Problem 1 (25 pts)

When you first show up at the after-final party, you notice that there is a Helium balloon with a radius of 18.0 cm. Before the host can actually find a radio station playing music, you hear the weather reporter say that the outside temperature is 74 °F (23 °C). Several hours later, you look up at the same balloon and notice that the radius has now decreased to 17.0 cm.

Assuming that none of the He leaked out, what is the temperature in °F outside now?

$r_1 = 18.0\text{ cm} = 0.18\text{ m}$

$T_1 = 23°C = 296\text{ K}$

$r_2 = 17.0\text{ cm}$
$= 0.17\text{ cm}$

°C

$PV = nRT$

Pressure is constant $(P_1 = P_2)$

moles is constant "nothing leaked out" $n_1 = n_2$

$$\Rightarrow \frac{V}{T} = \frac{nR}{P} = \text{constant}$$

$$\Rightarrow \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\frac{\cancel{\frac{4}{3}\pi} r_1^3}{T_1} = \frac{\cancel{\frac{4}{3}\pi} r_2^3}{T_2}$$

Note: The shape of the balloon doesn't really matter, but most will assume it is a sphere:

$$V_{sphere} = \frac{4}{3}\pi r^3$$

$$\Rightarrow T_2 = T_1\left(\frac{r_2}{r_1}\right)^3$$

$$= (296\text{ K})\left(\frac{17}{18}\right)^3 = \boxed{249\text{ K} = T_2}$$

must be in Kelvin (-2 otherwise)

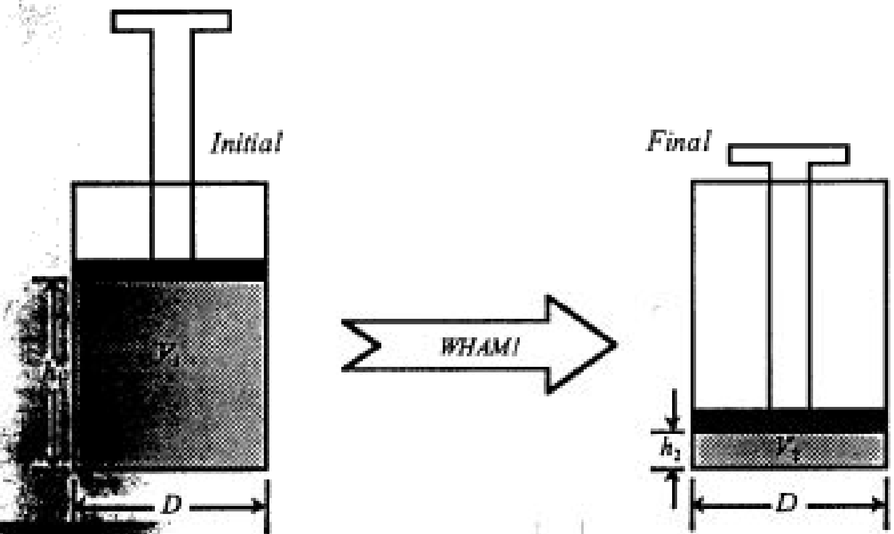
$T_2 = 249\text{K} \rightarrow -24°C \rightarrow 32° + \frac{9}{5}(-24°C) = -11°F$

$$\boxed{T_2 = -11\text{ °F}}$$

Final Exam CODE #________

Problem 2 (35 pts total)

One day Andrew comes around to your lab table with a cylindrical fire syringe, and he shows you he can fry things with it. Based on the diagram and quantities given below, you will calculate what was occurring. (Note: The diagram below shows the cross-section of the cylindrical syringe.)



h_1 = 20.0 mm
h_2 = 5.0 mm
D = 5.0 mm

(a) What was the initial volume of air inside the tube after he first put the plunger in? (6 pts)

$$V_{cylinder} = \pi r^2 h$$
$$= \frac{\pi D^2}{4} h_1 = \frac{\pi (5\,mm)^2}{4} (20\,mm)$$
$$\boxed{V_1 = 393\ mm^3} = 3.93 \times 10^{-7}\ m^3$$

(b) What was the final volume of air inside the tube after he pushed the plunger down? (6 pts)

$$V = \frac{\pi D^2}{4} h_2 = \pi \frac{(5\,mm)^2}{4} (5\,mm) = \boxed{98.2\ mm^3 = V_2}$$

(c) This volume change occurs over a very short period of time. Which of the four basic thermodynamic processes would you classify this as? Why? (4 pts)

