# *Quiz 2*

CODE # ___*KEY*___

***Score***

Problem 1:__________ (25 pts)

Problem 2: __________ (25 pts)

Problem 3: __________ (25 pts)

Problem 4: __________ (50 pts)

**TOTAL: __________ (125 pts)**

***To receive full credit, you must show all your work (including steps taken, calculations, and formulas used).***

If you do not wish your quiz to be placed on the public shelves in the Quiz Return Room, write your name *instead* of your code number at the top of each page. If you choose this option, your quiz will be held in a locked cabinet in the Quiz Return Room for pick-up with valid identification only.

Quiz Return Room

WLH 2126 (downstairs from the lab)

Mon - Fri, 10 AM - 12:30 PM, 1:00 PM - 2:00 PM

***Problem 1*** (25 pts)

A flat sheet of brass has a 3.00-cm-diameter circular hole when it is 20 °C. What would the diameter of the hole be if the brass sheet was dropped into a reservoir of boiling water and allowed to reach thermal equilibrium? (The coefficient of linear expansion of brass is $18.9 \times 10^{-6}$ K$^{-1}$.)

*Solution*: The brass sheet will undergo linear expansion, and so the diameter of hole will expand according to the linear expansion formula:

$$\Delta L = L_0 \alpha \Delta T$$

In this case, the characteristic length, *L*, is the diameter, so we may write instead,

$$\Delta D = D_0 \alpha \Delta T$$

where $D_0$ is the initial diameter, $\alpha$ is the coefficient of expansion, and $\Delta T$ is the change in temperature of the brass that causes the expansion.

We wish to find the new diameter of the brass, not just its change in length. The new diameter is simply the original diameter plus the additional change in length:

$$\begin{aligned} D_{\text{new}} &= D_0 + \Delta D \\ &= D_0 + D_0 \alpha \Delta T \end{aligned}$$

The brass is initially 20 °C, and is placed in boiling water (100 °C), and so

$$\Delta T = 100\ °\text{C} - 20\ °\text{C} = 80\ \text{K}$$

Now, inserting known quantities into the above equation gives,

$$\begin{aligned} D_{\text{new}} &= D_0 + D_0 \alpha \Delta T = 3.00\ \text{cm} + (3.00\ \text{cm})(18.9 \times 10^{-6}\ \text{K}^{-1})(80\ \text{K}) \\ &= 3.005\ \text{cm} \end{aligned}$$

That is, the new diameter of the hole is **3.005 cm** (or 3.01 cm). Note that the change in diameter is $4.54 \times 10^{-3}$ cm.

***Problem 2*** (25 pts)

You would like to cool down 250 g of water at an initial temperature of 30 °C by dropping some ice (at 0 °C) into it. If you want the final temperature of your water to be 0 °C, what is the minimum amount of ice you would have to add? (Ignore the heat capacity of the Styrofoam cup and any radiative, conductive, or convective heat losses.)

*Solution*: The combination of two materials, as in this problem, is an exercise in thermal equilibrium. In this case, the total energy of the ice and water (denoted by "I" and "W" subscripts) is conserved:

$$\Delta Q_I + \Delta Q_W = 0$$

So, in order to solve this problem, we must essentially determine how (and how much) heat will be transferred from one material to the other. Initially the water is at 30 °C and the ice is at 0 °C. The final mixture is water at 0 °C. Thus, the water must be cooled down (lose heat) and the ice must melt (gain heat). The heat the water must lose in order to cool down is

$$\Delta Q_W = m_W c_W \Delta T_W$$

where $c_W$ is the specific heat of water and is equal to 4.186 kJ/kg·K.
The heat the ice must gain to melt is

$$\Delta Q_I = +m_I L_f$$

where $L_f$ = 333.7 kJ/kg is the heat of fusion of water/ice. Note that since the ice is already at 0 °C, its temperature does not need to change, only its state (from ice to water).

