Quiz 4

CODE #___*KEY*___

Score

Problem 1:__________ (35 pts)

Problem 2: __________ (25 pts)

Problem 3: __________ (25 pts)

Problem 4: __________ (25 pts)

Problem 5: __________ (15 pts)

TOTAL: __________ (125 pts)

To receive full credit, you must show all your work
(including steps taken, calculations, and formulas used).

If you do not wish your quiz to be placed on the public shelves in the Quiz Return Room, write your name *instead* of your code number at the top of each page. If you choose this option, your quiz will be held in a locked cabinet in the Quiz Return Room for pick-up with valid identification only.

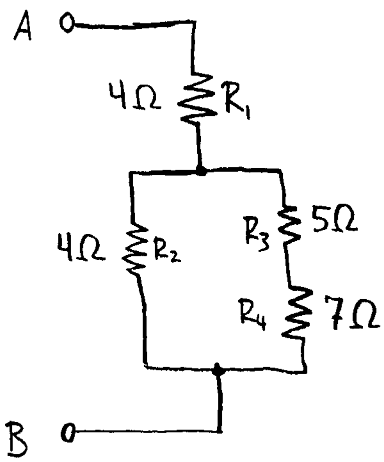
Quiz Return Room

WLH 2126 (downstairs from the lab)

Mon - Fri, 10 AM - 12:30 PM, 1:00 PM - 2:00 PM

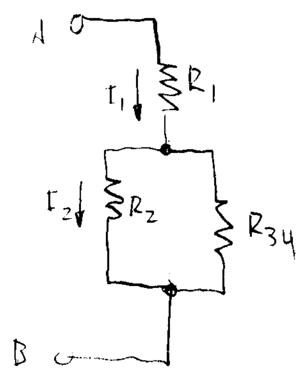
Problem 1 (35 pts total)

Given the following circuit:



(a) What is the equivalent resistance this circuit? (15 pts)

This circuit must be broken up into smaller circuits in order to determine the equivalent resistance.



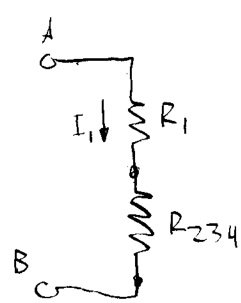
R_3 and R_4 are in series:

$$R_{34} = R_3 + R_4 = 5 + 7 = 12\ \Omega$$

R_2 and R_{34} are in parallel:

$$\frac{1}{R_{234}} = \frac{1}{R_2} + \frac{1}{R_{34}} = \frac{1}{4} + \frac{1}{12} = \frac{1}{3}$$

$$R_{234} = 3\ \Omega$$



R_1 and R_{234} are in series:

$$R_{eq} = R_1 + R_{234} = 4 + 3 = 7\ \Omega$$

The equivalent resistance of this circuit is 7 Ω.

Problem 1 continued:

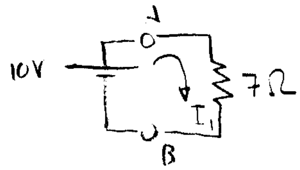
(b) Assume the terminals *A* and *B* of the circuit are now connected to a 10-V ideal battery. Find the currents through resistors R_1 and R_2. (15 pts)

For simplicity, we will denote the currents through R_1 and R_2 as I_1 and I_2, respectively.

The current I_1 is the same as the current through the equivalent resistance (which is also the same as the current supplied by the ideal battery). This current is given by Ohm's Law:

$$V = I_1 R_{eq} \quad \Rightarrow$$

$$I_1 = \frac{V}{R_{eq}} = \frac{10\ \text{V}}{7\ \Omega} = 1.43\ \text{A}$$



The current through R_1 is 1.43 A.

The current I_2 is a little more difficult to find. The current through R_2 is determined by the voltage across the branch that contains R_2. As shown in the circuit reduction, the voltage across R_2 is determined by the current through R_{234}:

$$V_2 = I_1 R_{234} = (1.43\ \text{A})(3\ \Omega) = 4.29\ \text{V}$$

Thus, the current through R_2 is simply the voltage across R_2 divided by its resistance:

$$I_2 = \frac{V_2}{R_2} = \frac{4.29\ \text{V}}{4\ \Omega} = 1.07\ \text{A}$$

The current through R_2 is 1.07 A.

