{e1} \quad (2)$$

where the $\rho_i^2 \nabla_\perp^2$ term is the polarization density.

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Flux-surface-averaged quasineutrality

$$\langle n_{i\vec{q}} \rangle = -\frac{n_0 e}{T_i}(1 + q^2\rho_i^2)\widetilde{\phi_{\vec{q}}} + \left\langle \int d^3v \; e^{-iQ} h_{\vec{q}} \right\rangle = \langle n_{e\vec{q}} \rangle = 0 \qquad (3)$$

Using the result derived above for $h_{\vec{q}}$ gives the potential:

$$n_0 \frac{e}{T_i} \chi_{\vec{q}}(\Omega) \widetilde{\phi_{\vec{q}}}$$
$$= \left\langle \int d^3v \; \frac{\overline{(e^{-iQ})} e^{iQ}}{(-i\Omega + \overline{d_{\vec{q}}(\Omega)})} \left[\mathcal{N}_{\vec{q}}^{(ic)}(\Omega) + f_{\vec{q}}(t=0) \right] \right\rangle \qquad (4)$$

where the renormalized polarization response is

$$\chi_{\vec{q}}(\Omega) = 1 + q^2\rho_i^2$$
$$-\frac{T_i}{n_0 e} \left\langle \int d^3v \; \frac{\overline{(e^{-iQ})} e^{iQ}}{(-i\Omega + \overline{d_{\vec{q}}(\Omega)})} \left(-i\Omega \frac{e}{T_i} F_i + \beta_{\vec{q}} \right) \right\rangle \qquad (5)$$

Assuming $\overline{d_{\vec{q}}(\Omega)} \ll \Omega$ and expanding in this ratio and in Q, the polarization response becomes

$$\chi_{\vec{q}}(\Omega) \simeq \chi_{\vec{q}}^{RH} + \frac{T_i}{n_0 e}\frac{i}{\Omega}\int d^3v \ \left[\frac{e}{T_i}F_i d_{\vec{q}}(\Omega) - \beta_{\vec{q}}(\Omega)\right] \qquad (6)$$

where the first term is the Rosenbluth-Hinton polarizability,

$$\begin{aligned} \chi_{\vec{q}}^{RH} &= q^2\rho_i^2 + \left\langle \int d^3v \ \frac{F_i}{n_0}\left[\overline{(Q^2)} - \left(\overline{Q}\right)^2\right]\right\rangle \\ &\simeq q^2\rho_i^2\left[1 + 1.6 q_s^2/\epsilon^{1/2}\right] \end{aligned} \qquad (7)$$

with q_s the safety factor.

After carrying out the $\boldsymbol{\omega'}$ integration, this becomes

$$\chi_{\vec{q}}(\Omega) \simeq \chi_{\vec{q}}^{RH} + \frac{iT_i}{2T_e\Omega B^2} \sum_{\vec{k}'} \left(\hat{b} \times \vec{k}' \cdot \vec{q}\right)^2 |\widetilde{\phi}_{\vec{k}'}(\Omega)|^2 \tag{8}$$

The second term is a damping term; the "dispersion relation" $\chi_{\vec{q}}(\Omega) = 0$ yields pure damping:

$$\Omega = -\frac{iT_i}{2T_e B^2 \chi_{\vec{q}}^{RH}} \sum_{\vec{k}'} \left(\hat{b} \times \vec{k}' \cdot \vec{q}\right)^2 |\widetilde{\phi}_{\vec{k}'}(0)|^2 \tag{9}$$

where the approximation $\widetilde{\phi}_{\vec{k}'}(\Omega) \simeq \widetilde{\phi}_{\vec{k}'}(0)$ has been used.

Because of the appearance of the factor q^2 in the damping rate, we might be tempted to interpret this as turbulent diffusion, but this factor is cancelled by the q^2 factor in $\chi_{\vec{q}}^{RH}$, so the damping is more like "drag" than "diffusion".

Incoherent Mode Coupling as a Noise Source

In Eq.(4), we neglect $\boldsymbol{d_{\vec{q}}(\Omega)}$ and $\boldsymbol{Q}$, so that we can write this equation in the form

$$\widetilde{\phi}_{\vec{q}}(\omega) = \Lambda_{\vec{q}}(\omega)\ \widetilde{S}_{\vec{q}}(\omega) \tag{10}$$

where $\Lambda_{\vec{q}}(\omega)$ is the susceptibility, which can be identified as

$$\Lambda_{\vec{q}}(\Omega) = \left(\frac{T_i}{n_i e}\right) \frac{i}{\Omega\ \chi_{\vec{q}}(\Omega)} \tag{11}$$

and $\widetilde{S}_{\vec{q}}(\omega)$ is the source, which can be identified as

$$\widetilde{S}_{\vec{q}}(\Omega) = \int d^3v\ \left[\mathcal{N}^{(ic)}_{\vec{q}}(\Omega, \vec{v}) + f_{\vec{q}}(\vec{v}; 0)\right] \tag{12}$$

We now neglect the particle discreteness term $f_{\vec{q}}(\vec{v}; 0)$.

Conclusions

The spectral density of zonal flow potentials has been expressed as the ratio of driving terms, due to particle discreteness and incoherent mode coupling from drift waves, and a renormalized polarization response. The polarization response is the sum of the Rosenbluth-Hinton polarization and a nonlinear damping term, which can be identified as due to a nonlinear drag effect of the drift waves on the zonal flows. Treating the incoherent mode-coupling drift-wave terms in the simplest way, with many approximations, yields the unsatisfactory result that the zonal flow source from drift-wave turbulence is zero.