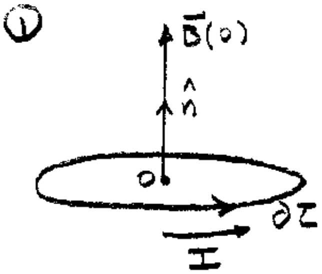
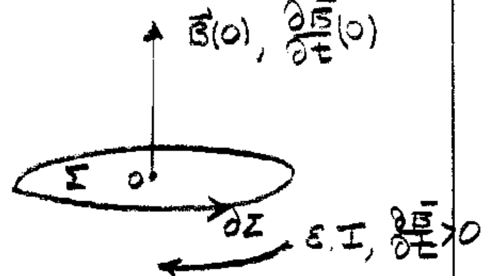
I. SELF-INDUCTANCE

TAKE ANOTHER LOOK AT THE CURRENT LOOP —

① 

Current I in conducting current loop $\partial\Sigma$ producing a magnetic field $\vec{B}(0)$ at "0".

$$\vec{B}(0) = \frac{\mu_0 I}{2a}\hat{n}$$

② 

Changing flux of a $\vec{B}$-field causing a "Back-Emf" around $\partial\Sigma$

$$\mathcal{E} = -\frac{\partial \Phi_B}{\partial t}$$

THE CHARGES IN ② HAVE NO IDEA WHERE THE $\vec{B}$-FIELD CAME FROM. ALL THEY SEE IS A FLUX $\Phi_B = \int_\Sigma \vec{B}\cdot d\vec{A}$ THROUGH Σ.

$$\mathcal{E} = -\frac{\partial \Phi_B}{\partial t}$$

$$= -\frac{\partial}{\partial t}\int_\Sigma \vec{B}\cdot d\vec{A}$$

$$\mathcal{E} = -\int \left(\frac{\partial \vec{B}}{\partial t}\right)\cdot d\vec{A}$$

IF WE APPROXIMATE THE $\vec{B}$-FIELD IN ② BY ITS VALUE FROM ①

$$\frac{\partial \vec{B}}{\partial t} = \frac{\mu_0}{2a}\frac{dI}{dt}\hat{n}$$

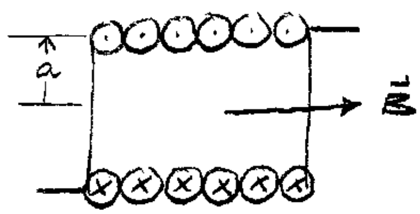
SO,

$$\mathcal{E} = -\frac{\mu_0}{2a}\frac{dI}{dt}\int \hat{n}\cdot dA\,\hat{n}$$

$$\mathcal{E} = -\frac{\mu_0 A}{2a}\frac{dI}{dt}$$

$$\mathcal{E} = -\left(\frac{\mu_0 \pi a}{2}\right)\frac{dI}{dt}$$

LIKEWISE, FOR THE SOLENOID,



$$B = \mu_0 n I = \mu_0 \frac{N}{L} I$$

$$\frac{\partial B}{\partial t} = \mu_0 \frac{N}{L}\frac{dI}{dt}$$

$$\mathcal{E}_{1\,loop} = -\frac{\partial \Phi_B}{\partial t}$$

$$= -\frac{\mu_0 N}{L}\frac{dI}{dt}\int dA$$

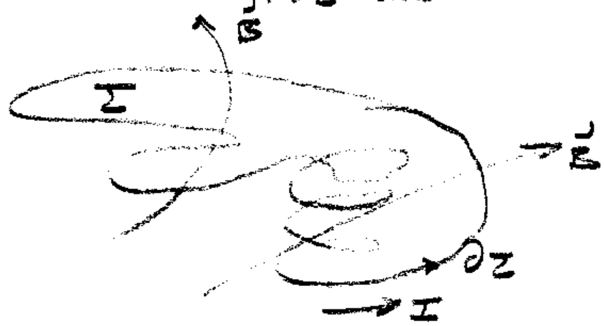
$$\mathcal{E}_{1\,loop} = -\left(\frac{\mu_0 N \pi a^2}{L}\right)\frac{dI}{dt}$$

FOR N- loops

$$\mathcal{E} = \mathcal{E}_N = N\mathcal{E}_1 = -\left(\frac{\mu_0 N^2 \pi a^2}{L}\right)\frac{dI}{dt}$$

