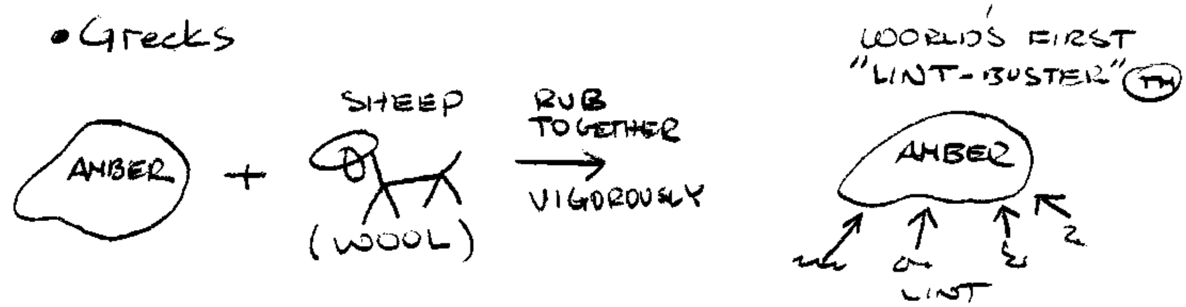
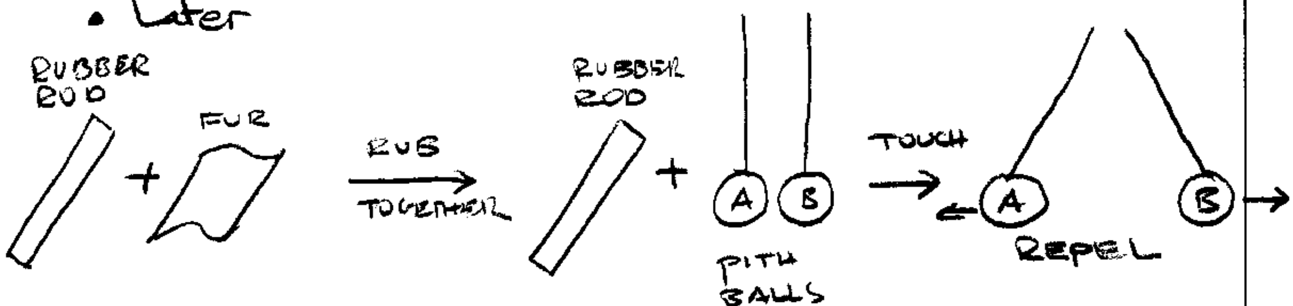
ELECTROSTATICS

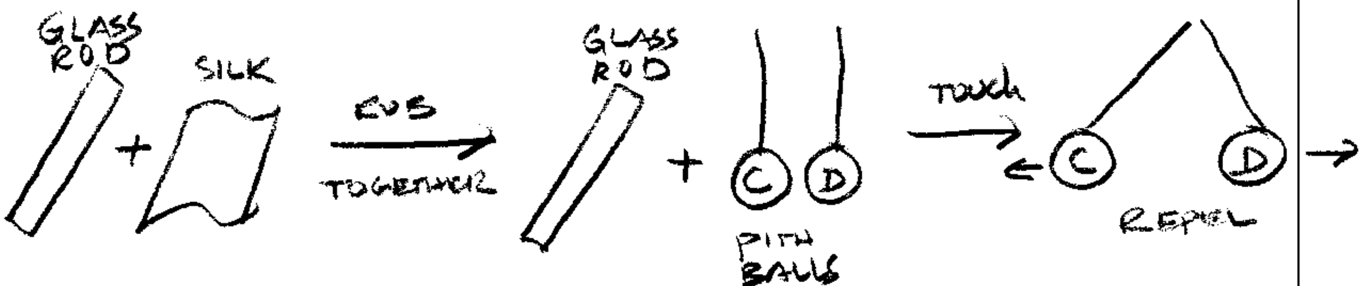
I. TWO KINDS OF CHARGE

• Greeks

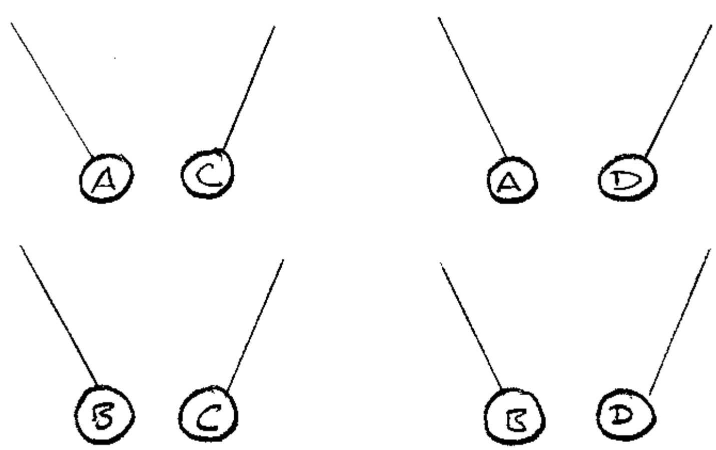


• Later





But Now,



ATTRACTION BETWEEN RUBBER/GLASS SETS.

