Quiz 1

CODE #___*KEY*___

Score

Problem 1:__________ (25 pts)

Problem 2: __________ (25 pts)

Problem 3: __________ (25 pts)

Problem 4: __________ (25 pts)

Problem 5: __________ (25 pts)

TOTAL: __________ (125 pts)

To receive full credit, you must show all your work
(including steps taken, calculations, and formulas used).

If you do not wish your quiz to be placed on the public shelves in the Quiz Return Room, write your name *instead* of your code number at the top of each page. If you choose this option, your quiz will be held in a locked cabinet in the Quiz Return Room for pick-up with valid identification only.

Quiz Return Room

WLH 2126 (downstairs from the lab)

Mon - Fri, 10 AM - 12:30 PM, 1:00 PM - 2:00 PM

Problems 1 (25 pts)

At a cruising altitude of 45,000 ft (13,707 m), the air pressure outside of a commercial jetliner is about 0.015 MPa. The inside cabin pressure is maintained at a comfortable 0.101 MPa. But, if there is a "sudden loss of cabin pressure" (as the flight attendant warned you about), this comfortable cabin pressure may rapidly decrease to the outside pressure.

If you were to try really hard to hold your breath, such that none of the air in your lungs could escape, your lungs would expand to a larger volume. What would be the ratio of the volume of your lungs *after* the incident to the volume of your lungs *before* the loss of cabin pressure? (Although the outside temperature is -51.5 °C, assume that the loss of pressure happens so quickly that the temperature in your lungs does not change.)

Solution: We wish to find a ratio between two volumes: the volume after the loss in cabin pressure (V_2) divided by the volume before the loss in cabin pressure (V_1). We are given P_1 = 0.101 MPa, and P_2 = 0.015 MPa.

The volume in any given state can be determined from the Ideal Gas Law:

$$PV = nRT \quad \Rightarrow \quad V = \frac{nRT}{P}$$

So, the desired volumes can be written, in general, as,

$$V_1 = \frac{n_1 R T_1}{P_1} \quad \text{and} \quad V_2 = \frac{n_2 R T_2}{P_2}$$

However, in this problem, we are told that the temperature does not change ($T_1 = T_2$) and that none of the air in your lungs escapes ($n_1 = n_2$). Including this information and dividing the two volumes gives,

$$\frac{V_2}{V_1} = \frac{\left(\dfrac{nRT}{P_2}\right)}{\left(\dfrac{nRT}{P_1}\right)} = \frac{P_1}{P_2}$$

Inserting given quantities into this equation yields,

$$\frac{V_2}{V_1} = \frac{P_1}{P_2} = \frac{0.101 \text{ MPa}}{0.015 \text{ MPa}} = 6.73$$

That is, **the volume of your lungs after the loss of cabin pressure would increase to about 6.73 times the size before the loss of cabin pressure**.

Notes on the units: Since the final equation did not involve temperature, but merely a ratio of two pressures, any pressure units are acceptable provided that they are the same in the numerator or denominator. In this case, both are MPa = 10^6 Pa. The final answer (a ratio) is unit-less.

Problem 2 (25 pts)

At 45,000 ft, the air temperature outside of a jetliner's cabin is about -51.5 °C. Given the pressures from Problem 1, how many moles per unit volume of air are there at an altitude of 45,000 ft compared to the moles per unit volume of air at standard cabin pressure and temperature (20 °C)?

Solution: We wish to compare the moles per unit volume in two different states: inside the cabin (P_1 = 0.101 MPa, T_1 = 20 °C), and outside the cabin (P_2 = 0.015 MPa, T_2 = -51.5 °C).