Adiabatic: The process occurs so quickly that no heat is transferred (via conduction, etc.) out of the syringe. (Adiabatic $\Leftrightarrow$ $Q = 0$)

Final Exam

CODE #__________

Problem 2 (cont'd):

(d) What were the initial temperature (in K) and pressure inside the tube when he first put the plunger in? Assume the temperature is San Diego's average of 25 °C. (4 pts)

Initial pressure is atmospheric: $\boxed{P_1 = 1\text{ atm}} = 1.01\times10^5\text{ Pa}$

$\boxed{T_1 = 25°C = 298\text{ K}}$

(e) What was the final temperature of the air ($\gamma = 5/3$) after the plunger was pushed down? (12 pts)

Adiabatic process:

$$T^{3/2}V = \text{constant}$$

$$\Rightarrow T_1^{3/2}V_1 = T_2^{3/2}V_2$$

$$T_2^{3/2} = T_1^{3/2}\left(\frac{V_1}{V_2}\right)$$

$$\Rightarrow T_2 = T_1\left(\frac{V_1}{V_2}\right)^{2/3}$$

must be in Kel

$$= (298\text{ K})\left(\frac{393}{98.2}\right)^{2/3}$$

$$\boxed{T_2 = 751\text{ K}}$$

see back for an alternate solution

(f) What is this temperature in degrees Fahrenheit? (3 pts)

$T_2 = 751\text{ K} \rightarrow 478\ °C \rightarrow 32 + \frac{9}{5}(478°C)$

$\boxed{T_2 = 892\ °F}$

Adiabatic process:

$PV^{\gamma} = \text{constant}$

$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$ but P_2 is not known or desired

$PV = nRT$ (always) $\Rightarrow P_1 = \dfrac{n_1 R T_1}{V_1}$ and $P_2 = \dfrac{n_2 R T_2}{V_2}$ (but $n_1 = n_2$ since no gas escapes)

$$\left(\frac{nRT_1}{V_1}\right) V_1^{\gamma} = \left(\frac{nRT_2}{V_2}\right) V_2^{\gamma}$$

$$T_1 \frac{V_1^{\gamma}}{V_1} = T_2 \frac{V_2^{\gamma}}{V_2} \Rightarrow T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \Rightarrow T_2 = T_1 \frac{V_1^{\gamma-1}}{V_2^{\gamma-1}}$$

now, $\gamma = 5/3$

$T_1 = 298\text{ K}$

$V_1 = 393\text{ mm}^3$

$V_2 = 98.2\text{ mm}^3$

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1} = (298\text{ K})\left(\frac{393}{98.2}\right)^{5/3-1}$$

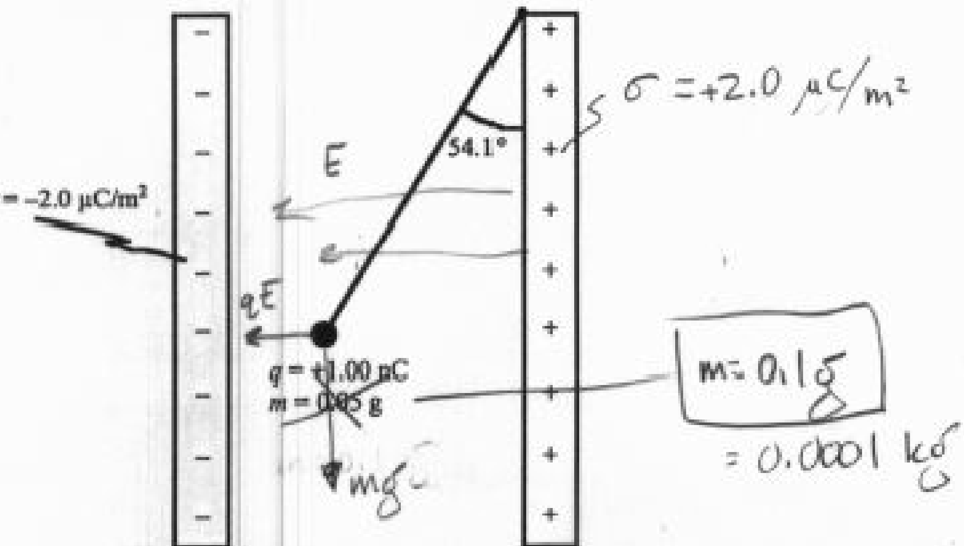
$$\boxed{T_2 = 751\text{ K}}$$

Final Exam

CODE #________

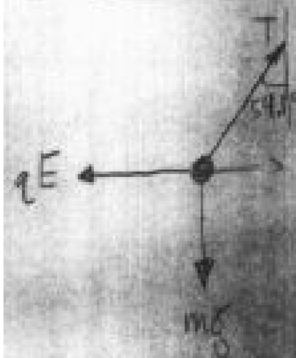
Problem 3 (30 pts)

