I. MAGNETIC FORCE

- LORENTZ FORCE LAW

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

FORCE ON A PARTICLE OF CHARGE q MOVING THOUGH ELECTROMAGNETIC FIELD $(\vec{E}, \vec{B})$, WITH VELOCITY $\vec{v}$.

$$\vec{F}_E = q\vec{E}$$

IS JUST THE PART DUE TO THE $\vec{E}$-FIELD THAT WE'VE ALREADY SEEN.

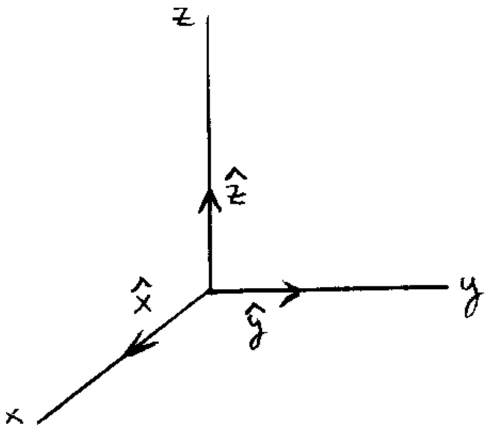
- HERE, WE WILL WANT TO CONCENTRATE ON THE FORCE DUE TO THE MAGNETIC FIELD $\vec{B}$

$$\boxed{\vec{F}_M = q\vec{v} \times \vec{B}}$$

$\vec{F}_M$ = MAGNETIC FORCE EXPERIENCED BY CHARGE q MOVING WITH VELOCITY $\vec{v}$ THROUGH EXTERNAL MAGNETIC FIELD $\vec{B}$.

- THE VELOCITY-DEPENDENCE AND THE CROSS-PRODUCT MAKES THESE PROBLEMS A BIT MORE COMPLICATED THAN ELECTROSTATICS.

- HANDY TABLE OF VECTOR MULTIPLICATION OF CARTESIAN UNIT BASIS VECTORS



$$\boxed{\begin{aligned}(\hat{x}\times\hat{y}) &= \hat{z}\\ (\hat{y}\times\hat{z}) &= \hat{x}\\ (\hat{z}\times\hat{x}) &= \hat{y}\end{aligned}}$$

In general,

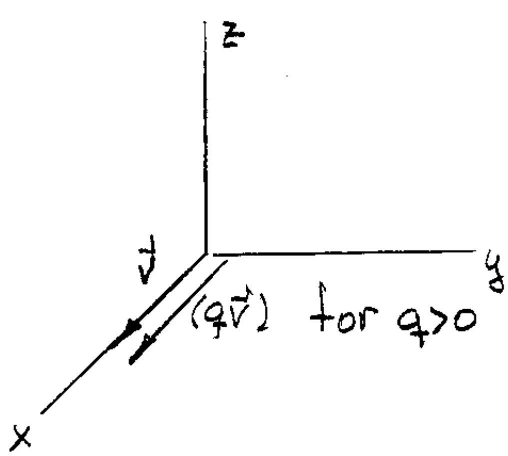
$$\vec{A}\times\vec{B} = -(\vec{B}\times\vec{A})$$

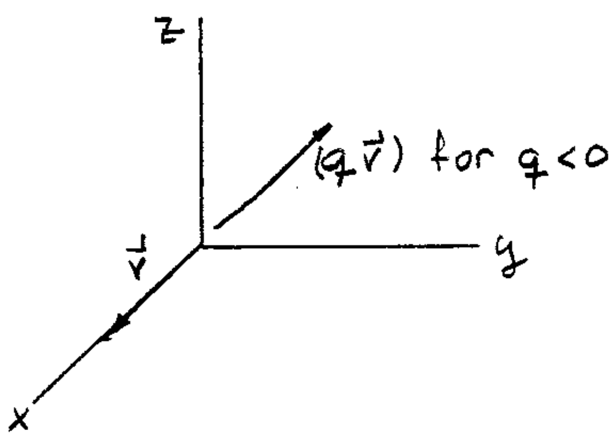
So have

$$(\hat{y}\times\hat{x}) = -\hat{z}$$
$$(\hat{z}\times\hat{y}) = -\hat{x}$$
$$(\hat{x}\times\hat{z}) = -\hat{y}$$

