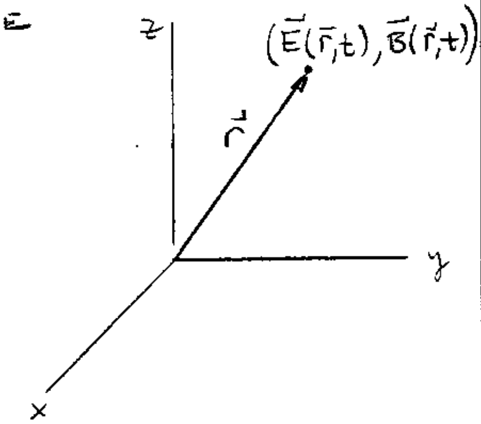
I. FARADAY'S LAW

FARADAYS LAW RELATES THE ELECTRIC AND MAGNETIC FIELDS PRESENT THROUGHOUT SOME REGION OF SPACE AND TIME

$$\boxed{\bar{\nabla} \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}}$$

$$\vec{E} = \vec{E}(\vec{r},t) \qquad \vec{B} = \vec{B}(\vec{r},t)$$



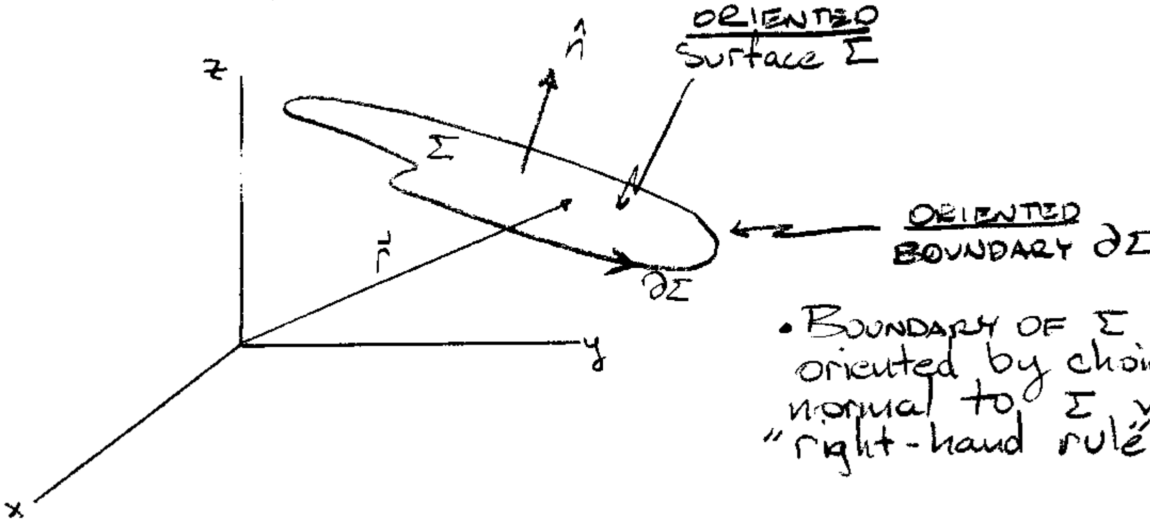
THIS IS A POWERFUL RESULT. THE TEXT SPENDS A GREAT DEAL OF EFFORT ATTEMPTING TO MOTIVATE IT. IN REALITY, IT CAN'T BE DERIVED FROM ANY MORE-BASIC PRINCIPLE — WE MUST ACCEPT IT AS A HYPOTHESIS. BUT, IT IS A VERY GOOD HYPOTHESIS WHICH HAS NEVER BEEN OBSERVED TO BE VIOLATED BY EXPERIMENT. HENCE, IT HAS BECOME PART OF THE THEORY OF ELECTROMAGNETISM (IT IS ONE OF FOUR EQUATIONS, KNOWN COLLECTIVELY AS MAXWELL'S EQUATIONS, WHICH UNIFY ELECTRICITY WITH MAGNETISM, AND CONTAIN EVERYTHING WE KNOW ABOUT E/M.)

THE TEXT DEVELOPS THE ABOVE EQN IN TERMS OF INTEGRALS, BUT BOTH FORMS ARE EQUIVALENT, BEING RELATED BY SIMPLE IDENTITIES.