FOR SOME ARBITRARY LOOP OF current $\partial\Sigma$ BOUNDING Σ, FIND THAT

$$\mathcal{E} = -\left(\begin{array}{c}\text{purely geometrical}\\ \text{factor relating}\\ \partial\Sigma \text{ and } \Sigma\end{array}\right)\frac{dI}{dt}$$



ALL HAVE THE FORM

$$\mathcal{E} = -L\frac{dI}{dt}$$

$L \equiv$ "SELF-INDUCTANCE" (OR "INDUCTANCE" FOR SHORT)

Unit of $L \equiv$ Henry $= \left(\frac{V \cdot s}{A}\right)$

- A <u>CHANGING</u> CURRENT IN ANY CURRENT LOOP WILL SELF-INDUCE AN EMF WHICH WILL OPPOSE THE CURRENT THAT CREATED IT.

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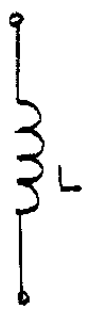
- SCHEMATIC SYMBOL FOR INDUCTANCE —

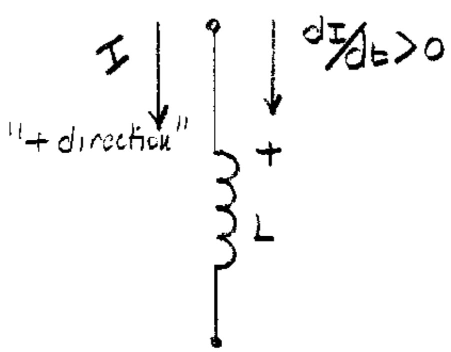


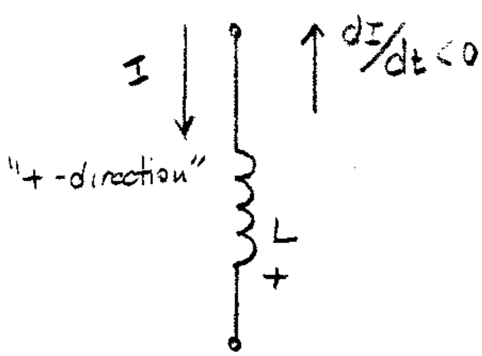
L REPRESENTS BOTH THE INDUCTANCE AND ITS VALUE. IT IS <u>ANYTHING</u> THAT HAS THE PROPERTY

$$\mathcal{E} = -L \frac{dI}{dt}$$

THE VALUE OF L DEPENDS ON THE PHYSICAL GEOMETRY OF THE OBJECT

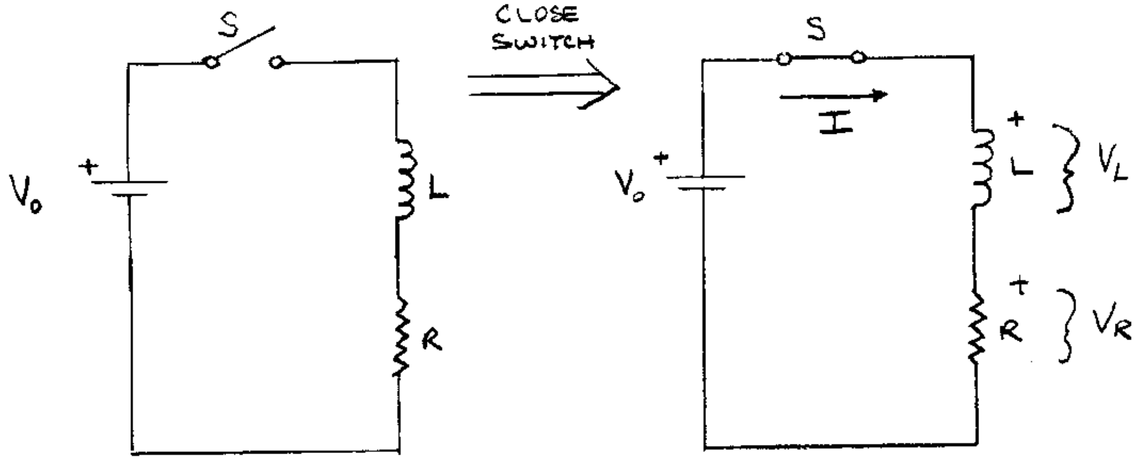
- ORIENTING L —





II. R-L CIRCUITS

THROW ONE OF THESE L-THINGS INTO A CIRCUIT LIKE SO —



$$V_0 - V_L - V_R = 0$$

$$V_0 - L\frac{dI}{dt} - IR = 0$$

$$L\frac{dI}{dt} + IR = V_0$$

$$\frac{dI}{dt} + \left(\frac{R}{L}\right)I = \frac{V_0}{L}$$

Differential Equation for $I(t)$

Solve:

