Quiz 3

CODE #___*KEY*___

Score

Problem 1:__________ (25 pts)

Problem 2: __________ (25 pts)

Problem 3: __________ (25 pts)

Problem 4: __________ (25 pts)

Problem 5: __________ (25 pts)

TOTAL: __________ (125 pts)

To receive full credit, you must show your work (including calculations and formulas used).

If you do not wish your quiz to be placed on the public shelves in the Quiz Return Room, write your name *instead* of your code number at the top of each page. If you choose this option, your quiz will be held in a locked cabinet in the Quiz Return Room for pick-up with valid identification only.

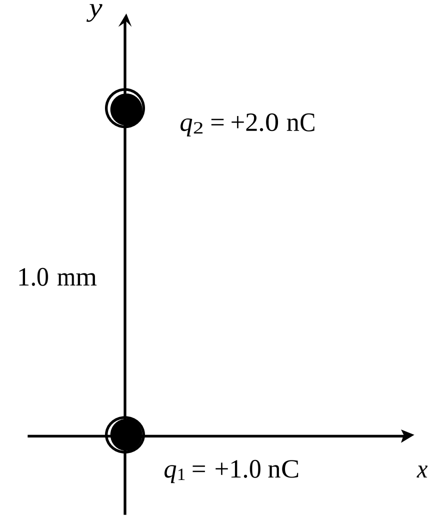
Quiz Return Room

WLH 2126 (downstairs from the lab)

Mon - Fri, 10 AM - 12:30 PM, 1:00 PM - 2:00 PM

Problem 1 (25 pts total)

Two pith balls (which are assumed to be point-like) are arranged as shown:



(a) What is the magnitude of the force exerted by charge q_1 on charge q_2? (15 pts)

The force between two point charges is defined by Coulomb's Law:

$$F_E = k_0 \frac{q_1 q_2}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

Inserting the given quantities into the equation gives the magnitude of this force:

$$F_E = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.0\times10^{-9}\ \text{C})(2.0\times10^{-9}\ \text{C})}{(0.001\,\text{m})^2} = 0.018\ \text{N}$$

$$= \mathbf{1.8\times10^{-2}\ N}$$

Since we are only concerned with the force exerted by one charge on another, gravitational effects are not relevant.

(b) What direction is this force in? (5 pts)

Since both charges are positive, they repel, so the force that q_2 *feels* due to the electric field created by q_1 is **upward** or in the +y-direction. (Likewise, the force that q_1 feels is downward.)

(c) What is the magnitude and direction of the force exerted by q_2 on q_1? (5 pts)

Newton's Third Law states that for every force acting on an object, there is a force equal in magnitude and opposite in direction acting on the other object. Thus, the force from q_2 on q_1 is **0.018 N downward**.

Problem 2 (25 pts)

An alpha particle (α) is a dangerous type of radiation which can easily strip electrons off of atoms that it gets close to. However, an α-particle is simply a fast-moving Helium nucleus, consisting of two protons (net charge = +2*e*).

Imagine that an α-particle has found its way into your body. Given that the dielectric constant (or relative permittivity) of flesh is 8.0, what is the magnitude of the electric field produced by the α-particle at a distance 10.0 nm from its center?

Solution: The magnitude of an electric field produced by a point is given by Eq. (15.5):

$$E = k_0 \frac{q}{r^2}$$

When the point charge (and so, electric field) is immersed in a dielectric material, the constant, k_0, is replaced by (see pg. 630),

$$k = \frac{1}{4\pi\varepsilon}$$

The relative permittivity, K_e, is related to the permittivity of free space, ε_0, by

$$K_e = \frac{\varepsilon}{\varepsilon_0}$$

and so the permittivity of flesh (the dielectric medium, in this case) is

$$\varepsilon = K_e \varepsilon_0$$

Combining these equations gives a formula for the electric field of a point charge immersed in a dielectric medium in terms of the relative permittivity, K_e:

$$E = \frac{1}{4\pi K_e \varepsilon_0} \frac{q}{r^2}$$

Now, in this problem, the charge q is the charge of the alpha particle (+2*e*), or twice the magnitude of the charge of an electron.
Inserting given quantities into the above equation gives

$$E = \frac{1}{4\pi(8)(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})} \frac{2(1.6\times10^{-19}\ \mathrm{C})}{(10.0\times10^{-9}\ \mathrm{m})^2}$$
$$= \mathbf{3.60\times10^{6}\ V/m}$$

Problem 3 (25 pts total)

The Van de Graaff generator that you used in lab can be charged up to a rather substantial voltage (of about 5000 Volts), but it doesn't kill you if you touch it (although it does tingle a bit when it discharges through your body). This is because there is only a very small amount of charge on it, so the current that passes through your body is not life-threatening.

