HEAT AND THERMAL ENERGY

I. CHANGE OF STATE

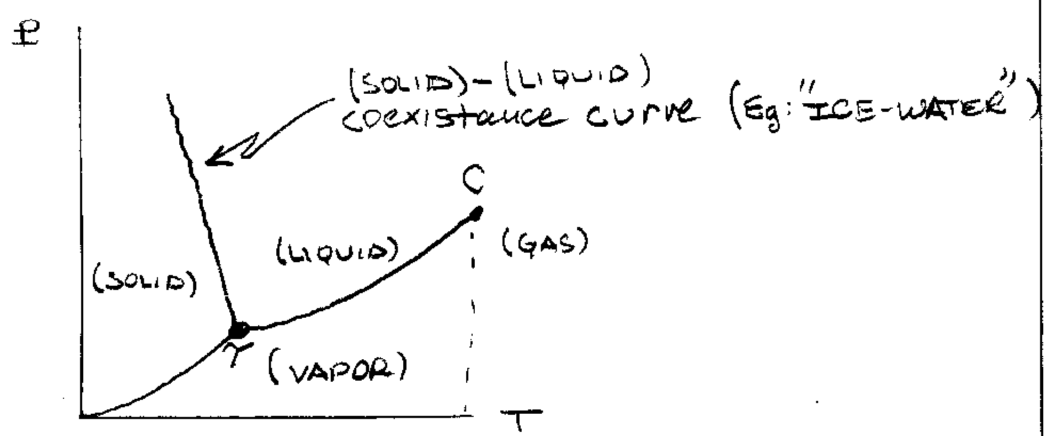
(A) Three basic phases of matter

- solid phase
- liquid phase
- gas (vapor) phase

(B) Phases transform into each other depending on P, T and energy exchange

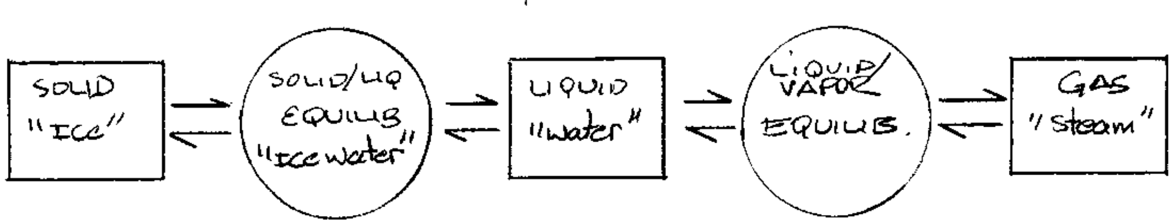
$$\text{SOLID} \rightleftharpoons \text{LIQUID} \rightleftharpoons \text{GAS}$$

Eg: H_2O



(C) Coexistence Curves

- Different phases in thermal contact exchange energy and transform from one to the other in an equilibrium phase. (They coexist in various proportions.)



HEAT AND THERMAL ENERGY

(D) What happens around phase transitions?

- We have a good "law" away from phase transitions, i.e.,

$$\boxed{\Delta Q = cm\Delta T.}$$

Now we want to know what happens in-between.

(E) Fusion (melting/solidifying) and Vaporization (evaporation/condensation)

- It turns out that the Temperature of an equilibrium phase remains unchanged as heat is added/removed from system until only one of the phases remains (after which the $\Delta Q = cm\Delta T$ "laws" take over until another phase transition is reached.)

$$\boxed{\begin{array}{l} \text{Fusion:} \quad \Delta Q = L_f \Delta m \\ \text{Vaporization:} \ \Delta Q = L_v \Delta m \end{array}}$$

"Latent heat of Fusion" $\equiv L_f = \frac{\Delta Q}{\Delta m}$ = amount of energy added/subtracted to completely melt/solidify a mass Δm of solid/liquid

"Latent heat of Vaporization" $\equiv L_v = \frac{\Delta Q}{\Delta m}$ = amount of energy added/subtracted to completely vaporize/condense a mass Δm of liquid/vapor

