I. An Ideal Model for Studying Thermodynamic Processes — An Ideal Gas.

- The first law, $dU = dQ - dW$, applies whether we're studying amoebas undulating in liquids or neutrons dissipating through an atomic pile. But, if we want to get results, we need to know how to calculate the internal energy and equation of state specific to the problem at hand.

- For a Monatomic Ideal Gas —

(A) Need - Internal Energy

- Got it: $U = \frac{3}{2} nRT$

(B) Equation of State

- Got it: $PV = nRT$

(C) Example Calculations

(1) $dT = 0$ (isothermal process)

- First Law: $dU = dQ - dW$
- Internal Energy: $U = \frac{3}{2} nRT$
- Eqn. of State: $PV = nRT$

$dU = \frac{3}{2} nR\,dT$ (for n = const)

$dT = 0 \Rightarrow dU = 0$

$dQ = dW$

$dW = p\,dV$

$p = \frac{nRT}{V}$

$$dQ = nRT\frac{dV}{V}$$

$$\Delta Q = nRT\int_{V_1}^{V_2}\frac{dV}{V} \qquad (T = \text{const.})$$

$$\boxed{\Delta Q = nRT\ln\left(V_2/V_1\right)}$$

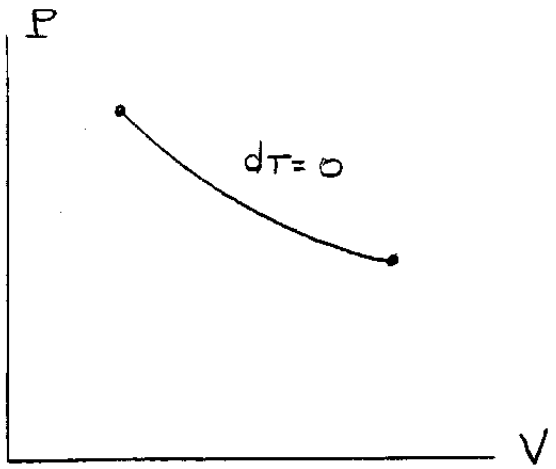
OR, $P_1V_1 = P_2V_2$

$$\boxed{\Delta Q = nRT\ln\left(P_1/P_2\right)}$$

$$dQ = dW$$

OR

$$\boxed{\Delta Q = \Delta W}$$



(2) $dP = 0$ (isobaric process)

- $dU = dQ - dW$
- $U = \frac{3}{2} nRT$

  $dU = \frac{3}{2} nR\,dT$ (n = const.)

- $PV = nRT$

  $P\,dV + V\,dP = nR\,dT$

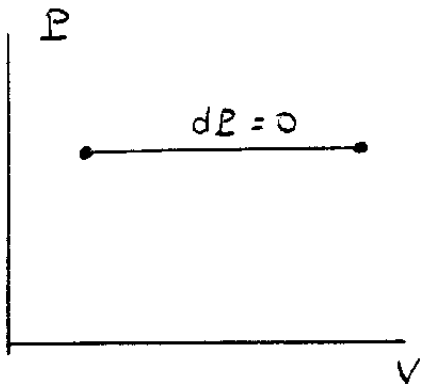
  $dW = P\,dV = nR\,dT$

$\frac{3}{2} nR\,dT = dQ - nR\,dT$

$\frac{5}{2} nR\,dT = dQ$

$$\Delta Q = \int dQ = \frac{5}{2} nR \int_{T_1}^{T_2} dT$$

$$\boxed{\Delta Q = \frac{5}{2} nR\,\Delta T}$$



P = const., so

$dW = P\,dV$

$$\Delta W = \int dW = P \int_{V_1}^{V_2} dV = P\,\Delta V$$

$$\boxed{\Delta W = P\,\Delta V}$$

(3) $dQ = 0$ (adiabatic process)

- $dU = dQ - dW$

  $dU = -dW$

- $U = \frac{3}{2} nRT$

  $dU = \frac{3}{2} nR\,dT$ (n = const.)