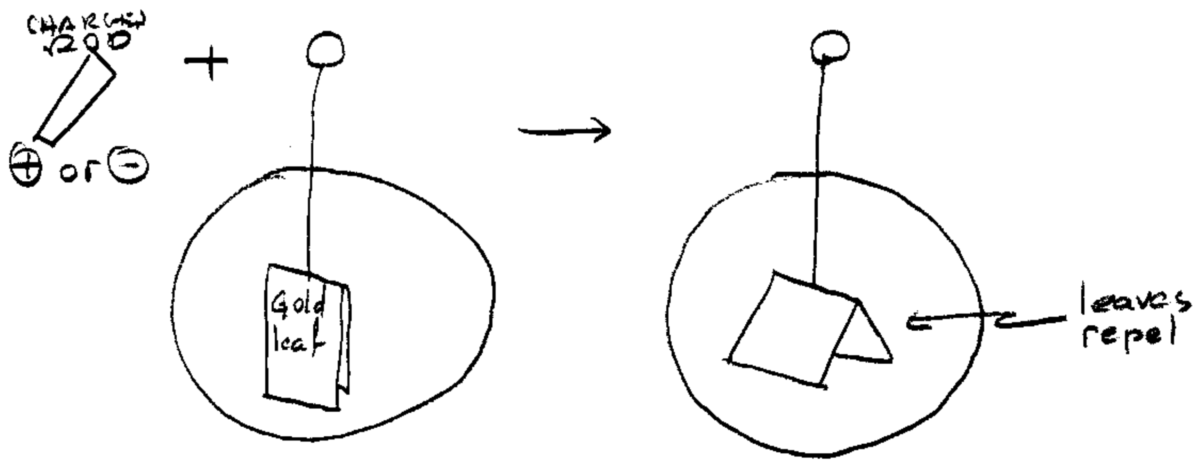
ELECTROSTATICS

- DO this WITH ALL SORTS OF THINGS AND FIND THE SAME BEHAVIOR ALL THE TIME (ATTRACTION / REPULSION / NOTHING), NEVER ANYTHING ELSE (LIKE JUMPING VERTICALLY, ETC).

- CONCLUSION: TWO TYPES OF CHARGE (OR NO CHARGE)

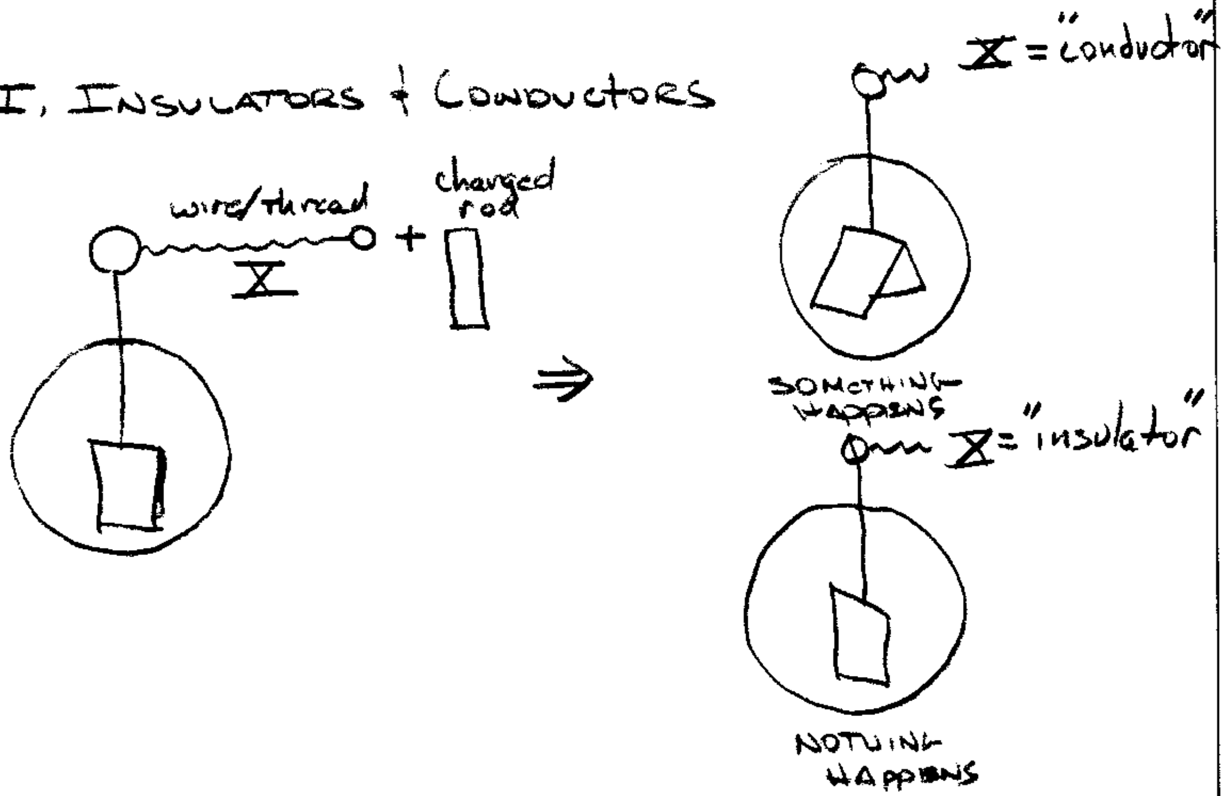
- By CONVENTION, CALL THE CHARGE THAT MAKES THINGS BEHAVE IN THE SAME WAY AS THE RUBBER ROD "NEGATIVE". ALL ELSE CALL "positive" (or neutral)