Your friend Luna, who lives on the Moon, is trying to figure out the acceleration due to gravity in her part of the universe. Not surprisingly, you suggest that she try an experiment by hanging a pith ball on a massless string between two equally and oppositely charged parallel plates (as shown below). Luna finds that the pith ball hangs at an angle of $\theta = 54.1°$ from the vertical.



What is the acceleration due to gravity on the Moon? Show all your work. (Simply writing an equation down is not sufficient for the solution to this problem.) Use the back of this page if necessary.

① FREE-BODY DIAGRAM



② FORCE BALANCE

horz: $qE = T\sin 54.1°$

$E = \frac{\sigma}{\varepsilon_0}$ (for a parallel-plate configuration)

$\Rightarrow \boxed{\frac{q\sigma}{\varepsilon_0} = T\sin\theta}$ vert: $\boxed{mg = T\cos\theta}$

③ SOLVE FOR UNKNOWN

Dividing the equations gives:

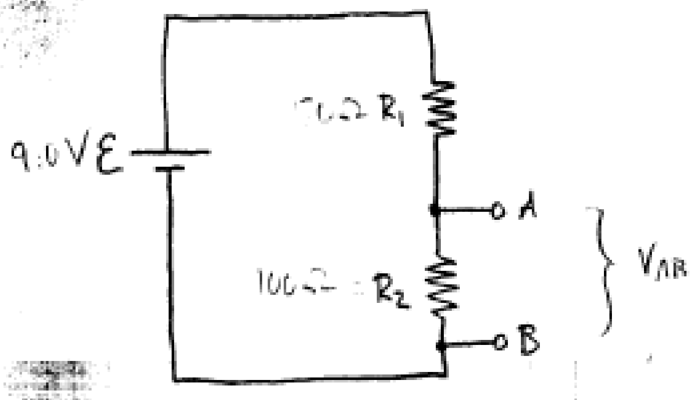
$$\frac{T\sin\theta = \frac{q\sigma}{\varepsilon_0}}{T\cos\theta = mg} \Rightarrow \tan\theta = \frac{q\sigma}{mg\varepsilon_0} \Rightarrow \boxed{g = \frac{q\sigma}{m\varepsilon_0 \tan\theta}}$$

$$g = \frac{(1\times10^{-9}\,C)(2\times10^{-6}\,C/m^2)}{(.0001\,kg)(8.85\times10^{-12}\,C^2/N\cdot m^2)\tan 54.1}$$

$$\boxed{g_{moon} = 1.64\,m/s^2}$$

Problem 4 (30 pts total)

Given the following circuit where $R_1 = 50\ \Omega$, $R_2 = 100\ \Omega$, and the *emf* of the ideal battery is 9.0 V.



(a) What is the voltage from terminal A to B? (Hint: This is just the voltage drop across R_2.) (10 pts)

$V_{AB} = IR_2$ (the voltage drop across R_2)

$I = \frac{V_{tot}}{R_{tot}} = \frac{\mathcal{E}}{R_1 + R_2}$ (since R_1 and R_2 are in series)

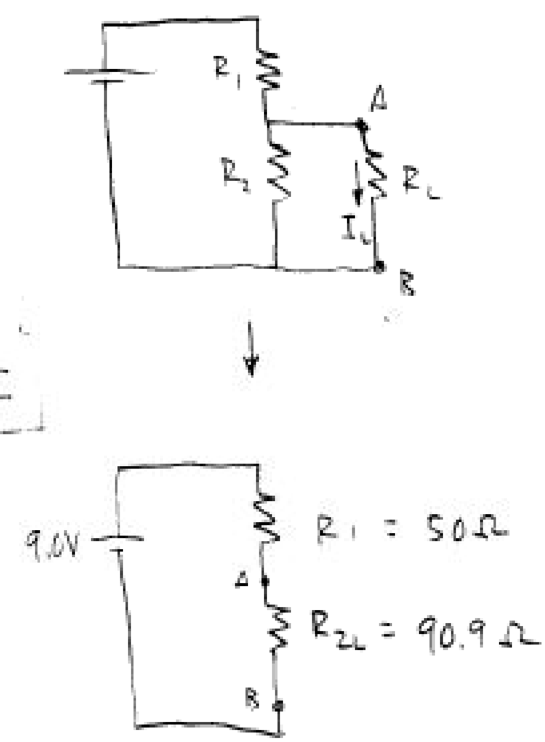
$\rightarrow V_{AB} = \left(\frac{\mathcal{E}}{R_1 + R_2}\right) R_2$