- $PV = nRT$

  $P\,dV + V\,dP = nR\,dT$

$dU = -p\,dV$

$\frac{3}{2} nR\,dT = -\frac{nRT}{V} dV$

$\frac{3}{2} \frac{dT}{T} = -\frac{dV}{V}$

$\frac{3}{2} \int \frac{dT}{T} = -\int \frac{dV}{V}$

$\frac{3}{2} \ln(T) = -\ln(V) + \text{const.}$

$\ln(T^{3/2}) + \ln(V) = \text{const.}$

$\ln(T^{3/2} V) = \text{const.}$

$$T^{3/2} V = \text{const.}$$

$$T_1^{3/2} V_1 = T_2^{3/2} V_2$$

ALSO HAVE THAT

$$T = \frac{PV}{nR}$$

$$T^{3/2} = \left(\frac{PV}{nR}\right)^{3/2}$$

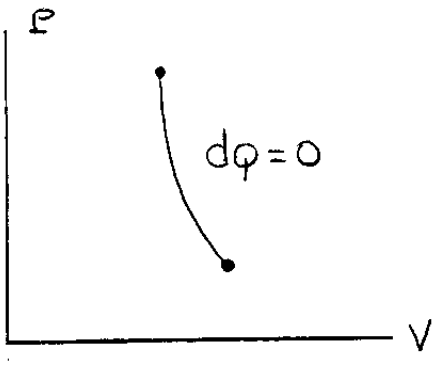
$$T^{3/2} V = \text{const.} \implies \left(\frac{PV}{nR}\right)^{3/2} V = \text{const.}$$

$$P^{3/2} V^{5/2} = \text{const.}$$

$$\left(P^{3/2} V^{5/2}\right)^{2/3} = \text{const.}$$

$$\boxed{PV^{5/3} = \text{const.}}$$

$$\boxed{P_1 V_1^{5/3} = P_2 V_2^{5/3}}$$

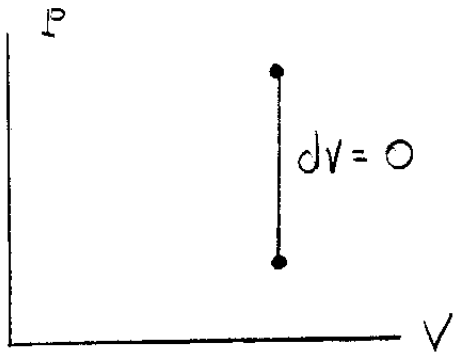


(4) $dV = 0$ (isochoric process)

- $dU = dQ - dW$
- $U = \frac{3}{2} nRT$
- $PV = nRT$

$$dU = dQ - dW$$
$$= dQ - pdV$$
$$dV = 0$$
$$dU = dQ$$
$$dQ = \frac{3}{2} nR dT$$
$$\Delta Q = \int dQ = \frac{3}{2} nR \int dT$$

$$\boxed{\Delta Q = \frac{3}{2} nR \Delta T}$$
$$\Delta W = 0$$



- SPECIFIC HEATS OF MONATOMIC IDEAL GASES
  - CONSIDER ADDING HEAT TO SAME SYSTEM UNDER TWO DIFFERENT CONDITIONS —

(1) $dP = 0$

(2) $dV = 0$

THE GAS GETS WARMER.

(1) $dP = 0$

$$PV = nRdT$$

$$P\,dV + V\cancel{dP}^{\,0} = nRdT$$

$$P\,dV = nRdT$$

$$dU = \tfrac{3}{2} nRdT$$

$$\tfrac{3}{2} nRdT = dQ - nRdT$$

$$dQ = \tfrac{5}{2} nRdT$$

Define C_p (specific molar heat capacity at constant pressure)

$$dQ \equiv n C_p dT$$

$$C_p = \tfrac{5}{2} R$$

(2) $dV = 0$

$$dU = dQ - dW$$

$$= dQ - p\cancel{dV}_{=0}$$

$$dU = dQ$$

$$\frac{3}{2} nR dT = dQ$$
$$\equiv n C_V dT$$

$C_V \equiv$ Specific molar heat capacity at constant volume.

We have then

$$C_p - C_V = \frac{5}{2}R - \frac{3}{2}R = R$$

So $\boxed{R = C_p - C_V}$

Now we know what R really is in terms of physical processes.