- LEAF ELECTROSCOPE



ELECTROSTATICS

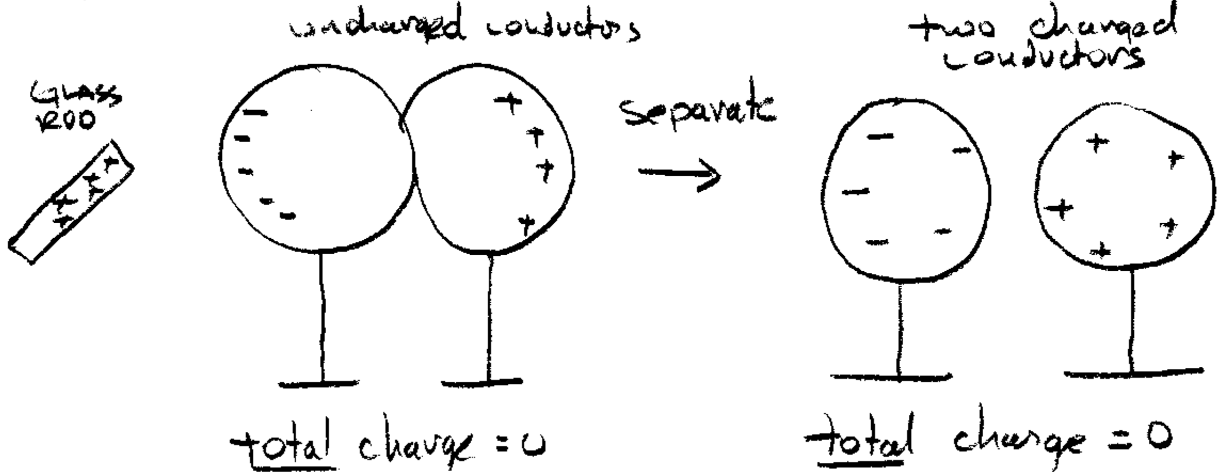
II. INSULATORS & CONDUCTORS



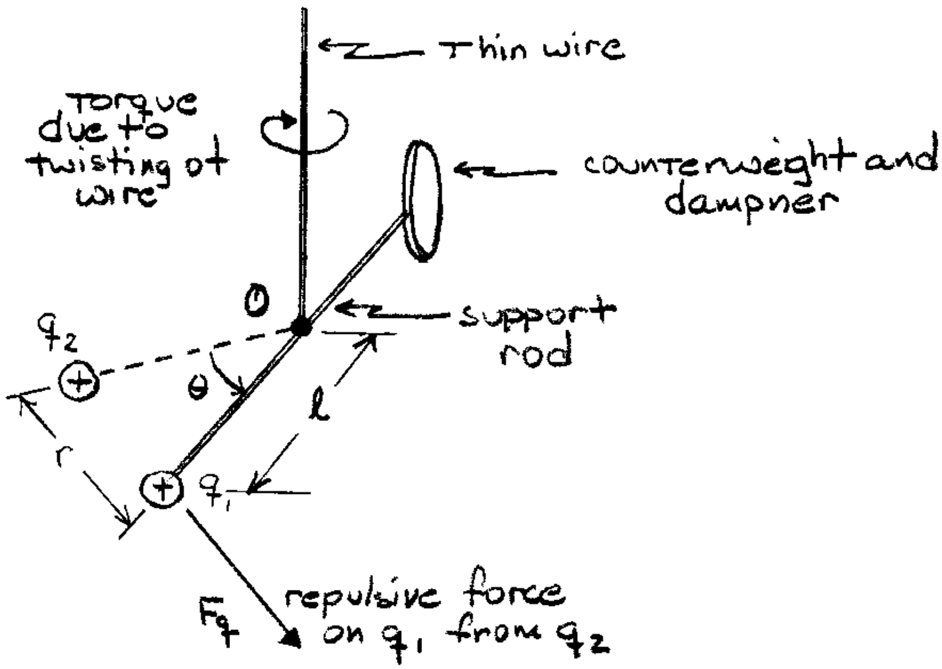
- Conductors can conduct charge from one place to another
- Insulators can not

(SEMI-CONDUCTORS later).