(c) How much power does the battery deliver to the circuit? (5 pts)

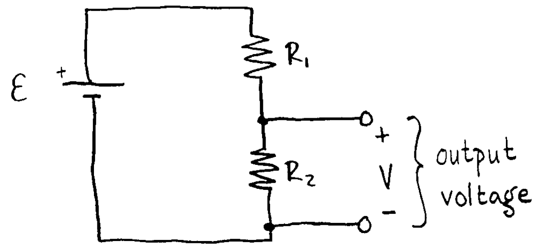
The power delivered by the battery is determined by the current supplied and the voltage of the battery. The current supplied by the battery was determined in part (b) to be I_1 = 1.43 A. Then, the power is,

$$P = VI = (10\ \text{V})(1.43\ \text{A}) = 14.3\ \text{W}$$

The power delivered by the battery to the circuit is 14.3 W.

Problem 2 (25 pts)

The following diagram illustrates a typical voltage divider useful for getting a smaller voltage from a larger voltage source.



(a) What is the output voltage, V, in terms of R_1, R_2, and the *emf*? (15 pts)

The output voltage in this problem, is simply the voltage drop across R_2. The voltage drop across any resistor is given by Ohm's Law:

$$V = IR_2$$

where I is the current through R_2. Since all the elements of the circuit are in series, the current everywhere is the same. This current depends on the *emf* due to the voltage source and the resistances of the circuit:

$$I = \frac{emf}{R_1 + R_2}$$

So, the voltage across R_2 is

$$V = IR_2 = \frac{emf}{R_1 + R_2} R_2$$

(b) How must R_1 and R_2 be related in order for the output voltage to be one-third of the *emf*? (10 pts)

Given $V = (1/3)emf$ and the answer to part (a), the resistances are related in the following manner:

$$V = \frac{1}{3} emf = \frac{emf}{R_1 + R_2} R_2$$

$$\Rightarrow \frac{1}{3} = \frac{R_2}{R_1 + R_2}$$

which, strictly speaking, is sufficient to answer the problem.
Rearranging the above equation tells us that $\boldsymbol{R_1 = 2R_2}$ **in order for the output voltage to be one-third of the *emf*.**

Problem 3 (25 pts)

How long does it take for a 9.0-V battery in series with a 2.0-kΩ resistor to charge up a 1.0-µF capacitor to 4.5 V? Assume that there was initially no charge (or voltage) on the capacitor before it was connected to the battery.

We wish to find the time it takes for the voltage across a capacitor to charge up to 4.5 V, given that the capacitor is in series with a resistor and battery.

The following equation provides a relationship for the voltage across the capacitor at any given time for charging up the capacitor:

$$V(t) = V_0(1 - e^{-t/RC})$$

where V_0 is the voltage of the battery (9.0 Volts, in this problem).

Note that there is a slightly different version of this equation without the "1–" term which corresponds to discharging a capacitor, and so is not immediately applicable to this problem.

For convenience, we will calculate the value of the time constant here:

$$t_0 = RC = (2 \times 10^3\ \Omega)(1 \times 10^{-6}\ \text{F}) = 0.002\ \text{sec}$$

Now, inserting known and given quantities into the voltage equation gives,

$$V(t) = 4.5 = 9.0(1 - e^{-t/0.002})$$

Dividing both sides by 9.0 and then subtracting 1 gives,

$$\frac{4.5}{9.0} = 0.5 = 1 - e^{-t\,/\,0.002}$$

$$0.5 = e^{-t\,/\,0.002}$$

Now, in order to solve the equation for time, we must take the natural logarithm of both sides:

$$\ln 0.5 = \ln(e^{-t\,/\,0.002})$$

$$\ln 0.5 = \frac{-t}{0.002}$$

$$t = -0.002 \times \ln\ 0.5$$

$$t = 0.0014\ \text{sec}$$

$$= 1.4\ \text{ms}$$

It takes 1.4 ms for the capacitor to charge up to 4.5 Volts.