- Now we can re-consider adiabatic processes

$$dQ = 0$$
$$p dV + V dp = nR dT$$
$$p dV + V dp = n(C_p - C_V) dT$$

$$\frac{p dV + V dp}{n(C_p - C_V)} = dT$$

$$n C_V dT = -p dV$$
$$dT = -\frac{p}{nC_V} dV$$

$$\frac{p dV + V dp}{n(C_p - C_V)} = -\frac{p}{nC_V} dV$$

$$P \frac{C_P}{C_V} dV + V dP = 0$$

$$\frac{dP}{P} + \left(\frac{C_P}{C_V}\right) \frac{dV}{V} = 0$$

$$\ln(P) + \left(\frac{C_P}{C_V}\right) \ln(V) = \text{const.}$$

$$P V^{(C_P/C_V)} = \text{const.}$$

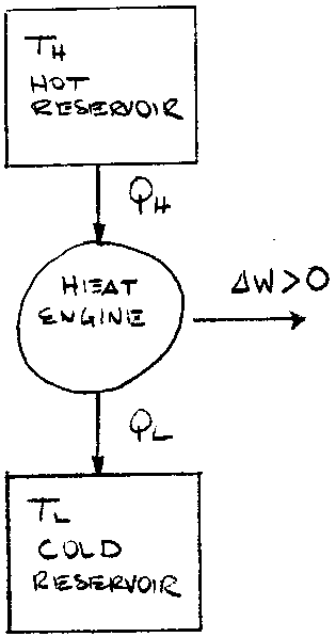
define $\gamma \equiv C_P/C_V$

$$\Rightarrow \boxed{\begin{aligned} P V^{\gamma} &= \text{const.} \\ P_1 V_1^{\gamma} &= P_2 V_2^{\gamma} \end{aligned}}$$

FOR ADIABATIC PROCESSES

IN CASE OF MONATOMIC IDEAL GAS, $\gamma = 5/3$

II. HEAT ENGINES



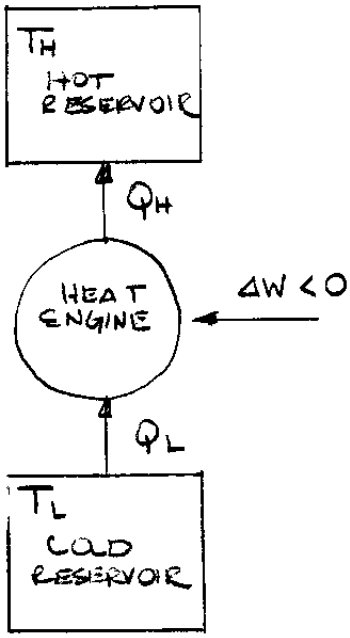
$$dU = dQ - dW$$

$$\oint dU = 0$$

$$0 = \Delta Q - \Delta W$$

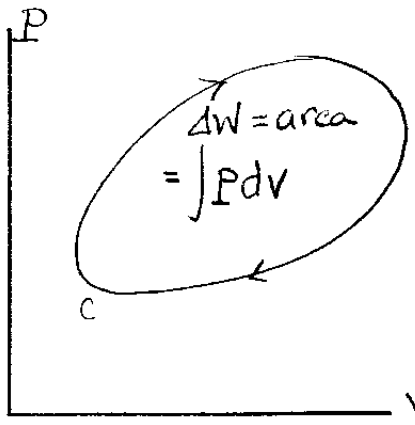
$$\boxed{\Delta Q = \Delta W}$$

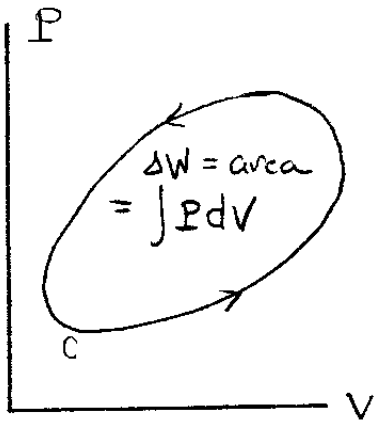
$$\boxed{\Delta Q = Q_H - Q_L}$$



ENGINE FOR GETTING USEFUL WORK OUT OF THE TRANSFER OF HEAT FROM A HOT TO A COLD RESERVIORS, E.g., THE MOTOR IN A CAR.