III. ELECTROSTATIC INDUCTION



III. COULOMB'S LAW



- Charges q_1 and q_2 repel each other. q_2 is fixed in space. q_1 is attached by a thin rod to a wire which it twists as q_1 is pushed-away by the force between q_1 and q_2. This force F_q exerts a torque around point O

$$\tau_1 = F\ell$$

- The wire exerts an opposite torque depending on how "strong" the wire is and the angle of twist, θ

$$\tau_2 = -k\theta$$

- When q_1 is in <u>equilibrium</u>, then

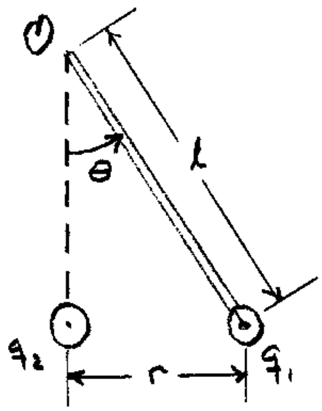
$$\tau_1 + \tau_2 = 0$$

$$\tau_1 = -\tau_2$$

$$F\ell = k\theta$$

$$F = \frac{k}{\ell}\theta$$

So, we can measure l and θ, and determine k by other means. This allows us to quantitativly measure the force between two charges as a function of θ, which we can translate into a function of r, the distance between the charges



$r = l \sin\theta$

Experimentally, Coulomb was able to determine that

(1) $F \sim q_1 q_2$

and

(2) $F \sim \frac{1}{r^2}$

Combining these two relations, and introducing a proportionality constant, we have

$$\boxed{F = K_0 \frac{q_1 q_2}{r^2}}$$

Coulomb's Law

where $\boxed{K_0 = 8.988 \times 10^9 \frac{N \cdot m^2}{q^2}}$ (IN VACUUM)

$q \equiv$ a unit of charge called the "Coulomb"

YOU'LL ALSO SEE K_0 EXPRESSED AS

$$\boxed{K_0 = \frac{1}{4\pi\epsilon_0}}$$

OR COULOMB'S LAW AS

$$\boxed{F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}}$$

WHERE

$\epsilon_0 \equiv$ "PERMITIVITY OF FREE SPACE"
$= 8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2}$

LATER ON (IN PHYSICS 1C), WE'LL FIND THAT CHARGE IS QUANTISED (IT COMES PACKAGED IN DISCRETE AMOUNTS). WE'LL THEN TAKE THE FUNDAMENTAL UNIT OF CHARGE AS THE AMOUNT OF CHARGE POSSESSED BY ONE ELECTRON. IT'S MAGNITUDE, TRANSLATED INTO COULOMB'S (A UNIT OF CHARGE MADE-UP BEFORE ELECTRONS WERE DISCOVERED) IS

$$\boxed{e \equiv 1.602 \times 10^{-19} C}$$

FUNDAMENTAL UNIT (quantum) OF CHARGE.

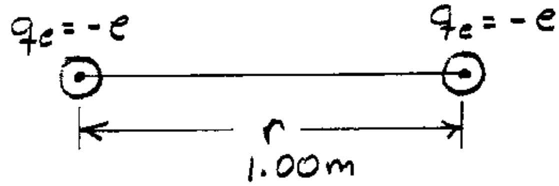
ELECTRONS HAVE A NEGATIVE AMOUNT OF THIS CHARGE

$$q_e = -e = -1.602 \times 10^{-19} C$$

Protons have a positive amount OF THIS CHARGE

$$q_p = +e = +1.602 \times 10^{-19} C$$

$\boxed{\text{EX:}}$ FORCE BETWEEN TWO ELECTRONS 1.00M APART



$$F = K_0 \frac{q_1 q_2}{r^2}$$

$$= K_0 \frac{(-e)^2}{r^2}$$

$$= K_0 \frac{e^2}{r^2}$$

$$= \left(9.0 \times 10^{9} \frac{N \cdot m^2}{C^2}\right) \frac{(1.602 \times 10^{-19} C)^2}{(1.00\,m)^2}$$

$$\boxed{F = 2.3 \times 10^{-28} N}$$

WOW! AT FIRST GLANCE, THAT'S A REALLY TINY FORCE. ONE MIGHT THINK THAT IT'S A NEGLIGIBLE FORCE, BUT IT'S DEFINITELY NOT!

THE MASS OF AN ELECTRON IS

$$m_e = 9.11 \times 10^{-31}\,kg$$

LET'S CALCULATE ITS ACCELERATION UNDER THIS FORCE:

$$F = ma$$

$$a = F/m_e$$

$$= (2.3 \times 10^{-28} N)/(9.11 \times 10^{-31} kg)$$

$$\boxed{a = 250\ m/s^2}$$

WHICH IS CERTAINLY NOT A NEGLIGIBLE ACCELERATION.