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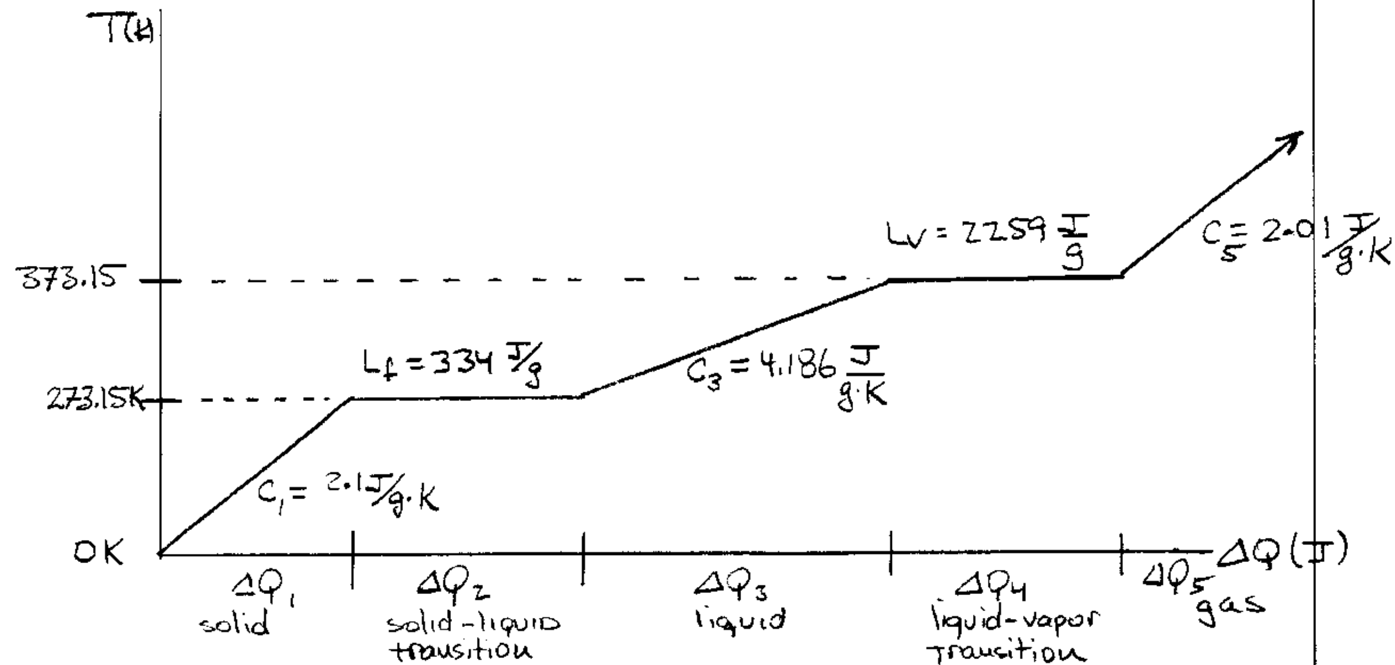
- Why is the temperature remaining constant? Recall that temperature is a measure of the average kinetic energy possessed by the atoms of the substance. E.g., the atoms of solid water (ice) are not free to move-around much - they mostly just vibrate around some fixed location in the solid. Their interaction energy is ~~stronger~~ than their kinetic energy (KE$\ll$I.E). Liquid water atoms can move around more freely - their kinetic energy is comparable to their interaction energy (KE$\approx$I.E.). To make one into the other, we need to add (subtract) enough energy to overcome the interaction energy. The temperature doesn't change until the kinetic energy can appreciably change, which means the interaction energy has to be supplied (taken away) by adding (subtracting) energy (heat) from the system.

HEAT AND THERMAL ENERGY

- ΔQ vs. T for H_2O



$\boxed{\text{Ex:}}$ 1.00g icecube making a trip from 0K to 473K:

$$\Delta Q_1 = c_1 m_1 \Delta T_1 = (2.1\ J/g \cdot K)(1.00g)(273.15K)$$
$$= 570J$$

$$\Delta Q_2 = L_f \Delta m_2 = (334\ J/g)(1.00g)$$
$$= 334J$$

$$\Delta Q_3 = C_3 M_3 \Delta T_3 = (4.186\ J/g \cdot K)(1.00g)(100.K)$$
$$= 419J$$

$$\Delta Q_4 = L_V \Delta m_4 = (2259\ J/g)(1.00g)$$
$$= 2260J$$

$$\Delta Q_5 = C_5 M_5 \Delta T_5 = (2.01\ J/g \cdot K)(1.00g)(100.K)$$
$$= 201.J$$

Total Energy for trip $= \Sigma \Delta Q_i = 3780J$

(60% of total by vaporization alone!)