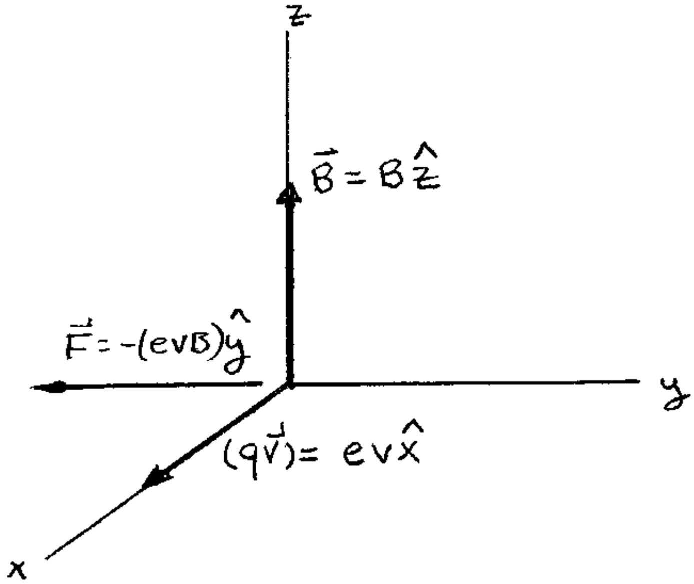
Really only need to remember first set (boxed set) and the cyclic ordering.

- HELPFUL TO CONSIDER MAGNITUDE OF VECTORS AS ALWAYS POSITIVE. THE SIGN TELLS YOU DIRECTION RELATIVE TO THE BASIS VECTORS



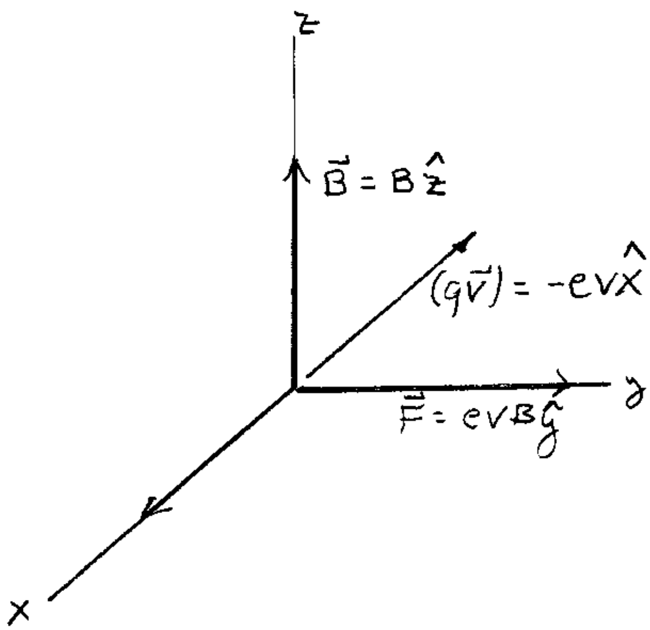


$\boxed{Ex:}$ Proton with charge $q = +e$, velocity $\vec{V} = V\hat{x}$, MOVING IN $\vec{B}$ field, $\vec{B} = B\hat{z}$



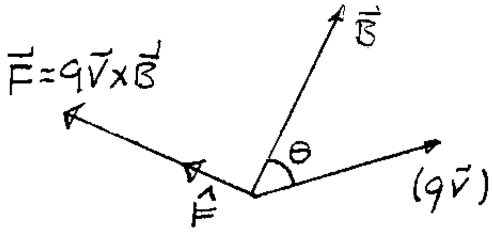
$$\vec{F} = q\vec{v} \times \vec{B}$$
$$= (ev\hat{x}) \times (B\hat{z})$$
$$= (evB)(\hat{x} \times \hat{z})$$
$$\vec{F} = -(evB)\hat{y}$$