- OBTAINING INTEGRAL FORM OF FARADAY'S LAW FROM ITS DIFFERENTIAL FORM

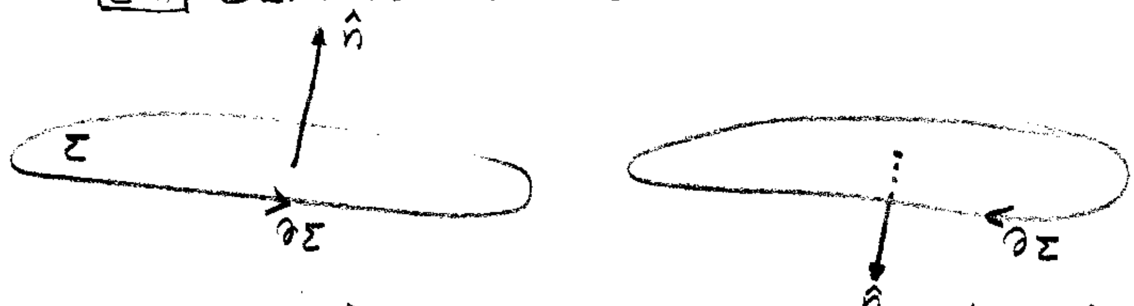
$$\bar{\nabla} \times \bar{E} = -\partial \bar{B}/\partial t$$

JUST INTEGRATE BOTH SIDES OVER SOME IMAGINARY SURFACE IN SPACE –



- Boundary of Σ is oriented by choice of normal to Σ via "right-hand rule".

$\boxed{\text{EX:}}$ ORIENTED SURFACE WITH ORIENTED BOUNDARY



- You are free to choose orientation of surface Σ by picking a direction for its unit normal (and sticking with it). Then, your choice of orientation for the boundary is fixed by the "right-hand-rule".

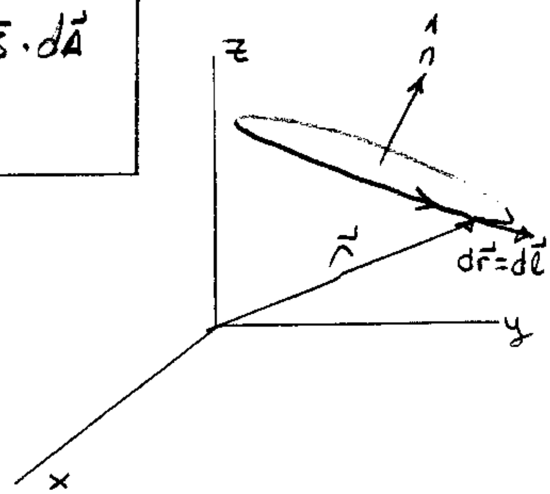
$$\bar{\nabla} \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

INTEGRATE THIS OVER SURFACE Σ

$$\int_{\Sigma} (\bar{\nabla} \times \bar{E}) \cdot d\bar{A} = -\frac{\partial}{\partial t} \int_{\Sigma} \bar{B} \cdot d\vec{A}$$

BY A VECTOR IDENTITY, WE HAVE

$$\boxed{\oint_{\partial \Sigma} \bar{E} \cdot d\vec{\ell} = -\frac{\partial}{\partial t} \int_{\Sigma} \bar{B} \cdot d\vec{A}}$$



Now, JUST MAKE A COUPLE OF STIPULATIVE DEFINITIONS —

$$\boxed{\mathcal{E} \equiv \oint_{\partial \Sigma} \bar{E} \cdot d\bar{\ell}}$$

$\mathcal{E}$ = "emf" (careful — Bad Name! ~~Not~~ a Force).

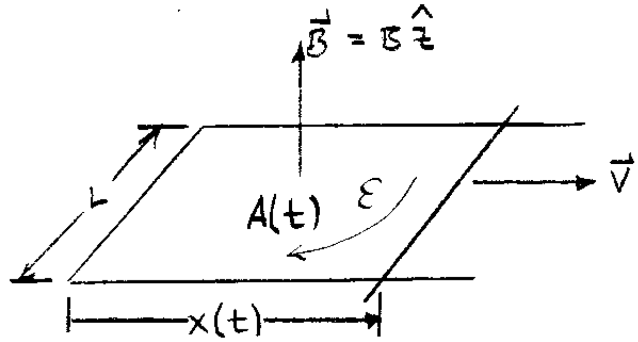
$$\boxed{\Phi_B = \int_{\Sigma} \bar{B} \cdot d\vec{A}}$$

"Flux" OF $\vec{B}$ through surface Σ.