HEAT AND THERMAL ENERGY

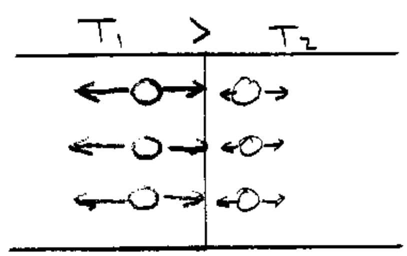
II. EXCHANGE OF THERMAL ENERGY

(A) Three basic processes

- Conduction
- Convection
- Radiation

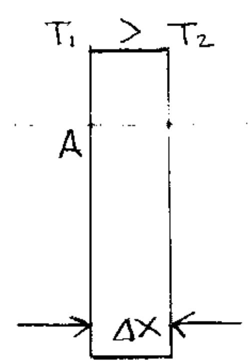
(B) Conduction

- Eg: Two metal bars at different temperatures in contact with each other. The atoms of the hotter one are losing energy, to the atoms of the cooler one via collisions at their interface. The hotter one cools – the cooler one warms



$H = \overrightarrow{\text{Energy Flow}} \rightarrow \text{Heat flow}$ ($J/s \equiv$ Watts)

- Eg: Thin metal plate with faces (Area A) at different temperatures



$H = \overrightarrow{\text{Energy flow}} = \text{Heat flow}$ ($J/s \equiv$ Watts)

HEAT AND THERMAL ENERGY

- What's relevant?

$H \sim \Delta T$ (Greater temperature difference means greater heat flow)

$H \sim \frac{1}{\Delta X}$ (takes longer for heat to flow across thicker material)

$H \sim A$ (larger surface area means more atoms can hit each other - more energy transfer per unit time).

Not knowing for sure what else to include, hide ignorance in a proportionality constant!

$$\boxed{H = -K_T A \frac{\Delta T}{\Delta X}}$$

Heat Conduction Equation

K_T = "Thermal Conductivity" - depends on the material ("other details we don't know").

$\boxed{\text{Ex!}}$ During a San Diego style "Cold-spell", (about 60°F), How much money is it costing a typical San Diegan these days to maintain a comfy temperature of 72°F inside their homes for each square-meter of household window they have?

- Reasonable assumptions
  - K_T glass $\simeq .80 \frac{W}{m \cdot K}$ (from Hecht)
  - $\Delta X \simeq 5.0$ mm (estimate)
  - $\Delta T \simeq -12°F = -6.6K$ (given)
  - $A = 1.00$ m (for example)

HEAT AND THERMAL ENERGY

- Calculate

$$H = -K_T A \frac{\Delta T}{\Delta X}$$

$$= -\left(.80 \frac{W}{m \cdot K}\right)\left(1.00 m^2\right)\left(\frac{-6.6 K}{5.0 \times 10^{-3} m}\right)$$