$\boxed{Ex:}$ Electron with charge $q = -e$, velocity $\vec{V} = V\hat{x}$, $\vec{B} = B\hat{z}$



$$\vec{F} = q\vec{v} \times \vec{B}$$
$$= (-ev\hat{x}) \times (B\hat{z})$$
$$= (-evB)(\hat{x} \times \hat{z})$$
$$= (-evB)(-\hat{y})$$
$$\vec{F} = evB\hat{y}$$

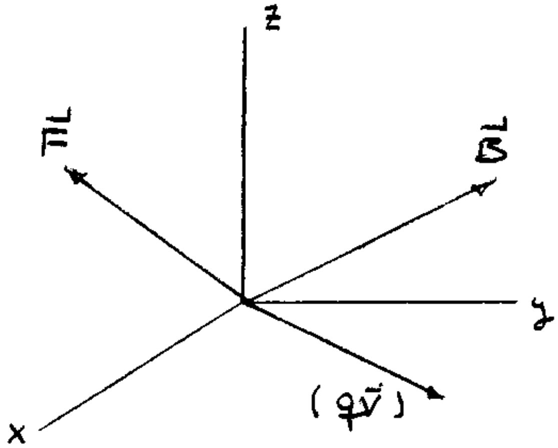
$\boxed{Ex:}$ WITHOUT REFERENCE TO ANY PARTICULAR COORDINATE SYSTEM



$$F = |F| = qvB\sin\theta$$

$$\hat F = \frac{\vec F}{|F|} = \frac{q\vec v\times\vec B}{|q\vec v\times\vec B|}$$

$\boxed{Ex:}$ FOR GENERAL VECTOR COMPONENTS IN ARBITRARY CARTESIAN COORDINATE SYSTEM



$$q\vec v = qv^x\hat x + qv^y\hat y + qv^z\hat z$$

$$\vec B = B^x\hat x + B^y\hat y + B^z\hat z$$

$$\vec F = F^x\hat x + F^y\hat y + F^z\hat z$$

$$\vec F = q\vec v\times\vec B$$

$$= \det\begin{vmatrix} \hat x & \hat y & \hat z \\ qv^x & qv^y & qv^z \\ B^x & B^y & B^z \end{vmatrix}$$

$$\vec F = \hat x\left(qv^yB^z - qv^zB^y\right) + \hat y\left(qv^zB^x - qv^xB^z\right) + \hat z\left(qv^xB^y - qv^yB^x\right)$$

$$= F^x\hat x + F^y\hat y + F^z\hat z$$

- $\vec{F} = q\vec{v} \times \vec{B}$

tells us how to calculate the instantaneous force. RECALL FROM 1A,

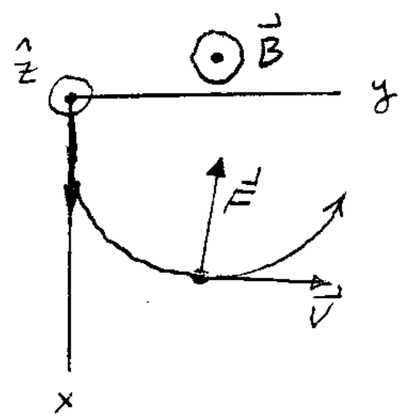
$$\vec{F} = m\vec{a}$$

SO $\qquad m\vec{a} = q\vec{v} \times \vec{B}$

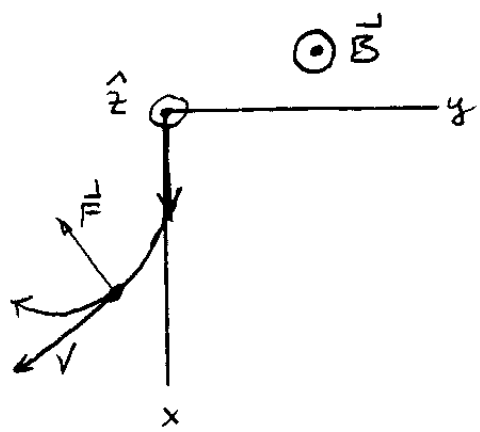
OR $\qquad m\dfrac{d\vec{v}}{dt} = q\vec{v} \times \vec{B}$

