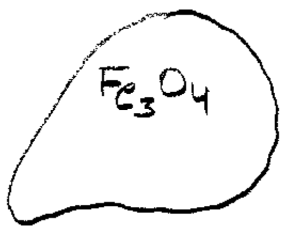
# I. MAGNETISM (QUALITATIVE)

## (A) MAGNETITE (MAGNETISM NOTICED)

- $Fe_3O_4$ (actually contains both ferrous and ferric states)

  $(Fe^{2+})(Fe^{3+})_2(O^{2-})_4$

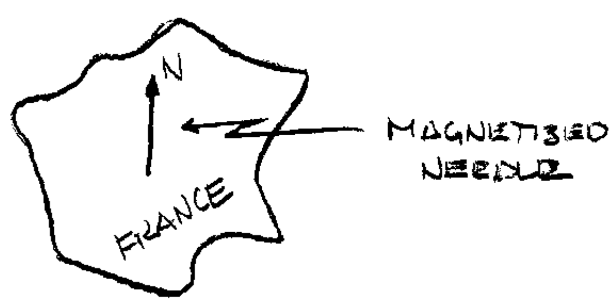
- CHINESE, ca. 2600 BC
  GREEKS, ca. 400 BC (LODESTONE)



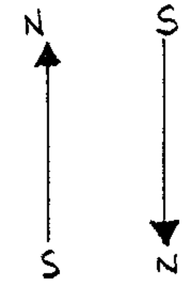

- attracts some other substances
- attractive behavior transferrable to substances it attracts

## (B) MAGNETIC COMPASS (MAGNETISM USED)

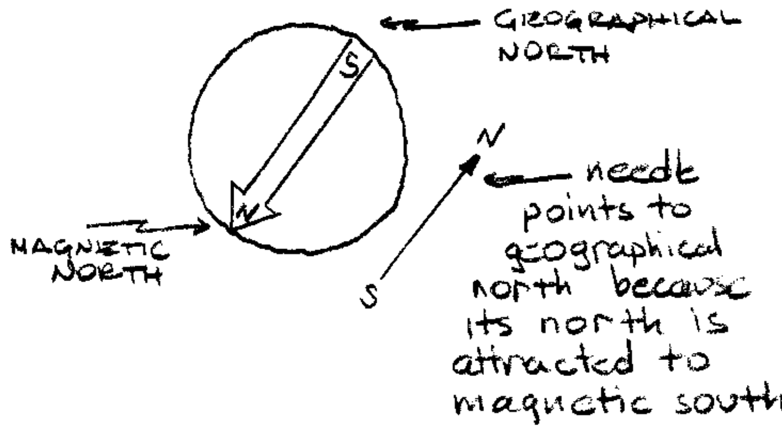
- MARICOURT (ca. 1269)
- RUB NEEDLE ON $Fe_3O_4$, FIND THAT NEEDLE ALIGNS ITSELF "NORTH-TO-SOUTH" RELATIVE TO EARTH.
- YOU ALREADY KNOW WHICH WAY NORTH IS, SO LABEL END POINTING NORTH "NORTH", LABEL OTHER END "SOUTH".



- Take two needles so-labelled, put near each other, and this happens




- Geographical North on Earth is a "South" magnetic pole (Kinda funny!)




- Now ready to make compasses, put one on each of your sailing ships, and go conquer the world.

II. AN ACCIDENT HAPPENS...

(A) OERSTED (ca. 1820, COPENHAGEN)

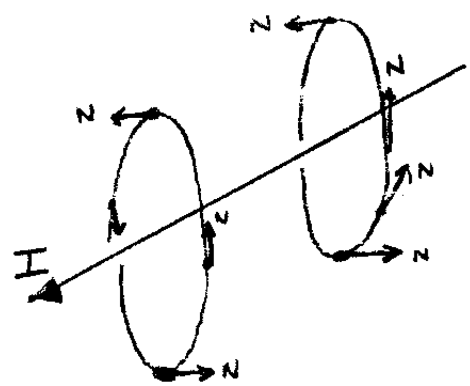
WHILE LECTURING, ACCIDENTALLY KNOCKS-OVER APPARATUS. NOTICES THAT CURRENT-CARRYING WIRE CAUSES NEARBY MAGNETIC COMPASS NEEDLE TO DEFLECT.

PUBLISHES OBSERVATIONS, AMPÈRE, BIOT AND SAVART GET-IN ON THE ACTION.