Ex:

LET'S COMPARE THIS FORCE OF ELECTROSTATIC (COULOMB) REPULSION TO THE GRAVITATIONAL FORCE OF ATTRACTION BETWEEN THE TWO ELECTRONS:

$$F_c = k_0 \frac{q_1 q_2}{r^2}$$

$$F_G = G_N \frac{m_1 m_2}{r^2}$$

$$\frac{F_c}{F_G} = k_0 \frac{q_1 q_2}{r^2} \cdot \frac{r^2}{G_N m_1 m_2}$$

$$= \frac{k_0 e^2}{G_N m_e^2}$$

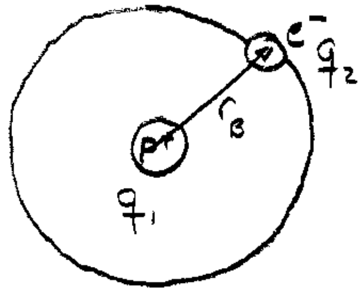
$$= \left(\frac{k_0}{G_N}\right)\left(\frac{e}{m_e}\right)^2$$

$$= \frac{\left(9.0 \times 10^{9} \ \frac{N \cdot m^2}{C^2}\right)}{\left(6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2}\right)} \cdot \left(\frac{1.602 \times 10^{-19}\, C}{9.11 \times 10^{-31}\, kg}\right)^2$$

$$\boxed{\frac{F_c}{F_G} = 4.2 \times 10^{42}}$$

HOLY COW! ELECTROSTATIC FORCES ARE ABOUT 10^{42} TIMES LARGER THAN GRAVITATIONAL FORCES BETWEEN THESE PARTICLES.

$\boxed{\text{Ex1}}$ COULOMB FORCE BETWEEN PROTON AND ELECTRON IN HYDROGEN ATOM



(Simple Planetary Model of Hydrogen)

Radius of electron orbit in ^1_1H is called "The Bohr Radius"

$$r = r_B = 5.3 \times 10^{-11} m$$

$$F = k_0 \frac{q_1 q_2}{r^2}$$

$$= k_0 \frac{(e)(-e)}{r_B^2}$$

$$= -k_0 \frac{e^2}{r_B^2}$$

$$= -\left(9.0 \times 10^9 \frac{N \cdot m^2}{C^2}\right) \frac{(1.602 \times 10^{-19} C)^2}{(5.3 \times 10^{-11} m)^2}$$

$$\boxed{F = -8.2 \times 10^{-8} N}$$

NOTE THAT $F/m_e = (-8.2 \times 10^{-8} N) / (9.11 \times 10^{-31} kg)$

$$= 9.0 \times 10^{22} m/s^2$$

THAT ELECTRON MUST REALLY BE MOVING-AROUND FAST TO KEEP FROM FALLING INTO THE NUCLEUS.

IV. FORCE IS A VECTOR QUANTITY

- VECTORS HAVE BOTH MAGNITUDE AND DIRECTION

- SO FAR, WE'VE ONLY BEEN CALCULATING THE MAGNITUDE OF COULOMB FORCES. NOW WE NEED TO LOOK AT THE DIRECTION OF COULOMB FORCES. WE'LL NEED TO REFRESH OURSELVES ON VECTOR ANALYSIS TO DO THIS (PERHAPS FROM A DIFFERENT POINT-OF-VIEW), WITH A FEW EXAMPLES SPECIFIC TO COULOMB'S LAW.

- COULOMB'S LAW IN VECTOR FORM

$$\boxed{\vec{F} = k_0 \frac{q_1 q_2}{r^2} \hat{r}}$$

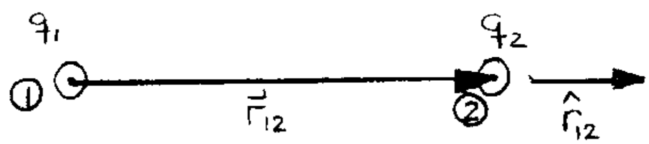
- MAGNITUDE OF FORCE IS

$$|\vec{F}| \equiv F = k_0 \frac{q_1 q_2}{r^2}$$

- DIRECTION OF FORCE

$$\mathrm{dir}(\vec{F}) = \hat{r}$$

WHERE $\hat{r}$ IS A UNIT VECTOR PARALLEL TO THE RADIUS VECTOR $\vec{r}$ JOINING THE TWO CHARGES



READ: $\vec{r}_{12}$ = "radius vector from ① to ②"

$\vec{F}_{12}$ = "Force of ① on ②"

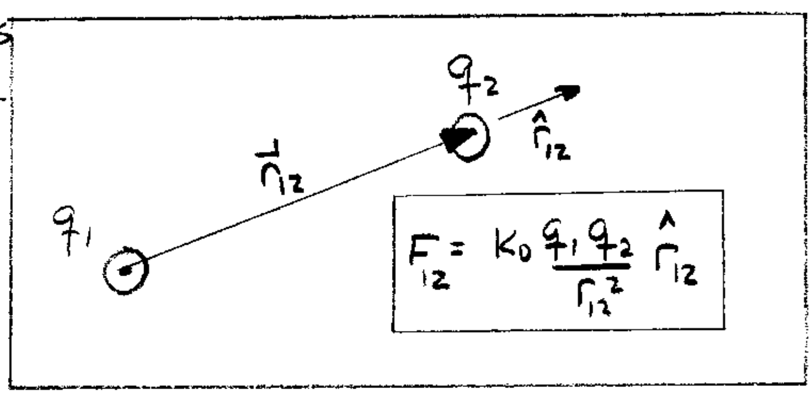
$$\vec{F}_{12} = \left(k_0 \frac{q_1 q_2}{r_{12}^2}\right) \hat{r}_{12}$$

NOTE THAT VECTORS ALLOW US TO EXPRESS PHYSICAL LAWS, SUCH AS COULOMB'S LAW, IN A VERY GENERAL WAY WITHOUT RESORT TO REFERENCE TO ANY PARTICULAR COORDINATE SYSTEM. THIS IS VERY POWERFUL.