Assume that the conductor on the top of the generator is a sphere with radius R = 15.0 cm.

(a) What is its capacitance? (9 pts)

The capacitance of a sphere is given by Eq. (16.13):

$$C = 4\pi\varepsilon_0 R$$
$$= 4\pi(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})(0.15\ \mathrm{m}) = 1.67\times10^{-11}\ \mathrm{F}$$
$$= \mathbf{16.7\ pF}$$

(b) How much charge is stored on it when V = 5000 V? (8 pts)

The charge stored on a capacitor is given by Eq. (16.12):

$$C = \frac{Q}{V} \Rightarrow$$
$$Q = CV = (1.67\times10^{-11}\ \mathrm{F})(5000\ \mathrm{V}) = 8.35\times10^{-8}\ \mathrm{C}$$
$$= \mathbf{83.5\ nC}$$

(c) When at this same voltage, how much energy is stored on it? (8 pts)

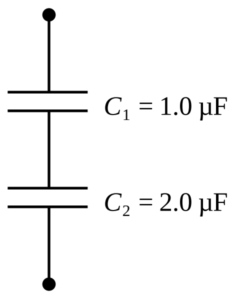
The energy stored on a capacitor is given by Eqs. (16.17-16.18):

$$U = \varepsilon_\mathrm{C} = \mathrm{PE_E} = \tfrac{1}{2}CV^2$$
$$= \tfrac{1}{2}(1.67\times10^{-11}\ \mathrm{F})(5000\ \mathrm{V})^2 = 2.1\times10^{-4}\ \mathrm{J}$$
$$= \mathbf{210\ \mu J}$$

Problem 4 (25 pts total)

Determine the equivalent capacitance for each of the circuits pictured below:

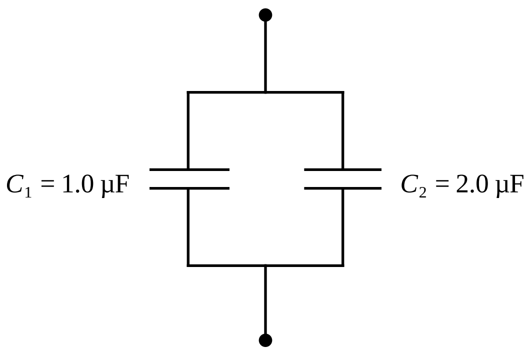
(a) (6 pts)



Series :

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$
$$= \frac{1}{1.0\ \mu\text{F}} + \frac{1}{2.0\ \mu\text{F}} = \frac{3}{2\ \mu\text{F}}$$
$$C_{eq} = \frac{2}{3}\ \mu\text{F} = \mathbf{0.67\ \mu F}$$

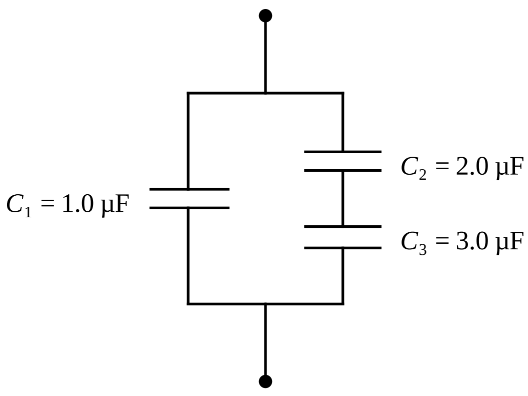
(b) (6 pts)



Parallel :

$$C_{eq} = C_1 + C_2$$
$$= 1.0\ \mu\text{F} + 2.0\ \mu\text{F} = \mathbf{3.0\ \mu F}$$

(c) (13 pts)