OR $\qquad m\dfrac{d^2\vec{r}}{dt^2} = q\dfrac{d\vec{r}}{dt} \times \vec{B}$

If $(q\vec{v} \times \vec{B}) \neq 0$, the charge will experience a force, causing it to accelerate



ELECTRON WITH Velocity $\vec{V}$ in $\vec{B} = B_0\hat{z}$



PROTON

- MAGNETIC FORCES DO NO WORK

$$dW = \vec{F}\cdot d\vec{r}$$
$$= \vec{F}\cdot \frac{d\vec{r}}{dt}\, dt$$
$$= \vec{F}\cdot\vec{v}\, dt$$

FOR MAGNETIC FORCE,

$$\vec{F} = q\vec{v}\times\vec{B}$$
$$dW = (q\vec{v}\times\vec{B})\cdot\vec{v}\, dt$$

Recall that $(q\vec{v}\times\vec{B})$ is either zero (if $\vec{v}$ and $\vec{B}$ are parallel) or it is a vector PERPENDICULAR TO BOTH $\vec{v}$ and $\vec{B}$.

So, $$(q\vec{v}\times\vec{B})\cdot\vec{v} = 0$$

OR
$$dW = 0$$

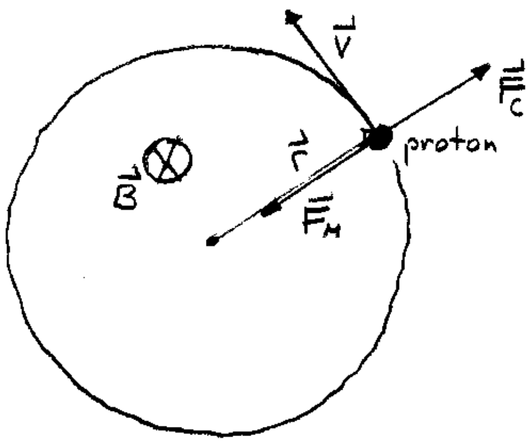
Hence, a magnetic field can cause a particle to accelerate by changing the direction of its velocity, but it can't change the Kinetic Energy of the particle

$$\Delta KE = \Delta W$$
$$= 0$$

- CIRCULAR ORBITS OF PARTICLES IN MAGNETIC FIELDS

FOR $\vec{F} \neq 0$ AND $\vec{v} \neq 0$, WE HAVE

$\vec{v} \cdot \vec{a} = 0 \Rightarrow$ PARTICLE MOVES IN A CIRCULAR ORBIT



$$\vec{F}_c = \frac{mv^2}{r}\hat{r}$$

$$\vec{F}_M = -qvB\,\hat{r}$$

$$\vec{F}_c + \vec{F}_M = 0$$

$$\frac{mv^2}{r} = qvB$$

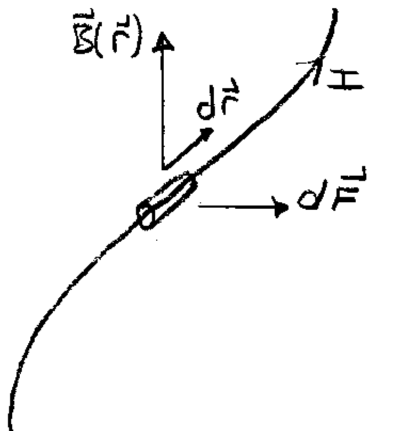
$$\boxed{r = \frac{mv}{qB}}$$

* Applications – Particle accelerators

SYNCHROTRON RADIATION
(USEFUL FOR RADIOLOGY & ONCOLOGY)

Accelerating charges emit radiation. Synchotron radiation can be custom-tuned and focussed right where you want it.