The moles per unit volume in any given state can be obtained from the Ideal Gas Law:

$$PV = nRT \quad \Rightarrow \quad \frac{n}{V} = \frac{P}{RT}$$

As in Problem 1, the moles per unit volume in both states 1 and 2 can be written as,

$$\left(\frac{n}{V}\right)_1 = \frac{P_1}{RT_1} \quad \text{and} \quad \left(\frac{n}{V}\right)_2 = \frac{P_2}{RT_2}$$

Unlike Problem 1, though, no claim is made about temperature being constant. Since the temperatures will not drop out of the equation they must be converted to Kelvin:

T_1 = 20 °C = 293 K $\qquad$ T_2 = -51.5 °C = 221.5 K

The moles per unit volume inside and outside are computed as follows

$$\left(\frac{n}{V}\right)_1 = \frac{P_1}{RT_1} = \frac{0.101\times10^6 \text{ Pa}}{(8.315 \text{ J/mol}\cdot\text{K})(293 \text{ K})} = 41.5 \text{ mol/m}^3$$

$$\left(\frac{n}{V}\right)_2 = \frac{P_2}{RT_2} = \frac{0.015\times10^6 \text{ Pa}}{(8.315 \text{ J/mol}\cdot\text{K})(221.5 \text{ K})} = 8.14 \text{ mol/m}^3$$

However, the problem asks for a comparison (or how does one relate to another). Dividing the two gives,

$$\frac{\left(n/V\right)_2}{\left(n/V\right)_1} = \frac{8.14 \text{ mol/m}^3}{41.5 \text{ mol/m}^3} = 0.19 \qquad \text{OR} \qquad \frac{\left(n/V\right)_1}{\left(n/V\right)_2} = \frac{41.5 \text{ mol/m}^3}{8.14 \text{ mol/m}^3} = 5.1$$

This means that **there are about one-fifth (0.2) as many moles of air per unit volume *outside* the cabin** (low pressure, high-altitude) **than at standard pressure** (sea level). Or, conversely, there are about 5 times as many moles of air per unit volume at standard pressure than at a pressure of 0.015 MPa.

(You do not need to plug in numbers until the final ratio; the numerical results of n/V inside and outside of the cabin are simply intermediate steps.)

Notes on the units: The units for "unit volume" are somewhat arbitrary and will not change the final ratio. However, they will affect the intermediate calculations. For instance, if you used volume units of liters, the intermediate calculations would be reduced by a factor of 10^{-3} m^3 (= 1 L). In any case, whether you use m^3, L, or even c.c. for unit volume, you must specify which you chose in any intermediate calculations.

Problem 3 (25 pts)

N_2 gas has a molar mass of 28.0 g/mol, and 1 mole = 6.03×10^{23} particles. What is the root-mean-square (rms) speed of an N_2 molecule inside your lungs at a temperature of 98.6 °F?

Solution: The rms velocity of a molecule is given by

$$v_{\text{rms}} = \sqrt{\frac{3k_B T}{m}}$$

where $k_B = 1.38 \times 10^{-23}$ J/K (Boltzmann's constant), T is in Kelvin, and m is in kg.

We are given a temperature in °F and so need to convert it to Kelvin:

$$T_C = \tfrac{5}{9}\left(T_F - 32°\right) = \tfrac{5}{9}\left(98.6° - 32°\right) = 37\ °\text{C}$$

$$T_K = T_C + 273 = 37 + 273 = \mathbf{310\ K}$$

We are not given the mass of the molecule, but rather the mass of 1 mole of molecules. The molar mass can be converted to mass of one molecule as follows:

$$m = \frac{28.0\text{ g}}{\text{mol}} \cdot \frac{1\text{ mol}}{6.02 \times 10^{23}\text{ molecules}} \cdot \frac{1\text{ kg}}{1000\text{ g}} = \mathbf{4.65 \times 10^{-26}\ kg/molecule}$$

Now, inserting known quantities into the equation gives,

$$v_{\text{rms}} = \sqrt{\frac{3k_B T}{m}} = \sqrt{\frac{3\left(1.38 \times 10^{-23}\text{ J/K}\right)\left(310\text{K}\right)}{4.65 \times 10^{-26}\text{ kg}}} = \mathbf{525\ m/s}$$

Notes on the units: The units reduce as follows:

$$v_{\text{rms}} = \sqrt{\frac{3k_B T}{m}} = \sqrt{\frac{(\text{J/K})(\text{K})}{\text{kg}}} = \sqrt{\frac{\text{J}}{\text{kg}}} = \sqrt{\frac{\text{N}\cdot\text{m}}{\text{kg}}} = \sqrt{\frac{\text{kg}\cdot\text{m}\cdot\text{m}}{\text{s}^2\cdot\text{kg}}} = \sqrt{\frac{\text{m}^2}{\text{s}^2}} = \frac{\text{m}}{\text{s}}$$

In order for the units to reduce in this manner, the mass and temperature must be in kg and Kelvin, respectively.