THESE DEFINITIONS ALLOW US TO COMPACTLY WRITE ABOVE EQUATION AS

$$\boxed{\mathcal{E} = -\frac{\partial \Phi_B}{\partial t}}$$

$\boxed{\mathcal{E}x:}$ EXAMPLE FROM TEXT, STARTING WITH GENERAL RESULT —



$$\mathcal{E} = -\frac{\partial \Phi_B}{\partial t}$$

$$\Phi_B = \int_\Sigma \vec{B}\cdot d\vec{A} \qquad \vec{B} = B\hat{z} \qquad d\vec{A} = dA\,\hat{z} = L\,dx\,\hat{z}$$

$$\Phi_B = \int_\Sigma (B\hat{z})\cdot(L\,dx\,\hat{z})$$

$$= BL\int_\Sigma dx$$

$$\Phi_B = BL\,x(t)$$

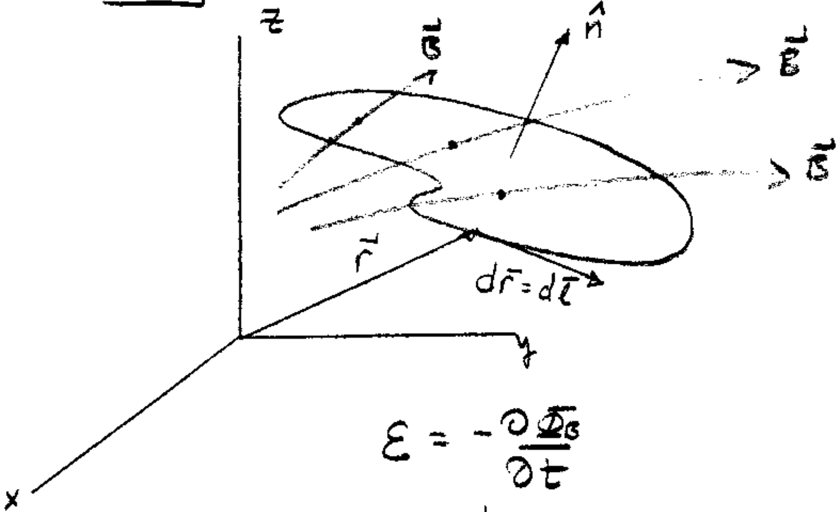
$$\frac{\partial \Phi_B}{\partial t} = BL\frac{\partial x}{\partial t} = BLV^x$$

$$\boxed{\mathcal{E} = -BLV^x}$$

NOTE THAT OUR RESULT DIFFERS BY A SIGN. THIS SIGN IS IMPORTANT — IT TELLS US THAT THE ORIENTATION OF $\mathcal{E}$ IS OPPOSITE TO OUR CHOICE OF BOUNDARY ORIENTATION

$$\mathcal{E} = \int_{\partial\Sigma} \vec{E}\cdot d\vec{l} < 0 \Rightarrow$$ $\mathcal{E}$ is oriented opposite to our choice of $d\vec{l}$, which followed from our choice of $\hat{n}$ $(=\hat{z})$

$\boxed{\text{Ex!}}$ ARBITRARY LOOP



$$\mathcal{E} = -\frac{\partial \Phi_B}{\partial t}$$

$$\Phi_B = \int_{\Sigma} \vec{B} \cdot d\vec{A}$$

$$\mathcal{E} = -\frac{\partial \Phi_B}{\partial t} = -\int_{\Sigma} \left[\left(\frac{\partial \vec{B}}{\partial t} \right) \cdot d\vec{A} + \vec{B} \cdot \left(\frac{\partial \vec{A}}{\partial t} \right) \right]$$

- THREE WAYS TO CREATE EMF $\mathcal{E}$ –
  - KEEP GEOMETRY OF SURFACE AND BOUNDARY FIXED, AND CHANGE $\vec{B}$-field

    $$\mathcal{E} = -\int_{\Sigma} \left(\frac{\partial \vec{B}}{\partial t} \right) \cdot d\vec{A} \qquad (d\vec{A} = 0)$$

  - Keep $\vec{B}$-field fixed, and change geometry of Σ (e.g., make loop bigger/smaller with time)

    $$\mathcal{E} = -\int_{\Sigma} \vec{B} \cdot d\vec{A} \qquad (d\vec{B} = 0)$$

  - KEEP BOTH GEOMETRY AND $\vec{B}$-FIELD FIXED, AND CHANGE RELATIVE ORIENTATION OF SURFACE IN $\vec{B}$-FIELD

    $$\mathcal{E} = -\int_{\Sigma} \frac{\partial}{\partial t} (\vec{B} \cdot \vec{A})$$