$$\boxed{H = 1.1 \times 10^3 J/s = 1.1 kW}$$

Typical San Diego energy cost these days is around \$.07/kW·hr

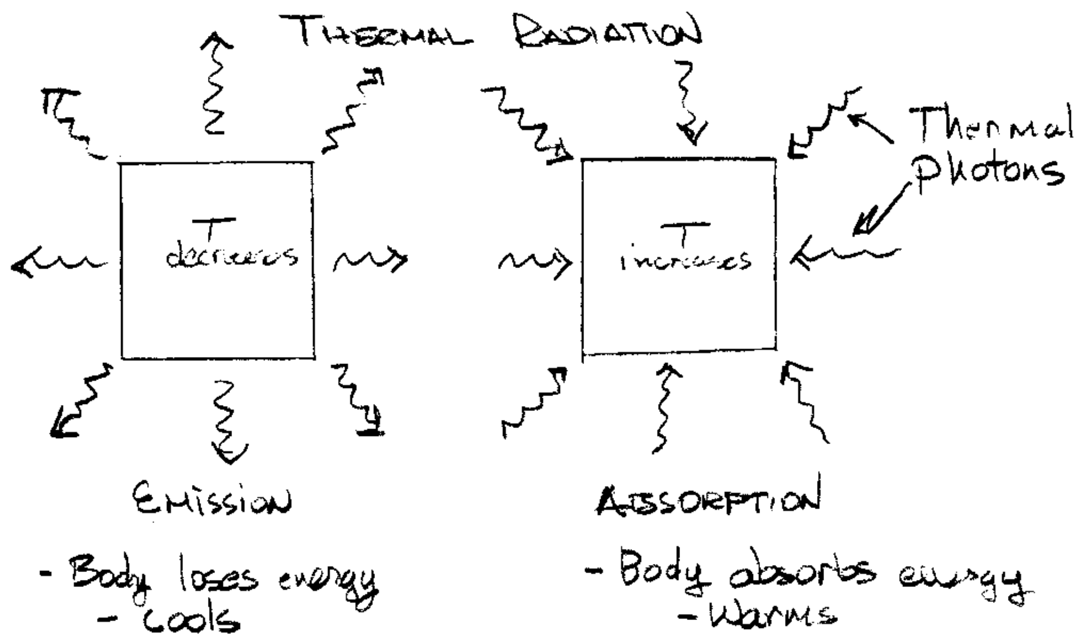
$$(1.1 kW)\left(\frac{\$.07}{kW \cdot hr}\right)\left(\frac{24 hr}{1 day}\right)$$

$= \$1.90/day$ per each square meter of window!

HEAT AND THERMAL ENERGY

(c) Radiation

Thermal radiation is energy lost/gained by a body due to the emission/absorption of massless particles of light, called photons. Most "low-temperature" things, such as human beings, radiate most of this energy in the infrared – it is light, but it's of a frequency too low for our eyes to detect.



HEAT AND THERMAL ENERGY

- What's relevant?
  - $P\,(J/s) \sim A$ (more area, the more total energy radiated by a surface)
  - $P \sim T^4$ (The hotter an object, the more it radiates. The T^4 can't be guessed. It follows from deeper principles.

We need more advanced methods to get all the details, so we'll just state the final equation here:

$$\boxed{P = \epsilon \sigma A T^4}$$

P = total energy radiated per second (J/s)

T = Temperature (K)

$\sigma = 5.67 \times 10^{-8} \frac{W}{m^2 \cdot K^4}$

$0 < \epsilon < 1$ = "emissivity" → depends on the details of the particular material and surface texture one has

bad emitter (a mirror)

good emitter ("black hole")

$\epsilon = 1 \Rightarrow$ "perfect blackbody"

- As a general rule, good absorbers are good emitters, and vice-versa

Good absorber (e.g. black paint) ⇔ Good emitter

Bad absorber (e.g. aluminum foil) ⇔ Bad emitter

HEAT AND THERMAL ENERGY

(D) Convection

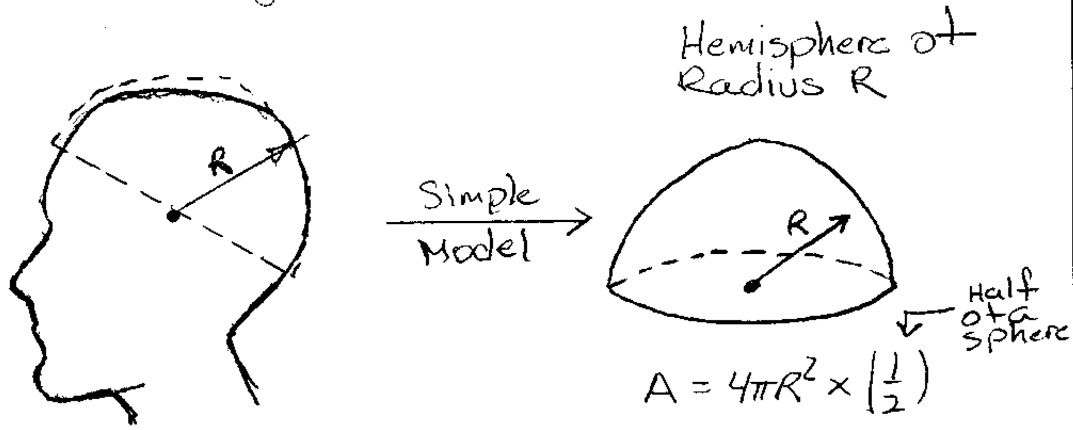
Convection is taking place when the bulk motion of a fluid or gas is transporting energy from one place to another. Modeling convection is extremely difficult (Fluid dynamics, turbulence, etc.), and we won't be attempting it here. Hecht's explanation is about as good as we'll get.