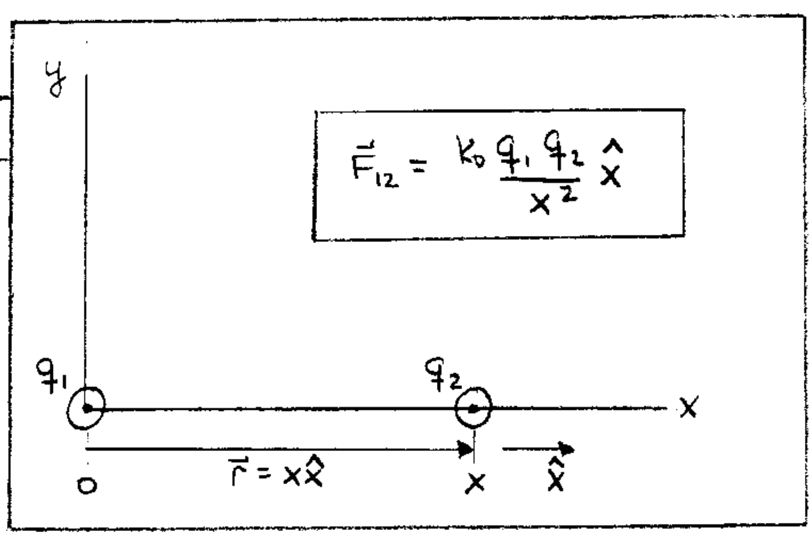
TO GET USED TO WORKING WITH MANIPULATING VECTORS, WE'LL TAKE A LOOK AT SOME SIMPLE EXAMPLES. WE WILL MAKE REFERENCE TO PARTICULAR COORDINATE SYSTEMS SINCE MANY ARE USED TO THIS. THE POINT IS TO FOCUS ON THE MORE POWERFUL TECHNIQUES OF VECTOR ANALYSIS, BUT TO SHOW IT IN-USE IN WHAT IS PROBABLY A MORE FAMILIAR SETTING (i.e., A SPECIFIC COORDINATE SYSTEM).

$\boxed{\text{Ex:}}$ COULOMB FORCE BETWEEN TWO CHARGES ORIENTED ARBITRARILY IN SPACE

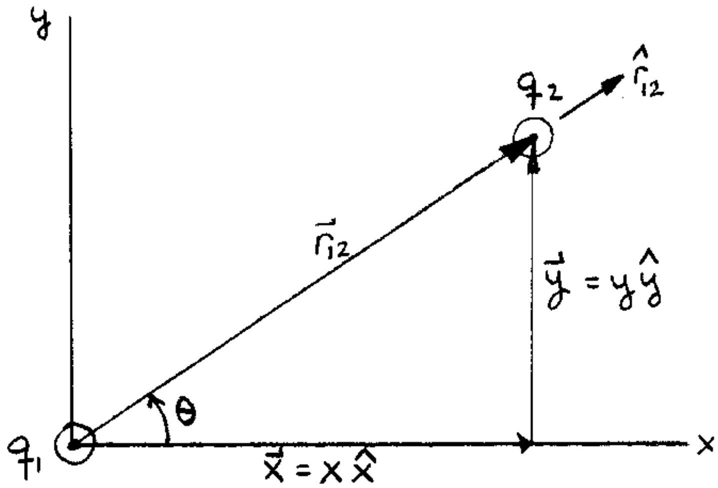
No Coordinates Necessary



SAME PROBLEM AFTER CHOOSING SOME CONVENIENT COORDINATE SYST.



$\boxed{\text{Ex:}}$ THE SAME PROBLEM WITH A DIFFERENT CHOICE OF COORDINATE SYSTEM.