Problem 4 (25 pts total)

Because the resistivity of a material changes with temperature, so does its resistance. Even though the temperature coefficient of resistivity for a carbon resistor at 20 °C is small (α_0 = –5.0 x 10^{-3} °C^{-1}), the resistance of large resistors can change as they experience Joule heating.

(a) The resistivity of a carbon resistor at T_0 = 20 °C is ρ_0 = 1.7 x 10^{-8} Ω·m. What is its resistivity at T = 100 °C? (10 pts)

The new resistivity (*not* the same as resistance) of a material due to a change in temperature is given by

$$\begin{aligned}\rho &= \rho_0\left(1+\alpha_0\Delta T\right)\\ &= (1.7\times10^{-8}\ \Omega\cdot\text{m})\left(1+(-5.0\times10^{-3}\ {}^\circ\text{C}^{-1})(100-20\ {}^\circ\text{C})\right)\\ &= \mathbf{1.02\times10^{-8}\ \Omega\cdot m}\end{aligned}$$

(b) If a carbon resistor had a resistance of 10 MΩ at T_0 = 20 °C, what resistance would it have at 100 °C? Assume that the length and diameter of the wire do not change with temperature. (15 pts)

As stated in the problem, the resistance changes with temperature (along with resistivity). The equation that relates resistance to resistivity is

$$R = \frac{\rho L}{A}$$

But the length and diameter of the wire are not known. However, since we assume that the length and diameter (and so, area) remain constant over the change in temperature, we may write

$$R_1 = \frac{\rho_1 L}{A} \qquad \text{and} \qquad R_2 = \frac{\rho_2 L}{A}$$

Dividing the second equation by the first (or by substitution, whichever you prefer) gives

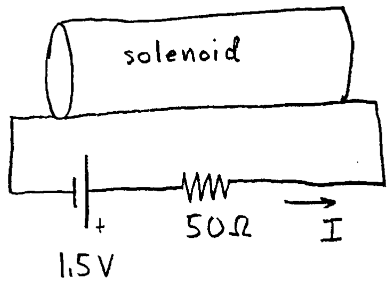
$$R_2 = \rho_2\left(\frac{L}{A}\right) = \rho_2\left(\frac{R_1}{\rho_1}\right) = R_1\left(\frac{\rho_2}{\rho_1}\right)$$

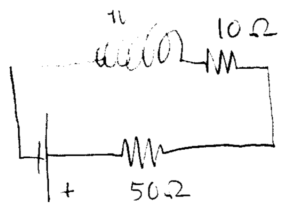
In this problem the "1" and "2" subscripts refer to properties at 20 °C and 100 °C, respectively. Substituting the given resistance (R_1) and resistivity (ρ_1) with the new resistivity (ρ_2) calculated in part (a) gives

$$R_2 = 10\ \text{M}\Omega\left(\frac{1.02\times10^{-8}}{1.7\times10^{-8}}\right) = \mathbf{6\ M\Omega}$$

Problem 5 (15 pts)

A long solenoid with 5 turns per meter is attached to a 1.5-V ideal battery and 50-Ω resistor as shown below (not to scale):





Assume that the solenoid itself has an additional small resistance of 10 Ω. What is the magnitude of the magnetic field inside of the solenoid?

The magnetic field inside of a solenoid is,

$$B = \mu_0 n I$$

where $\mu_0 = 1.26 \times 10^{-6}$ T·m/A is the permeability of free space,
n is the number of turns (coils) per meter (5, in this problem), and
I is the current through the wire.

The current through the wire is affected by the resistances (all in series) and the voltage powering the circuit:

$$I = \frac{V}{R + r} = \frac{1.5\text{ V}}{10 + 50\ \Omega} = 0.025\text{ A}$$

Thus,

$$B = \mu_0 n I = (1.26 \times 10^{-6}\text{ T·m/A})(5\text{ m}^{-1})(0.025\text{ A})$$
$$= 1.58 \times 10^{-7}\text{ T}$$

and so, **the strength of the magnetic field is 1.58 × 10^{-7} Tesla.**