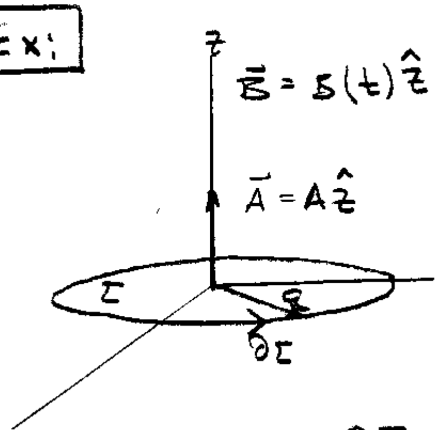
    ($d\vec{B} = 0$, $dA = 0$ but $d\vec{A} \neq 0$ $\vec{B} \cdot d\vec{A} = B\,dA \cos\theta$, angle changes)

- OF COURSE, ALL ABOVE COULD BE HAPPENING SIMULTANEOUSLY

$$\mathcal{E} = -\frac{\partial \Phi_B}{\partial t}$$

TRUE REGARDLESS.

$\boxed{\text{Ex:}}$



← time-dependent $\vec{B}$-field

← loop of fixed geometry radius a.

$$\mathcal{E} = -\frac{\partial \Phi_B}{\partial t}$$

$$\Phi_B = \int_\Sigma \vec{B}\cdot d\vec{A}$$

$$= \left(B(t)\hat{z}\right)\cdot \int_\Sigma dA\,\hat{z}$$

$$= B(t)\int_\Sigma dA$$

$$\Phi_B = B(t)\,\pi a^2$$

$$\frac{\partial \Phi_B}{\partial t} = \pi a^2 \frac{\partial B(t)}{\partial t}$$

$$\mathcal{E} = -\pi a^2 \frac{\partial B(t)}{\partial t}$$

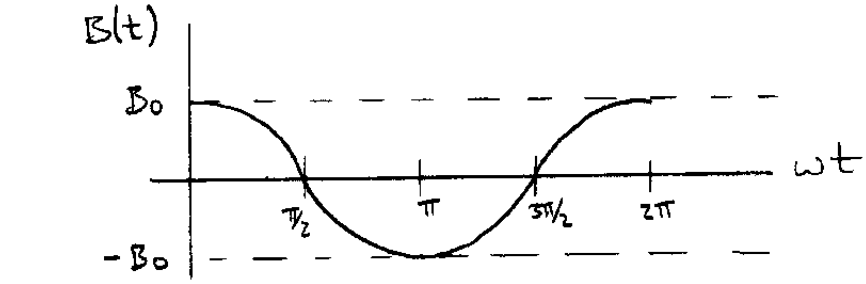
- if $\frac{\partial B}{\partial t} > 0$, $\mathcal{E}$ is oriented opposite to $\xrightarrow{\partial\Sigma}$ shown.
- if $\frac{\partial B}{\partial t} < 0$, $\mathcal{E}$ is oriented in same direction as $\xrightarrow{\partial\Sigma}$ shown.

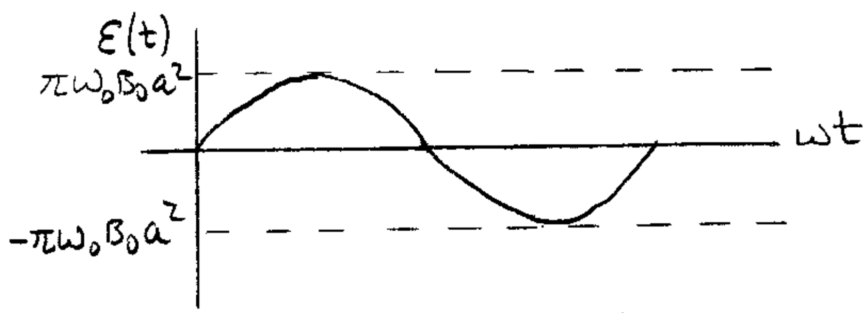
$\boxed{\text{Ex!}}$ ABOVE EXAMPLE WITH $B(t) = B_0 \cos(\omega t)$

$$\mathcal{E} = -\pi a^2 \frac{\partial B(t)}{\partial t}$$

$$\frac{\partial B(t)}{\partial t} = -\omega B_0 \sin(\omega t)$$

$$\mathcal{E}(t) = \pi \omega_0 B_0 a^2 \sin(\omega t)$$





- Oscillating magnetic field through Σ generates an oscillating flux through surface, which generates an oscillating emf around boundary. If boundary is a wire with free charge carriers, we have an oscillating ("alternating") current.