- COULOMB'S LAW IS THE SAME:

$$\vec{F} = k_0 \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12}$$

- THIS CHOICE OF COORDINATE SYSTEM JUST MAKES ABOVE PROBLEM HARDER

$$\vec{r}_{12} = \vec{x} + \vec{y}$$
$$= x\hat{x} + y\hat{y}$$

$$r_{12} = |\vec{r}_{12}| = \left[\vec{r}_{12}\cdot\vec{r}_{12}\right]^{1/2} = \left[(x\hat{x}+y\hat{y})\cdot(x\hat{x}+y\hat{y})\right]^{1/2}$$
$$= \left[x^2(\hat{x}\cdot\hat{x}) + 2xy(\hat{x}\cdot\hat{y}) + y^2(\hat{y}\cdot\hat{y})\right]^{1/2}$$

But $\hat{x}\cdot\hat{x} = 1$, $\hat{x}\cdot\hat{y} = 0$, $\hat{y}\cdot\hat{y} = 1$

$$r_{12} = \left[x^2 + y^2\right]^{1/2} \quad \text{(Just Pythagorean Theorem)}$$

SO, $$\hat{r}_{12} = \frac{\vec{r}_{12}}{|\vec{r}_{12}|}$$

$$\hat{r}_{12} = \frac{(x\hat{x} + y\hat{y})}{[x^2+y^2]^{1/2}}$$

$$\vec{F} = K_0 \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12}$$

$$= \frac{K_0 q_1 q_2}{(x^2+y^2)} \left(\frac{x\hat{x} + y\hat{y}}{(x^2+y^2)^{1/2}} \right)$$

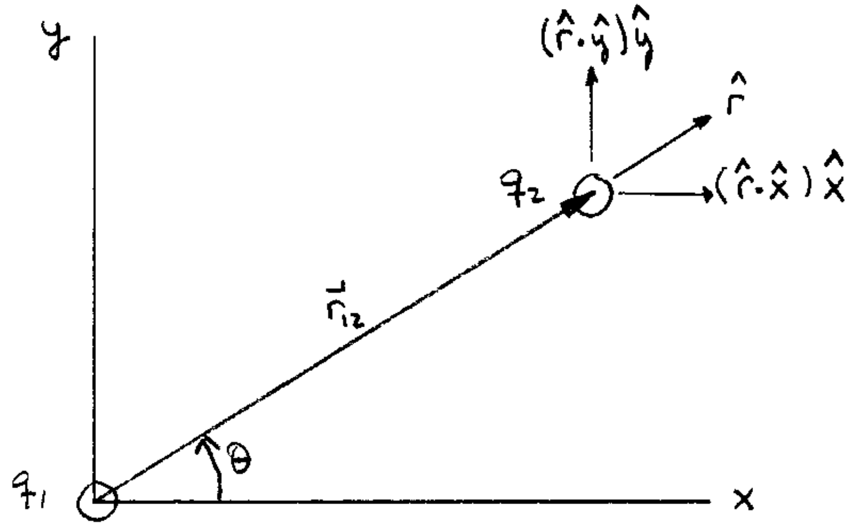
$$= \left(\frac{K_0 x q_1 q_2}{(x^2+y^2)^{3/2}} \right) \hat{x} + \left(\frac{K_0 y q_1 q_2}{(x^2+y^2)^{3/2}} \right) \hat{y}$$

OR $$\vec{F} = F^x \hat{x} + F^y \hat{y}$$

where $$F^x = \frac{K_0 q_1 q_2 x}{(x^2+y^2)^{3/2}} \qquad F^y = \frac{K_0 q_1 q_2 y}{(x^2+y^2)^{3/2}}$$

NOTICE THAT THIS SOLUTION IS EXACTLY THE SAME AS THE FIRST ONE, BUT IT LOOKS MORE COMPLICATED BECAUSE OF OUR CHOICE OF COORDINATE SYSTEM.

ALTERNATIVELY, WE COULD HAVE CHOSEN $r_{12} = |\vec{r}_{12}|$ AND θ AS COORDINATES —



$$\vec{F}(\vec{r}) = k_0 \frac{q_1 q_2}{r^2}\hat{r}$$

$$\vec{F}_x(\vec{r}) = (\vec{F}\cdot\hat{x})\hat{x}$$

$$= \left(k_0 \frac{q_1 q_2}{r^2}\hat{r}\cdot\hat{x}\right)\hat{x}$$

$$= \left(k_0 \frac{q_1 q_2}{r^2}\cos\theta\right)\hat{x}$$

$$\vec{F}_y(\vec{r}) = (\vec{F}\cdot\hat{y})\hat{y}$$

$$= \left(k_0 \frac{q_1 q_2}{r^2}\hat{r}\cdot\hat{y}\right)\hat{y}$$

$$= \left(k_0 \frac{q_1 q_2}{r^2}\sin\theta\right)\hat{y}$$

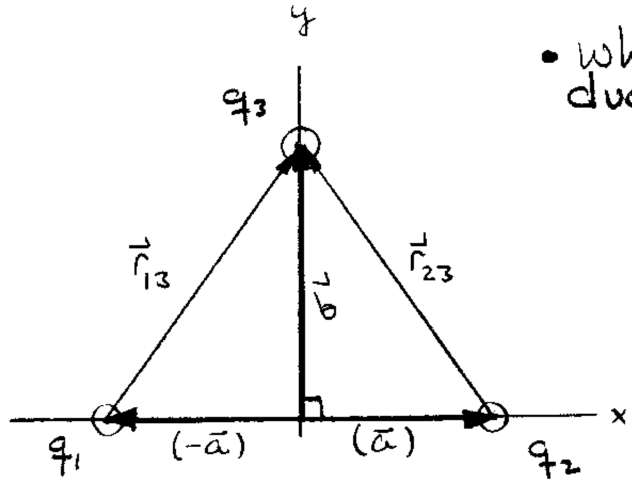
So, $\vec{F} = \vec{F}_x + \vec{F}_y$

$$= \left(k_0 \frac{q_1 q_2}{r^2}\cos\theta\right)\hat{x} + \left(k_0 \frac{q_1 q_2}{r^2}\sin\theta\right)\hat{y}$$

$$F_x = k_0 \frac{q_1 q_2}{r^2}\cos\theta \qquad F_y = k_0 \frac{q_1 q_2}{r^2}\sin\theta$$