Problem 4 (25 pts)

From a physicist's point of view, the fundamental reason that a person's body temperature increases above normal is that the body contains an excess amount of energy that it was not able to get rid of through normal physiological functions (such as sweating). The enzymes in a human being begin to break down chemically when the body temperature reaches about 108 °F. Normal body temperature is about 98.6 °F.

The average specific heat capacity for a human being is 3.47 kJ/kg·K. Suppose a 143-kg (320-lb) football player's body temperature has increased to 108 °F. What excess energy is still in his body?

Solution: The excess energy in the football player's body is the energy that you would need to remove to cool off the player to body temperature of 98.6 °F = 37 °C (see previous problem for calculation).

The amount of energy needed to change the temperature of some mass is given by

$$Q = mc\Delta T$$

where m is the mass, c is the specific heat of the mass, and both are given.

The change in temperature must be in Kelvin (in order to cancel properly with the specific heat). A change in temperature is the same in Kelvin and in °C, so we need to find the initial temperature in °C, at least:

$$T_C = \tfrac{5}{9}\left(T_F - 32°\right) = \tfrac{5}{9}\left(108° - 32°\right) = 42.2\ °\text{C}$$

Now, the change in temperature is

$$\Delta T = (42.2 - 37) = 5.2\ \text{K}$$

Inserting known quantities into the equation gives,

$$Q = mc\Delta T = \left(143\ \text{kg}\right)\left(3.47\ \text{kJ/kg}\cdot\text{K}\right)\left(5.2\ \text{K}\right)$$

$$= \mathbf{2580\ kJ} = 2.6\ \text{MJ}$$

Or, in terms of kilocalories,

$$Q = \frac{2580\ \text{kJ}}{} \cdot \frac{1\ \text{kcal}}{4.186\ \text{kJ}} = 616\ \text{kcal}$$

Problem 5 (25 pts)

Suppose you have a 143-kg person whose body temperature is 43 °C, and you want to cool him down to a normal body temperature of 37 °C by dropping him into a tub containing 1000 kg of pure water. What would the initial temperature of the water have to be in order for both systems (person + water) to reach the final desired temperature of 37 °C?

Solution: This problem describes two bodies in thermal contact that reach thermal equilibrium. Conservation of Energy can be used to solve the problem.

Conservation of energy states,

$$\Delta Q_P + \Delta Q_W = 0$$

where the person and water are denoted by "*P*" and "*W*" subscripts, respectively.

The change in heat is due to changes in temperature of each body. This is stated as,

$$\Delta Q_P = m_P c_P \Delta T_P \qquad \Delta Q_W = m_W c_W \Delta T_W$$

The initial temperatures of the person and water are T_P and T_W. Since the two bodies are in thermal equilibrium, their final temperatures are the same, T_f.

Plugging the above relations into the conservation of energy equation gives,

$$m_P c_P \Delta T_P + m_W c_W \Delta T_W = 0$$

$$m_P c_P \left(T_f - T_P\right) + m_W c_W \left(T_f - T_W\right) = 0$$

We wish to determine the initial temperature of the water, T_W. So, solving the above equation for the unknown, T_W, yields,

$$T_W = \frac{m_P c_P \left(T_f - T_P\right)}{m_W c_W} + T_f$$

Inserting known and given quantities gives,

$$T_W = \frac{(143\ \text{kg})(3.47\ \text{J/kg}\cdot\text{K})\left(37 - 43\ ^\circ\text{C}\right)}{(1000\ \text{kg})(4.186\ \text{J/kg}\cdot\text{K})} + 37\ ^\circ\text{C}$$

$$= \mathbf{36.3\ ^\circ C}$$