- CURRENT-CARRYING WIRES

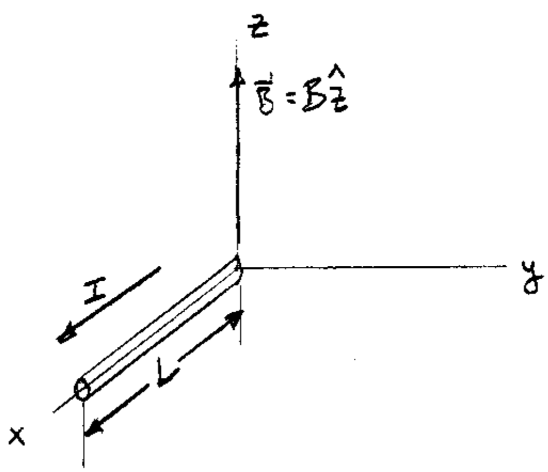


$$\vec{F} = q\vec{v} \times \vec{B}$$
$$= q\frac{d\vec{r}}{dt} \times \vec{B}$$
$$d\vec{F} = dq\frac{d\vec{r}}{dt} \times B$$
$$= \left(\frac{dq}{dt}\right) d\vec{r} \times \vec{B}$$

Force $d\vec{F}$ on infinitesimal length $d\vec{r}$ of wire carrying current I in external magnetic field $\vec{B}$.

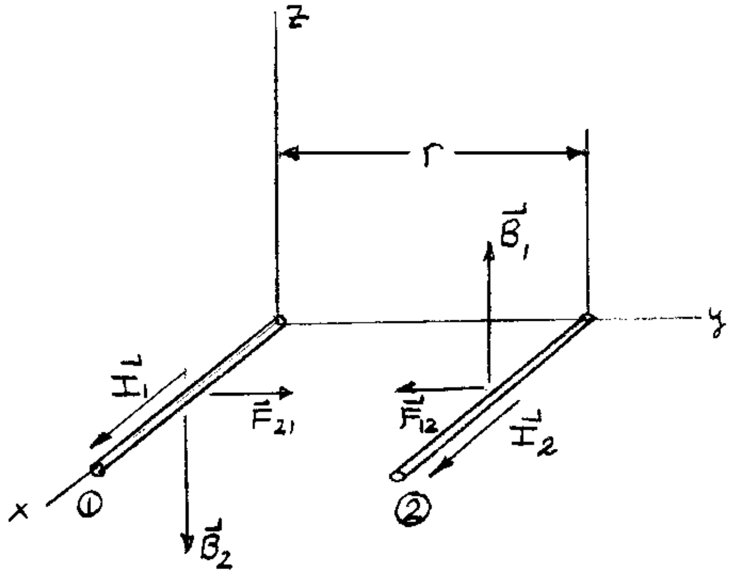
$$\boxed{d\vec{F} = I\, d\vec{r} \times \vec{B}}$$

$\boxed{\text{Ex}}$



$$d\vec{F} = I\, d\vec{r} \times \vec{B}$$
$$= I(dx\,\hat{x}) \times (B\hat{z})$$
$$= (I\,dx\,B)(\hat{x} \times \hat{z})$$
$$d\vec{F} = -(IB\,dx)\,\hat{y}$$
$$\vec{F} = \int d\vec{F} = \int_0^L -IB\,dx\ \hat{y}$$
$$\vec{F} = -IBL\hat{y}$$
$$\boxed{F = IBL}$$