What we would like to do, though, is take a detailed look at a problem combining both thermal radiation and conduction - Your Head.

HEAT AND THERMAL ENERGY

$\boxed{\text{Ex:}}$ A Human Head Exchanging Energy with its Environment

- What's Relevant?
  - Radiation (head has a temperature)
  - Conduction (head's temp. is different than environment)
  - Convection (Hot Head causes surrounding air to expand. ~~Less dense~~ air heats and rises. We'll ignore this.)
  - Evaporation (Sweat evaporates into environment, carrying away energy. We'll also ignore this.)
- Relevant Equations
  - Radiation: $P = \epsilon \sigma A T^4$
  - Conduction: $H = -K_T A \frac{\Delta T}{\Delta X}$
- Simple ("Toy") Model of Head



$A = 4\pi R^2 \times \left(\frac{1}{2}\right)$

$A = 2\pi R^2$

Heat AND THERMAL ENERGY

- One step at a time –
  (A) Radiation
  - $P = \epsilon \sigma A T^4$
    $= 2\pi \epsilon \sigma R^2 T^4$
  - Make reasonable assumptions
    - $\epsilon \simeq .97$ (Hecht: Human Flesh)
    - $R \simeq 8.0\,cm$
    - $T \simeq 98.6°F$
    - $\sigma = 5.67 \times 10^{-8} \frac{W}{m^2 \cdot K^4}$
  - Calculate
    - Get everything into same units
      - $\sigma = 5.67 \times 10^{-8} \frac{W}{m^2 \cdot K^4}$ (← clue)
      - $R = 8.0\,cm = 8.0 \times 10^{-2}\,m$
      - $T = 98.6°F = 310.\,K$
      - $\epsilon = .97$ (dimensionless)

$P = 2\pi \epsilon \sigma R^2 T^4$

$= (2)(3.1415)(.97)(5.67 \times 10^{-8} \frac{W}{m^2 \cdot K^4})(8.0 \times 10^{-2}\,m)^2 (310.\,K)^4$

$$\boxed{P = 21\,W = 21\,(J/s)}$$

HEAT AND THERMAL ENERGY

- Check
  - Dimensions okay? $[\text{Power}] = \left[\frac{\text{ENERGY}}{\text{TIME}}\right]$ ✓
  - Units okay? $(\text{J/s}) = (\text{J/s}) = \text{Watts}$ ✓
  - Is answer <u>reasonable</u>?
    - Let's see: That's as much energy as a 20W lightbulb! (About four typical "night-lights".) Why isn't my head <u>glowing</u>!?
      - <u>It IS</u>!!

        Almost all of this energy is being radiated in the infrared, below the threshold of human vision. Look at it through an infrared camera (like Police Helicopters do at night), and it glows bright like a light-bulb!
        (More about this in Physics 1C.)
    - Let's see: How many daily food calories would it take to support this energy loss without decreasing the temperature of the skull?

$$\left(21\ \frac{\text{J}}{\text{s}}\right)\left(\frac{1\,\text{kcal}}{4186\,\text{J}}\right)\left(\frac{60\,\text{sec}}{1\,\text{min}}\right)\left(\frac{60\,\text{min}}{1\,\text{HR}}\right)\left(\frac{24\,\text{HR}}{1\,\text{day}}\right)$$

$$= 433\ \text{Cal/day}$$

Reasonable? Sure! Doesn't exceed average daily caloric intake of 2000 Cal.

HEAT AND THERMAL ENERGY

- What did we neglect?
  - Rate at which energy is absorbed from environment:
    (will lower this estimate.)
    Daytime / Nightime will be different
    (another reason its "colder" at night!)

(B) Conduction

- Our brains are protected by a thin layer of material (skull – bone/flesh)
  Inside the skull, we have $\approx 98.6°F$.
  Outside, the environment often has a lower air-temperature.
  Heat thus is conducted out of our Hot heads, through our skulls, to the surrounding air.