("A.C. GENERATOR")

$\boxed{Ex:}$ KEEP $\vec{B}$-field constant, but change area of loop

$$\mathcal{E} = -\frac{\partial \Phi_B}{\partial t}$$

$$\Phi_B = \int_{\Sigma} \vec{B} \cdot d\vec{A}$$

$$d\vec{A} = dA\,\hat{z} \qquad \vec{B} = B\hat{z}$$

$$= 2\pi r\,dr\,\hat{z}$$

$$\vec{B} \cdot d\vec{A} = 2\pi B\, r\,dr$$

$$\Phi_B = 2\pi B \int_0^a r\,dr$$

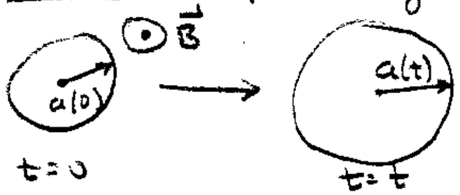
$$= \pi B a^2$$

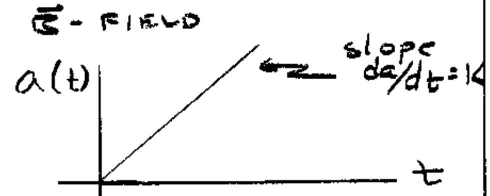
$$\frac{\partial \Phi_B}{\partial t} = 2\pi B a \frac{da}{dt}$$

$$\mathcal{E} = -2\pi B a \frac{da}{dt}$$

- if $\frac{\partial a}{\partial t} > 0$, then $\mathcal{E}$ is opposite to chosen boundary orientation
- if $\frac{\partial a}{\partial t} < 0$, the $\mathcal{E}$ is in same direction as chosen $\partial\Sigma$ orientation

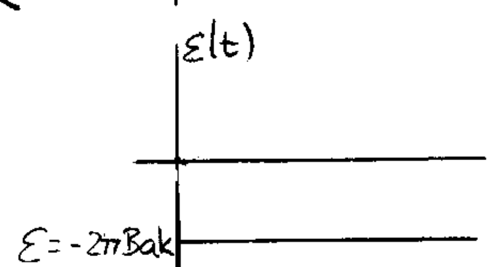
$\boxed{Ex:}$ Expanding loop: $a(t) = kt \quad 0 \le t$
IN CONSTANT $\vec{B}$-FIELD





$$a(t) = kt \qquad \frac{da}{dt} = k$$

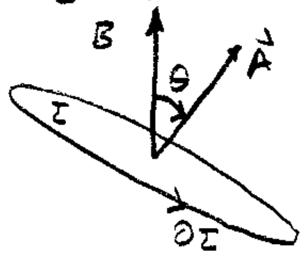
$$\mathcal{E} = -2\pi B a k$$



("D.C. GENERATOR")

$\boxed{Ex!}$ LOOP GEOMETRY FIXED, BUT ROTATING; $\vec{B}$-field FIXED

$\theta = \theta(t)$



$$\mathcal{E} = -\frac{\partial \Phi_B}{\partial t}$$

$$\Phi_B = \int_\Sigma \vec{B}\cdot d\vec{A} = \int_\Sigma B\,dA\cos\theta$$

$$= B\cos\theta \int_\Sigma dA$$

$$\Phi_B = AB\cos\theta$$

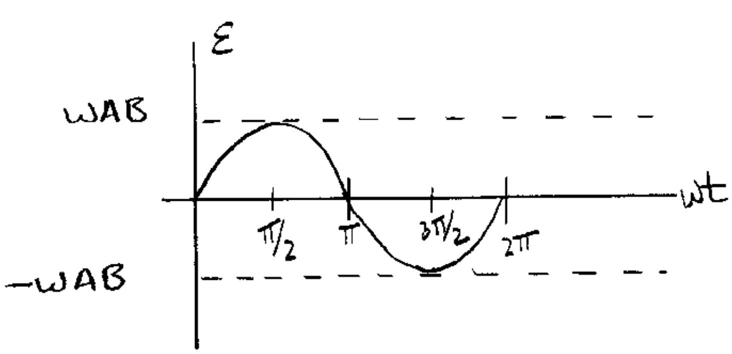
$$\frac{\partial \Phi_B}{\partial t} = -AB\sin\theta\,\frac{d\theta}{dt}$$

$$\mathcal{E} = AB\sin\theta\,\frac{d\theta}{dt}$$

$\boxed{Ex!}$ SUPPOSE THAT LOOP IS rotating relative to $\vec{B}$-field such that $\theta(t) = \omega t$