Series : C_2 and C_3

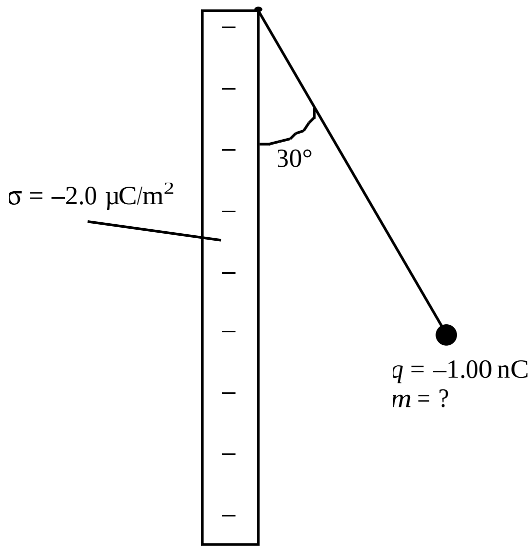
$$\frac{1}{C_{23}} = \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{2.0\ \mu\text{F}} + \frac{1}{3.0\ \mu\text{F}} = \frac{5}{6\ \mu\text{F}}$$
$$C_{23} = \frac{6}{5}\ \mu\text{F} = 1.2\ \mu\text{F}$$

Parallel : C_1 and C_{23}

$$C_{eq} = C_1 + C_{23}$$
$$= 1.0\ \mu\text{F} + 1.2\ \mu\text{F} = \mathbf{2.2\ \mu F}$$

Problem 5 (25 pts)

A tiny pith ball with charge q = –1.00 nC is attached by a cotton string to an infinite-plane sheet of charge with surface charge density σ = –2.0 μC/m^2. Under the influence of gravity, the *E*-field of the sheet, and the tension of the string, the ball hangs at an angle θ = 30° from the plane as shown below.

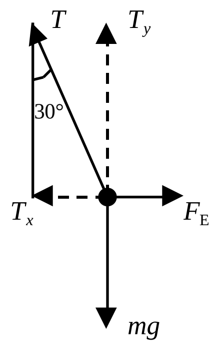


What is the mass of the pith ball? Show all your work. (Simply writing an equation down is not sufficient for the solution to this problem.) Use the back of this page if necessary.

See the following pages for the solution.

Problem 5 Solution

Step 1: Draw Free-Body Diagram



where the components of the tension in the string, T, are

$$T_x = T\sin 30°$$
$$T_y = T\cos 30°$$

In general, the force on a charge, q, in the presence of an electric field, E, has magnitude

$$F_E = qE$$

In this specific case, the electric field is due to the infinite plane. An infinite plane has an electric field whose magnitude is given by

$$E = \frac{\sigma}{2\varepsilon_0} \qquad \text{(Hecht, 15.17)}$$

and so the force on the (charged) pith ball due to the electric field from the infinite plane is

$$F_E = q\frac{\sigma}{2\varepsilon_0}$$

Since both the flat plane and pith ball have a positive charge, they will repel each other, as demonstrated by the force due to their interaction, F_E, pointing to the right.

Continued on next page…

Step 2: Balance Forces

Since the system depicted is in equilibrium, the sum of forces in both the vertical (y) and horizontal (x) directions must be zero:

Horizontal components:

$$T_x = F_{\mathrm{E}}$$

or, substituting for both of these quantities,

$$\boxed{T\sin 30° = q\frac{\sigma}{2\varepsilon_0}} \qquad (1)$$

Vertical components:

$$T_y = mg$$

or, substituting for the left-hand-side,

$$\boxed{T\cos 30° = mg} \qquad (2)$$

Step 3: Solve for the Unknown

There are many acceptable tactics used to solve two equations for an unknown. It is often a matter of preference or comfort. In this case, it seems logical to divide Eq. (1) by Eq. (2), giving

$$\frac{T\sin 30°}{T\cos 30°} = \frac{q\dfrac{\sigma}{2\varepsilon_0}}{mg}$$

which, after canceling the T's, and using the definition of the tangent, reduces to

$$\tan 30° = q\frac{\sigma}{2\varepsilon_0}\frac{}{mg}$$

Solving this for the unknown mass, m, and inserting the quantities given in the problem yields

$$\begin{aligned} m &= q\frac{\sigma}{2\varepsilon_0}\frac{}{g\tan 30°} \\ &= (-1.0\times 10^{-9}\ \mathrm{C})\frac{(-2.0\times 10^{-6}\ \mathrm{C/m^2})}{2(8.85\times 10^{-12}\ \mathrm{C^2/N\cdot m^2})}\frac{}{(9.8\ \mathrm{m/s^2})\tan 30°} \\ &= \mathbf{2.0\times 10^{-5}\ kg} = 0.020\ \mathrm{g} \end{aligned}$$