(SAME PROBLEM - CHOICE OF COORDINATES MAKES IT LOOK DIFFERENT.)

$\boxed{\text{Ex:}}$ THREE CHARGES ARRANGED AT VERTICES OF ISOCELES TRIANGLE (& LINEAR SUPERPOSITION)



• What's the force on q_3 due to q_1 & q_2?

• Consider forces $\vec{F}_{13}$ & $\vec{F}_{23}$ separately, then add them to get total resultant force.

$$\vec{F}_{13} = k_0 \frac{q_1 q_3}{r_{13}^2} \hat{r}_{13}$$

$$\vec{F}_{23} = k_0 \frac{q_2 q_3}{r_{23}^2} \hat{r}_{23}$$

$$\vec{r}_{13} = \vec{b} - (-\vec{a}) = \vec{b} + \vec{a}$$

$$r_{13}^2 = \vec{r}_{13} \cdot \vec{r}_{13} = (\vec{b}+\vec{a}) \cdot (\vec{b}+\vec{a})$$
$$= b^2 + a^2 + 2\vec{b}\cdot\vec{a}$$
$$= a^2 + b^2 \qquad (\vec{b} \perp \text{ to } \vec{a}, \text{ so } \vec{b}\cdot\vec{a} = 0)$$

$$\vec{r}_{23} = \vec{b} - \vec{a}$$

$$r_{23}^2 = (\vec{b} - \vec{a}) \cdot (\vec{b} - \vec{a})$$
$$= b^2 + a^2 - 2\vec{a}\cdot\vec{b}$$
$$= a^2 + b^2$$

$$\hat{r}_{13} = \frac{\vec{r}_{13}}{|\vec{r}_{13}|} = \frac{(\vec{a}+\vec{b})}{(a^2+b^2)^{1/2}}$$

$$\vec{r}_{23} = \frac{\vec{r}_{23}}{|\vec{r}_{23}|} = \frac{(\vec{b}-\vec{a})}{(a^2+b^2)^{1/2}}$$

$$\vec{F} = \vec{F}_{13} + \vec{F}_{23}$$

$$= \left[\frac{k_0 q_1 q_3}{(a^2+b^2)}\right]\left[\frac{\vec{a}+\vec{b}}{(a^2+b^2)^{1/2}}\right] + \left[\frac{k_0 q_2 q_3}{(a^2+b^2)}\right]\left[\frac{-\vec{a}+\vec{b}}{(a^2+b^2)^{1/2}}\right]$$

$$\boxed{\vec{F} = \frac{k_0 q_3}{(a^2+b^2)^{3/2}}\left[(q_1-q_2)\vec{a} + (q_1+q_2)\vec{b}\right]}$$

NOTE THAT THE VECTORS $\vec{a}$ AND $\vec{b}$ ARE STILL COMPLETELY GENERAL. BUT, IF WE EXPRESS THEM IN TERMS OF x AND y COORDINATES AS DRAWN (CHOSEN), WE HAVE

$$\vec{a} = a\hat{x} \qquad \vec{b} = b\hat{y}$$

SO,

$$\boxed{\vec{F} = \frac{k_0 q_3}{(a^2+b^2)^{3/2}}\left[a(q_1-q_2)\hat{x} + b(q_1+q_2)\hat{y}\right]}$$

OKAY. SO IT LOOKS LIKE WE'VE DONE A LOT OF UNNECESSARY MATH-STUFF FOR THIS PROBLEM, BUT THAT'S NOT ENTIRELY TRUE. THIS EXPRESSION IS CAPABLE OF ANSWERING MANY DIFFERENT QUESTIONS ABOUT CHARGES DISTRIBUTED THIS WAY.

FOR EXAMPLE:

- If $q_1 = q_2$, there is no x-component of force on q_3, and all of the force is parallel (or antiparallel) to the y-axis.
- If $q_1 = -q_2$, there is no y-component of force on q_3, etc.
- The angle of the total resultant force with respect to the x-axis is given generally by

$$\theta = \tan^{-1}\left[\frac{F_y}{F_x}\right]$$

$$\theta = \tan^{-1}\left[\frac{b(q_1+q_2)}{a(q_1-q_2)}\right]$$

OUR VECTOR ANALYSIS HAS ACTUALLY RATHER EASILY GIVEN US THE ANSWERS TO AN INFINITE NUMBER OF POSSIBILITIES ALL-AT-ONCE.