($\omega \equiv d\theta/dt \rightarrow$ angular velocity)

$$\mathcal{E} = \omega AB\sin(\omega t)$$



("A.C. GENERATOR")

- NOW, GO BACK AND TAKE A LOOK AT EACH OF THE ABOVE EXAMPLES AND TAKE NOTE OF THE ORIENTATIONS OF $\mathcal{E}$ IN EACH CASE. THESE ORIENTATIONS COME FROM

$$\mathcal{E} = \oint_{\partial \Sigma} \vec{E} \cdot d\vec{l}$$

WE'VE CHOSEN THE ORIENTATION OF $\partial\Sigma$, SO WE KNOW WHICH WAY IS "positive". SO WE HAVE SOME $\vec{E}$-FIELD THAT IS ORIENTED ACCORDING TO THE SIGN OF $\vec{E}$ RELATIVE TO $d\vec{l}$ $(\partial\Sigma)$.

POSITIVE CHARGE CARRIERS MOVE IN THE DIRECTION OF $\vec{E}$, SO THE CURRENTS IN SOME CONDUCTOR CONSTITUTING SOME $\partial\Sigma$ ARE IN THE DIRECTION OF $\vec{E}$ (or $\mathcal{E}$).

THESE CURRENTS PRODUCE MAGNETIC FIELDS OF THEIR OWN, WITH ORIENTATION GIVEN BY THE "RIGHT-HAND RULE".

NOTE THAT IN EACH CASE, THESE <u>INDUCED</u> MAGNETIC FIELDS OPPOSE THE ORIGINAL $\vec{B}$-FIELDS THAT CREATED THEM.

THIS IS

$\boxed{\text{LENZ'S LAW}}$

- An induced EMF will produce a current that always acts opposite to the change that caused it.

- NOTE THAT IF THIS WERE NOT THE CASE, THEN WE COULD ENHANCE THE ORIGINAL FIELD JUST BY PUTTING A CONDUCTING LOOP IN IT. WE COULD THEN MAKE AN ELECTRIC GENERATOR THAT COULD PRODUCE AN INFINITE AMOUNT OF ENERGY FOR FREE. THIS WOULD VIOLATE CONSERVATION OF ENERGY. THIS DOESN'T HAPPEN, WHICH LENDS FURTHER EVIDENCE FOR THE LAW.

- SOME ESOTERICA

EVERYTHING THAT WE HUMANS KNOW ABOUT CLASSICAL ELECTROMAGNETISM CAN BE SUMMARIZED BY FIVE SIMPLE EQUATIONS —

DIFFERENTIAL FORM	same equation $\longleftrightarrow$ different language	INTEGRAL FORM
$\vec{\nabla}\cdot\vec{D} = \rho_f$	(MAXWELL'S EQUATIONS)	$\int \vec{D}\cdot d\vec{A} = \int \rho_f \cdot d^3x$
$\vec{\nabla}\times\vec{E} = -\partial\vec{B}/\partial t$		$\oint \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\int \vec{B}\cdot dt$
$\vec{\nabla}\cdot\vec{B} = 0$		$\int \vec{B}\cdot d\vec{A} = 0$
$\vec{\nabla}\times\vec{H} = \vec{J}_f + \partial\vec{D}/\partial t$		$\int \vec{H}\cdot d\vec{l} = \int \vec{J}_f\cdot d\vec{A} + \frac{\partial}{\partial t}\int \vec{D}\cdot d\vec{A}$

and

$$\boxed{\vec{F} = q(\vec{v}\times\vec{B})}$$ (Lorentz Force Law)

TO DATE, IT APPEARS THAT THE SOLUTION TO ANY PROBLEM IN CLASSICAL E&M RESIDES IN THESE FIVE SIMPLE EQUATIONS. ANY PROBLEM YOU WILL FACE WITH E&M CAN BE UNDERSTOOD IN TERMS OF JUST FIVE EQUATIONS.

THAT'S BEAUTY

THAT'S POWER

THAT'S PHYSICS