- $$H = -K_T A \frac{\Delta T}{\Delta x}$$
  $$= -2\pi K_T R^2 \frac{\Delta T}{\Delta x} \quad \text{(in our model).}$$

- Reasonable Assumptions
  - $K_T \simeq .21 \frac{W}{m \cdot K}$ (Hedat: Flesh-no blood Bone data not handy, but could be found.)
  - $R \simeq 8.0\,cm$ (estimate by looking)
  - $T_1 = 98.6°F$ (given)
    $T_2 = 70°F$ (for example)
  - $\Delta x \simeq 5.0\,mm$ (guess)

HEAT AND THERMAL ENERGY

- Calculate
  - get everything into same units
    - $K_T = .21 \frac{W}{m \cdot K}$ ← the clue
    - $R = 8.0\ cm = 8.0 \times 10^{-2} m$
    - $\Delta T = (T_2 - T_1) = (-28.6°F) = -16K$
    - $\Delta X = 5.0mm = 5.0 \times 10^{-3} m$

$$H = -2\pi K_T R^2 \frac{\Delta T}{\Delta X}$$

$$= -(2)(3.1415)(.21 \frac{W}{m \cdot K})(8.0 \times 10^{-2} m)^2 \left(\frac{-16K}{5.0 \times 10^{-3} m}\right)$$

$$\boxed{H = 27W \quad (J/s)}$$

- Check:
  - dimensions okay? $[Power] = \left[\frac{ENERGY}{TIME}\right]$ ✓
  - Units okay? $(J/s) = (J/s) = $ Watts ✓
  - Reasonable?

    Sure! Still within our average daily caloric intake, even with the radiation calculated above

    $$\left(433 \frac{Cal}{day}\right) + \left(\frac{557\ Cal}{day}\right) = 990 \frac{Cal}{day}$$

  - about $(990\ Cal/day)(2000 \frac{Cal}{day}) \to 50\%$

    But, we neglected hair, etc. This number would be <u>increased</u> by convection/evaporation.

HEAT AND THERMAL ENERGY

- What if our skulls were instead made of linen ($K_T = .088\ W/m \cdot K$)? How much of our daily diets would be lost to conduction, all else the same?

$$H = -K_T A \frac{\Delta T}{\Delta X}$$

$$= -2\pi K_T R^2 \frac{\Delta T}{\Delta X}$$

$$= -(2)(3.1415)(.088\ \tfrac{W}{m \cdot K})(8.0 \times 10^{-2} m)^2 \left(\frac{-16 K}{5.0 \times 10^{-3} m}\right)$$

$$\boxed{H = 11\ W \ll 27\ W}$$

$$(11\ J/s)\left(\frac{3600\ sec}{1\ HR}\right)\left(\frac{24\ HR}{1\ day}\right)\left(\frac{1\ kcal}{4186\ J}\right)\left(\frac{1\ day}{2000\ kcal}\right)$$

$\Rightarrow$ $\boxed{11\% \text{ of total caloric intake}}$ Much better than bare skull @ 28%.

(This is why we like to wear beanies on cold days!)

HEAT AND THERMAL ENERGY

(C) A SPECULATIVE QUESTION:

- Recall how we neglected to account for the amount of radiation absorbed by the skull from its surrounding environment. Air is not a very good absorber of infrared-through-visible radiation (that's why we can see through it.)

The intensity of solar radiation at the surface of the earth is about 1000 W/m^2 on an average, sunny day. Let's say your head absorbs about 50% of the sunlight that hits it on average:

hemisphere — 50%

$$A = 2\pi R^2 \times \left(\tfrac{1}{2}\right)$$

$$= (2)(3.1415)(8.0 \times 10^{-2}\,m)^2\left(\tfrac{1}{2}\right)$$

$$= .020\,m^2$$

$$(.020\,m^2)\left(\frac{1000\,W}{m^2}\right) = 20\,W \text{ absorbed}$$

Compare this to the amount of energy radiated by the skull as we calculated above (i.e., 21W) $\Rightarrow$ almost Balanced

$$P_{rad}/P_{abs} \sim 1$$

$$\frac{\epsilon\sigma T_{head}^4}{I_{sun}} \sim 1$$

Is our body temperature 98.6°F because the sun is at the distance away from us that it is?