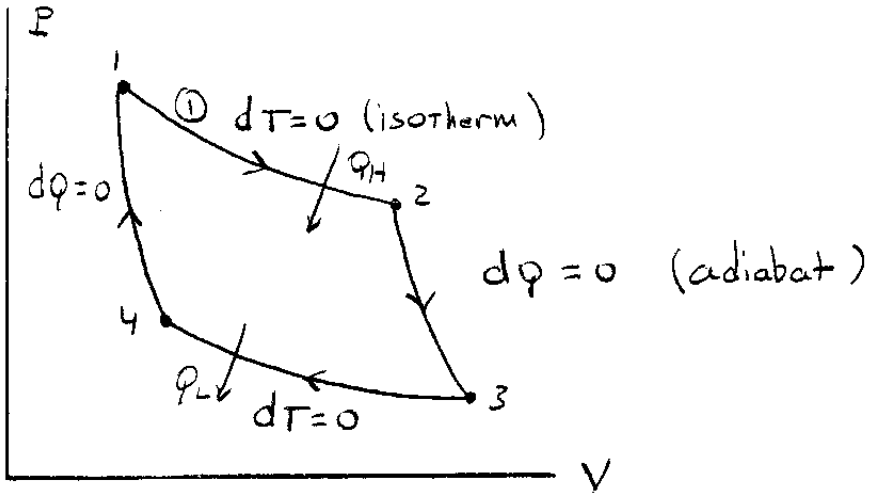
ENGINE FOR PUMPING HEAT FROM A COLD TO A HOT RESERVOIRS. WORK IS DONE ON ENGINE BY EXTERNAL SOURCE. E.g., A REFRIDGERATOR OR AIR CONDITIONER





NOTE ORIENTATION OF PV-cycle.

- CARNOT ENGINE (CARNOT CYCLE)
  - THE ARCHETYPAL HEAT ENGINE



Process:

- $(1\to2)$: $dT = 0$

$$\Delta Q_{12} = \Delta W_{12} = nRT_1 \ln(V_2/V_1)$$

- $(2\to3)$ $dQ = 0$

$$\Delta W_{23} = -\Delta U_{23} = -\frac{3}{2} nR(T_3 - T_2)$$

- $(3\to4)$: $dT = 0$

$$\Delta Q_{34} = \Delta W_{34} = nRT_3 \ln(V_4/V_3)$$

- $(4\to1)$: $dQ = 0$

$$\Delta W_{41} = -\Delta U_{41} = -\frac{3}{2} nR(T_1 - T_4)$$

- THERMODYNAMIC EFFICIENCY
  - IN GENERAL

$$(\text{EFFICIENCY}) = \frac{(\text{OUT})}{(\text{IN})}$$

e.g., MECHANICAL EFFICIENCY

$$e = \frac{\text{Work Out}}{\text{Work In}}$$

Thermodynamic Efficiency

$$e = \frac{(\text{useful Work Out})}{(\text{NET HEAT IN})}$$

TURNS OUT THAT

$$\boxed{e = \frac{(Q_H - Q_L)}{(Q_H)} = 1 - Q_L/Q_H}$$

CAN ALSO BE EXPRESSED AS

$$\boxed{e = 1 - T_L/T_H}$$

Note that <u>percent efficiency</u> is just

$$\boxed{e\,\% = 100 \cdot (1 - T_L/T_H)}$$

- THE CARNOT CYCLE IS "MAXIMALLY EFFICIENT." ANY REAL ENGINE <u>MUST</u> HAVE AN EFFICIENCY $\leq$ CARNOT EFFICIENCY.

- ONLY TWO WAYS TO HAVE 100% EFFICIENCY

$$T_L \to 0$$

OR

$$T_H \to \infty$$

BOTH OF WHICH ARE IMPOSSIBLE, SO

IT IS IMPOSSIBLE TO HAVE A 100% EFFICIENT HEAT ENGINE

(YOUR ENGINE WILL <u>ALWAYS</u> COST YOU SOMETHING).