Inserting these heat equations into the conservation of energy gives

$$m_I L_f + m_W c_W \Delta T_W = 0$$

Solving this equation for the unknown mass of ice, $m_I$, and inserting known quantities yields

$$m_I = \frac{-m_W c_W \Delta T_W}{L_f} = -\frac{m_W c_W (T_f - T_W)}{L_f}$$

$$= -\frac{(250 \text{ g})(4.186 \text{ kJ/kg·K})(0 - 30 \text{ K})}{333.7 \text{ kJ/kg}}$$

$$= 94.1 \text{ g}$$

Thus, the minimum amount of ice at 0 °C you would have to add is **94.1 g**. This final mixture is entirely water. (However, if you added more ice, the initial mass of water would still be cooled to 0 °C, but not all the ice you added would melt, and you would have some ice-water mixture all at 0 °C.) Indeed, the final mixture is actually 344.1 g of liquid water at 0 °C.

*Notes on the units*: You may convert the masses to kg, and the constants ($c_W$ and $L_f$) to use in Joules (instead of kJ). However, since there are kilojoules in both the denominator and numerator of the final equation, the energy units cancel out. The units you use for the mass of ice will be the units of the final answer. In this case, grams seem more convenient. Also, remember a change in temperature is the same in Kelvin and in Celsius, so no conversion is necessary.

***Problem 3*** (25 pts)

The brick siding of a house is 20.0 cm thick. The temperatures inside and outside of the house are 72 °F and 32 °F, respectively. What is the rate of energy loss per square meter of the brick siding? (The thermal conductivity of brick is about 0.63 W/m·K.)

Energy will be lost through conduction through the brick siding of the house. The rate of conductive heat loss is

$$\frac{Q}{t} = H = -k_T A \frac{\Delta T}{\Delta L}$$

We wish to find this rate of energy loss per square meter (or per unit area), and dividing by the area, *A*, gives

$$\frac{H}{A} = -k_T \frac{\Delta T}{\Delta L}$$

where $k_T$ is the thermal conductivity of brick (0.63 W/m·K), $\Delta T$ is the temperature change on either side of the brick siding, and $\Delta L$ is the thickness of the brick.

However, the temperatures are given in °F, and so they must be converted to Kelvin in order for the units to properly cancel with $k_T$. The inside and outside temperatures are converted in the following manner:

$$72\ ^\circ\text{F} \rightarrow \tfrac{5}{9}(72 - 32) = 22.2\ ^\circ\text{C}$$
$$32\ ^\circ\text{F} \rightarrow \tfrac{5}{9}(32 - 32) = 0\ ^\circ\text{C}$$

The change in temperature is simply the difference of these two temperatures, or 22.2 K.

Since all the desired quantities are known, inserting them into the above equation yields,

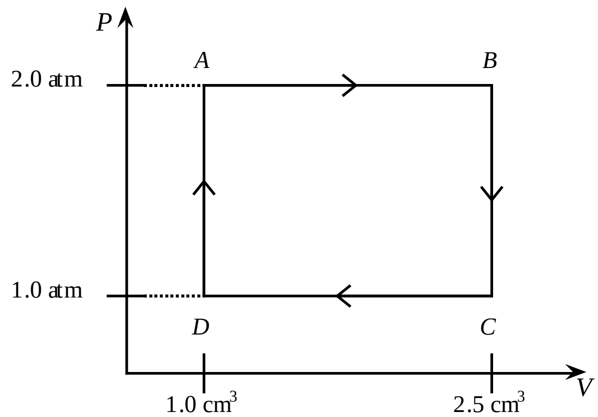
$$\frac{H}{A} = -k_T \frac{\Delta T}{\Delta L} = -(0.63\ \text{W/m·K})\frac{22.2\ \text{K}}{0.20\ \text{cm}}$$
$$= 69.9\ \text{W/m}^2$$

**The rate of (heat) energy loss per square meter of the brick siding is 69.9 W/m$^2$.**

Note that the shape of the brick siding is irrelevant, since the result is per unit area. The only requirement of the brick siding is that it has a uniform thickness of 20.0 cm.

***Problem 4*** (50 pts total)

The following *PV*-diagram shows the cyclic thermodynamic process of a certain heat engine that is operating with an ideal gas in a well-sealed cylinder. The temperature of the gas at point *A* is 150 °C.