$= \frac{(9.0\text{V})}{(100+50)}(100) = \boxed{6.0\ \text{V} = V_{AB}}$

Problem 4 continued:

(b) Suppose now that a load resistor $R_L = 1.0\ \text{k}\Omega$ is connected across terminals A and B. What is the voltage across this load resistor? (15 pts)

Now, $V_{AB} = I R_{2L}$



$$\frac{1}{R_{2L}} = \frac{1}{R_2} + \frac{1}{R_L} = \frac{1}{100} + \frac{1}{1000}$$

$$\boxed{R_{2L} = 90.9\ \Omega}$$

$$\frac{V_{tot}}{R_{tot}} = \frac{9.0\text{V}}{141\ \Omega} = \boxed{0.064\text{ A} = I}$$

$$V_{AB} = I R_{2L}$$
$$= (0.064\text{ A})(90.9\ \Omega)$$

$$\boxed{V_{AB} = 5.82\text{ V}}$$

(which is less than V_{AB} before the load was added)

(c) What is the power dissipated by the load resistor? (5 pts)

$$P = V_{AB} I_L = \frac{V_{AB}^2}{R_L} = \frac{(5.82\text{ V})^2}{1\text{k}\Omega}$$

$$\boxed{P = 0.034\text{ W}}$$

Or, they can calculate the current, I_L:

$$I_L = \frac{V_{AB}}{R_L} = \frac{5.82\text{ V}}{1000\ \Omega} = 0.0058\text{ A}$$

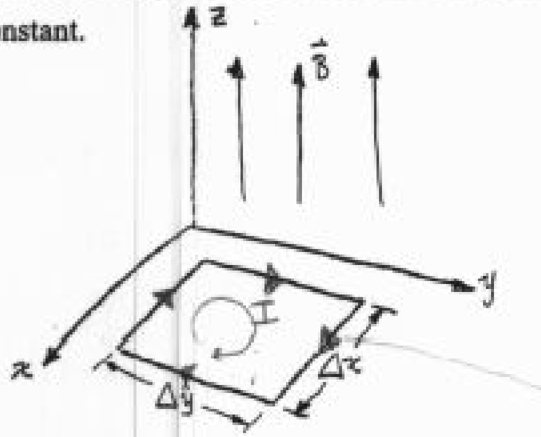
$$P = I_L^2 R_L = (0.0058\text{A})^2 (1000\ \Omega) = \boxed{0.034\text{ W} = P}$$

Final Exam

CODE #________

Problem 5 (30 pts total)

Suppose there is a rectangular loop of wire (with side lengths Δx and Δy) lying flat on the horizontal xy-plane as shown below. There is a uniform magnetic field directed in the positive z-direction whose magnitude is $B(t) = kt^2$, where k is a constant.



since $\vec{B}$ varies with time

(a) What is the magnitude of the induced *emf* in the wire? (25 pts)

Induced emf:

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \quad \text{where} \quad \Phi_B = \int \vec{B} \cdot d\vec{A} = B(t)\,A$$

$$\mathcal{E} = -\frac{d}{dt}(B(t)A) = -A\frac{d}{dt}B(t) = -A\frac{d}{dt}kt^2$$

$A = \Delta x \Delta y$

The magnitude of $\mathcal{E}$ is

$$\mathcal{E} = 2(\Delta x)(\Delta y)kt$$

$$\boxed{\mathcal{E} = -2Akt}$$

$$\boxed{\mathcal{E} = -2(\Delta x)(\Delta y)kt}$$

The signed magnitude also indicates orientation, so $\mathcal{E} = -2(\Delta x)(\Delta y)kt$ is also okay

(b) In what direction is the induced current in the wire? (5 pts)

Since the existing B-field is increasing upward, the induced current must create a B-field that opposes the existing B-field. So, the induced current creates a downward B-field which, by the right-hand rule, means the current must be CLOCKWISE. (see picture)