- FORCES BETWEEN CURRENT-CARRYING WIRES



① Produces a magnetic field at ②'s location

$$\vec{B}_1 = \frac{\mu_0 I_1}{2\pi r}\,\hat{z}$$

Charge moving down ② feels a force

$$d\vec{F}_{12} = I_2\, d\vec{r}_2 \times \vec{B}_1$$

$$\vec{F}_{12} = -(I_2 L_2 B_1)\,\hat{y}$$

$$= -\left(\frac{\mu_0 I_1 I_2}{2\pi r} L_2\right)\hat{y}$$

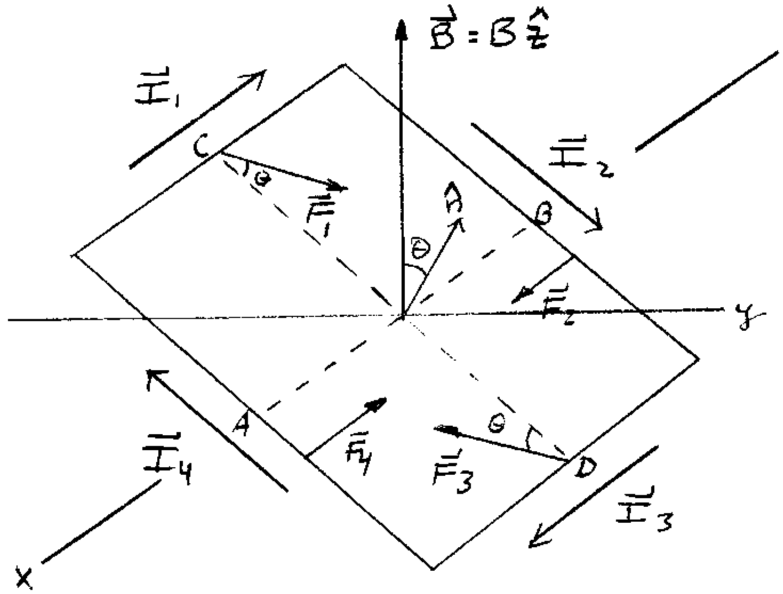
Likewise, ② PRODUCES MAGNETIC FIELD AT ①

$$\vec{F}_{21} = \left(\frac{\mu_0 I_1 I_2}{2\pi r} L_1\right)\hat{y}$$

- Currents traveling in same direction attract each other.
- Currents traveling in opposite directions repel
- FORCE PER UNIT LENGTH IS

$$F/L = \frac{\mu_0 I_1 I_2}{2\pi r}$$

- FORCE AND TORQUE ON CURRENT LOOPS IN EXTERNAL FIELDS



- square loop of side L

$$d\vec{F}_1 = I\,(-dx_1\,\hat{x}) \times (B\hat{z})$$
$$= -I\,B\,dx\,\hat{y}$$
$$\vec{F}_1 = -I\,BL\,\hat{y}$$
$$d\vec{F}_2 = I\,d\vec{r}_2 \times \vec{B}$$
$$\vec{F}_2 = I\,LB\sin\theta\,\hat{x}$$
$$\vec{F}_3 = IBL\,\hat{y}$$
$$\vec{F}_4 = -ILB\sin\theta\,\hat{x}$$

TOTAL FORCE ON LOOP:

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$$
$$= 0$$

- Current LOOP EXPERIENCES NO NET FORCE IN <u>UNIFORM</u> MAGNETIC FIELD

* NOTE IF $\vec{B}$ IS NOT UNIFORM, THERE WILL BE SUCH A FORCE.

BUT LOOK at $\vec{F}_1$ AND $\vec{F}_4$. THEY EXERT A <u>TORQUE</u> ABOUT THE LINE (A-B)

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = rF\sin\theta$$

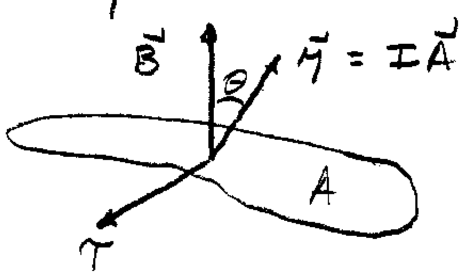
$$= \left(\frac{L}{2}\right)(2IBL)\sin\theta$$

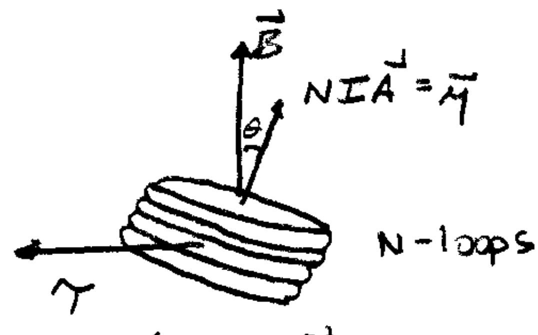
$$\tau = IL^2 B\sin\theta$$

Let $\vec{\mu} \equiv I\vec{A} = IA\hat{n} = I(L^2)\hat{n}$

$$\boxed{\vec{\tau} = \vec{\mu} \times \vec{B}}$$

This is a general result. $\bar{\mu}$ is called the *magnetic dipole moment*. Any current loop will experience a torque in an external magnetic field — it will rotate, attempting to line-up $\bar{\mu}$ with $\vec{B}$.