$$\frac{dI}{dt} = \left(\frac{V_0}{L}\right) - \left(\frac{R}{L}\right)I$$

$$= \left(\frac{R}{L}\right)\left[\left(\frac{V_0}{R}\right) - I\right]$$

$$= -\left(\frac{R}{L}\right)\left[I - \left(\frac{V_0}{R}\right)\right]$$

$$\frac{dI}{I-(V_0/R)} = -(R/L)\,dt$$

Have that $I(0)=0$

$$\int_0^I \frac{dI'}{I'-(V_0/R)} = -(R/L)\int_0^t dt'$$

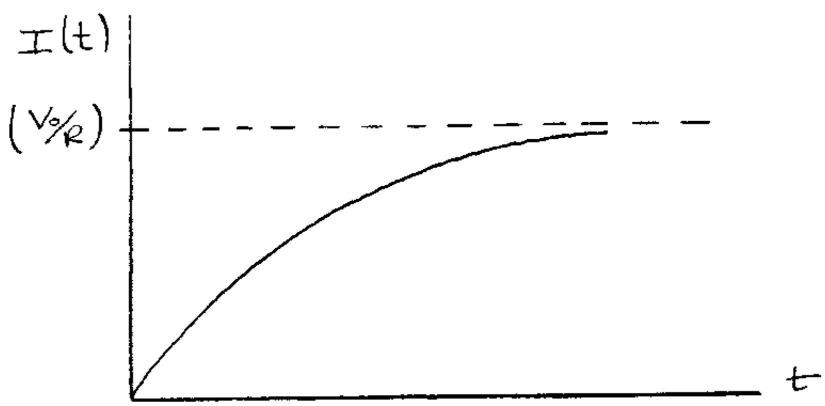
$$\ln\left|I'-(V_0/R)\right|\Big|_0^I = -(R/L)\,t$$

$$\ln\left|\frac{I-(V_0/R)}{-(V_0/R)}\right| = -(R/L)\,t$$

$$\frac{I-(V_0/R)}{-(V_0/R)} = e^{-(R/L)t}$$

Finally,

$$\boxed{I(t) = \left(\frac{V_0}{R}\right)\left[1-e^{-t/(L/R)}\right]}$$



- LR TIME CONSTANT

$$\tau \equiv L/R \qquad \left(\frac{V \cdot s}{A}\right)\left(\frac{A}{V}\right) = (\text{sec.})$$

IS THE LR ANALOGUE OF $t_0 = RC$

III. ENERGY STORED IN B-FIELD OF INDUCTOR

$$dW_L = dq\, \mathcal{E}_L$$

$$\frac{dW_L}{dt} = \frac{dq}{dt}\mathcal{E}_L$$

$$dW_L = \frac{dq}{dt}\mathcal{E}_L\, dt$$

$$= \left(\frac{dq}{dt}\right)\left(L\frac{dI}{dt}\right)dt$$

$$dW_L = I\, L\, dI$$

$$\Delta W_L = \int dW_L = L\int I\, dI$$

$$\boxed{U_L = \Delta W_L = \frac{1}{2}LI^2}$$

IV. ENERGY STORED IN ANY CONFIGURATION OF $\vec{E}$- AND $\vec{B}$- fields —

Energy Density: $u = U/\text{Vol.}$

$$\boxed{u_E = \frac{1}{2}\epsilon_0 E^2}$$

$$\boxed{u_B = \frac{1}{2\mu_0} B^2}$$

Total Energy density

$$u = u_E + u_B$$

$$\boxed{u = \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0} B^2\right)}$$

Total Energy in some region $\mathcal{R}$ of space —

$$U = \int_{\mathcal{R}} u \, d^3r$$

$$\boxed{U = \frac{1}{2}\int \left(\epsilon_0 E^2 + \frac{1}{\mu_0} B^2\right) d^3r}$$