(a) Calculate the total amount of work done on/by the system during one complete cycle. Was this work done on or by the system? (30 pts)

*Solution*: The work done during one complete cycle is the sum of the work done during each step:

Step *A*-*B*: Isobaric ($dP = 0$),

$$W_{AB} = P_A \Delta V_{AB} = \left( \frac{2 \text{ atm}}{} \cdot \frac{0.101\times10^6 \text{ Pa}}{1 \text{ atm}} \right) \left( \frac{(2.5-1.0)\text{ cm}^3}{} \cdot \left( \frac{1 \text{ m}}{100 \text{ cm}} \right)^3 \right) = 0.30 \text{ J}$$

Step *B*-*C*: Isochoric ($dV = 0$),
Since $dW = PdV$ (pg. 577) and $dV = 0$, this implies $dW = 0$, so no work is done ($W_{BC} = 0$).

Step *C*-*D*: Isobaric,

$$W_{CD} = P_C \Delta V_{CD} = \left( \frac{1 \text{ atm}}{} \cdot \frac{0.101\times10^6 \text{ Pa}}{1 \text{ atm}} \right) \left( \frac{(1.0-2.5)\text{ cm}^3}{} \cdot \left( \frac{1 \text{ m}}{100 \text{ cm}} \right)^3 \right) = -0.15 \text{ J}$$

Step *D*-*A*: Isochoric,
Same as *B*-*C*: $W_{DA} = 0$.

Adding up the work done in each step gives,

$$W_{\text{tot}} = W_{AB} + W_{BC} + W_{CD} + W_{DA} = 0.30 + 0 - 0.15 + 0$$
$$= 0.15 \text{ J}$$

The **total work done in one complete cycle is 0.15 Joules**. Since this work is positive, this is **work done by the system** on the environment (work out).

***Problem 4 cont'd:***

(b) Calculate the amount of heat that entered or left the system during the process $A \to B$. Did the heat enter or leave the system? (20 pts)

*Solution*: The *A*-*B* process is isobaric since pressure is constant ($dP = 0$). The amount of heat transferred during any isobaric process is given by the following equation for a monatomic ideal gas,

$$\Delta Q = \tfrac{5}{2} nR\Delta T \qquad \text{(isobaric process, monatomic ideal gas)}$$

This equation was given in lecture. Please see Tom or Andrew, or refer to the lecture notes for the derivation of this equation.

Specifcally, for this problem, we may re-write the equation as follows,

$$\Delta Q_{AB} = \tfrac{5}{2} nR\left(T_B - T_A\right) = \tfrac{5}{2}\left(nRT_B - nRT_A\right)$$

Now, $T_A$ is given in the problem (150 °C), but $T_B$ and the number of moles, $n$, are not.

Recall that the ideal gas law always applies to an ideal gas at any given instant (for example, at states *A* or *B*). That is, in general,

$$P_A V_A = n_A R T_A \qquad P_B V_B = n_B R T_B \qquad \text{(ideal gas law)}$$

In this problem, the system is sealed so $n_A = n_B = n$, and during the *A*-*B* process, $P_A = P_B = P$ (since $dP = 0$). Thus, we may modify the above equations to give,

$$PV_A = nRT_A \qquad PV_B = nRT_B$$

Notice that the right-hand-side of both of these equations constitutes the terms in parenthesis of the heat equation above. Straightforward substitution of these last two equations into the above equation for heat transfer gives,

$$\Delta Q_{AB} = \tfrac{5}{2}\left(nRT_B - nRT_A\right)$$
$$= \tfrac{5}{2}\left(PV_B - PV_A\right) = \tfrac{5}{2} P\Delta V_{AB}$$

All the quantities in this final equation are given on the *PV*-diagram. (Furthermore, $P\Delta V_{AB}$ was already calculated in part (a).) Thus, inserting known quantities and making unit conversions (from $cm^3$ to $m^3$ and atm to Pa, as shown in part (a)), yields,

$$\Delta Q_{AB} = \tfrac{5}{2} P\Delta V_{AB} = \tfrac{5}{2}(0.30 \text{ J})$$
$$= 0.75 \text{ J}$$

The amount of **heat transferred during the *A*-*B* process was 0.75 Joules**. Since this is positive, **heat entered the system**. (This also makes physical sense since during this process, the system expanded, which would require heat input.)

Note: You could have explicitly solved for the number of moles and $T_B$ and still obtained the same results. As it turns out, $n = 5.74 \times 10^{-5}$ moles, and $T_B = 1058$ K, so $\Delta T = 635$ K.