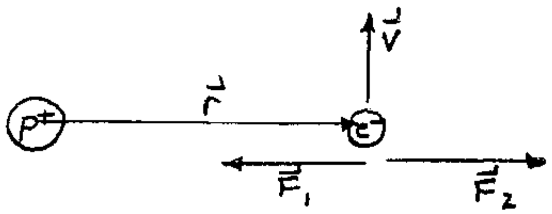


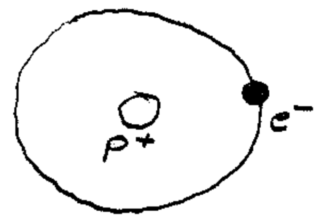


$$\vec{\tau} = \vec{\mu} \times \vec{B}$$
$$= I\vec{A} \times \vec{B}$$
$$\tau = IAB\sin\theta$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$
$$= NI\vec{A} \times \vec{B}$$
$$\tau = NIAB\sin\theta$$

$\boxed{\text{Ex}}$ ORBITAL MAGNETIC MOMENT OF HYDROGEN ATOM (SIMPLE MODEL)





$$\vec{F}_1 = -\frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2}\hat{r} \qquad \vec{F}_2 = \frac{mv^2}{r}\hat{r}$$

$$\vec{F} = \vec{F}_1 + \vec{F}_2 = 0 \quad \text{for circular orbit}$$

$$\vec{F} = \left(\frac{mv^2}{r} - \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2}\right)\hat{r} = 0$$

$$\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2}$$

$$v^2 = \frac{e^2}{4\pi\epsilon_0 m r}$$

$$v^2 = \frac{(1.602\times10^{-19}\,q)^2}{(4\pi)(8.85\times10^{-19}\,\frac{q^2}{N\cdot m^2})(9.11\times10^{-31}\,kg)(5.3\times10^{-11}\,m)}$$

$$v = 2.2\times10^{6}\ m/s$$

$$I = \frac{dq}{dt} = e/T \qquad T = 2\pi r/v$$

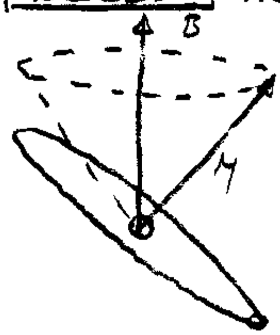
$$I = \frac{ev}{2\pi r}$$

$$\mu = IA = \frac{ev}{2\pi r}\cdot\pi r^2 = \frac{1}{2}evr$$

$$\mu = \left(\frac{1}{2}\right)(1.602\times10^{-19}\,q)(2.2\times10^{6}\,m/s)(5.3\times10^{-11}\,m)$$

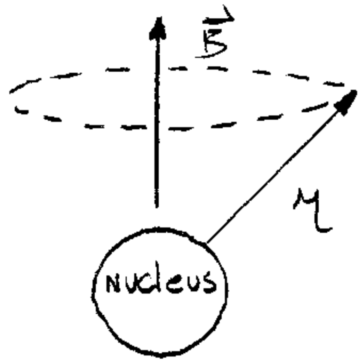
$$\mu = 9.3\times10^{-24}\ A\cdot m^2$$

- BECAUSE OF THIS TORQUE, ATOMS WILL PRECESS IN A MAGNETIC FIELD



← The atom wobbles like a tiny top in $\vec{B}$-field

- EVEN THE PARTICLES THEMSELVES HAVE THEIR OWN MAGNETIC MOMENTS (DUE TO THEIR Quantum-mechanical "SPIN").



Nuclei "wobble" in MAGNETIC FIELD AT CHARACTERISTIC RATES

BY WIGGLING THE $\vec{B}$-field at their favorite wobbling rates, we can "see" them. THIS IS

$\boxed{\text{NMR}}$