(B) AMPÈRE'S LAW

AMPERE CLOSELY STUDIES THE EFFECT, AND NOTES THAT

- COMPASS NEEDLE ORIENTS ITSELF PERPENDICULAR TO CURRENT DIRECTION, WITH A SENSE OF "CIRCULATION" AROUND WIRE

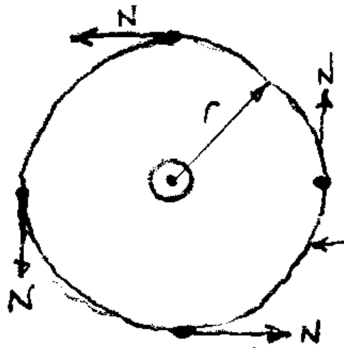



- NEED SOME NEW NOTATION NOW —

  $\odot$ = VECTOR ARROW COMING OUT OF THIS PAPER

  $\otimes$ = VECTOR ARROW GOING INTO THIS PAPER

• LOOKING DOWN INTO ABOVE CURRENT FROM POINT I ABOVE —




CIRCLE REPRESENTS LOCUS OF BASE-POINTS (WHERE YOU PUT THE COMPASS)

Direction of compass IN DIRECTION OF MAGNETIC FIELD

• CLOSER COMPASS IS TO I (THE SMALLER r), COMPASS ALIGNS ITSELF WITH MAGNETIC FIELD MORE STRONGLY

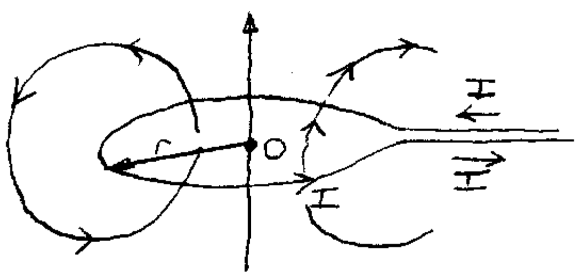
$$B = B(r) \sim 1/r$$

• MORE CURRENT I, STRONGER THE EFFECT

$$B \sim I$$

SO,
$$B(r) \sim \frac{I}{r}$$

• NOW, WIND current WIRE INTO A LOOP —




$$B(0) \sim I$$

$$B(0) \sim \frac{1}{r}$$

$$B(0) \sim \frac{I}{r}$$

- KEEP DOING ALL SORTS OF WIRE/LOOP/FIELD MEASUREMENTS, AND EVENTUALLY NOTICE THAT

$$\boxed{\mu_0 I = \oint_{\text{closed loop}} \vec{B}(\vec{r})\cdot d\vec{r}}$$

AMPÈRE'S LAW

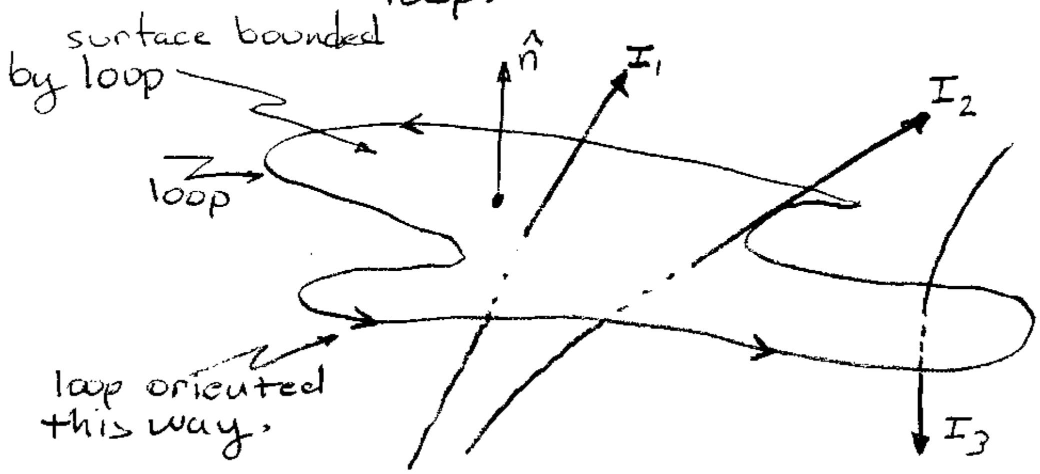
$\mu_0$ = "PERMEABILITY" OF SURROUNDING VACUUM
$= 4\pi \times 10^{-7}$ T·m/A

(T = "Tesla", a unit of magnetic field strength

$1\ \text{Tesla} = \frac{1\ \text{N}\cdot\text{s}}{\text{q}\cdot\text{m}}$ )

$I$ = <u>total</u> <u>net</u> current passing through imaginary surface bounded by imaginary loop being integrated over

$\vec{B}(\vec{r})$ = MAGNETIC FIELD at point $\vec{r}$ on loop.

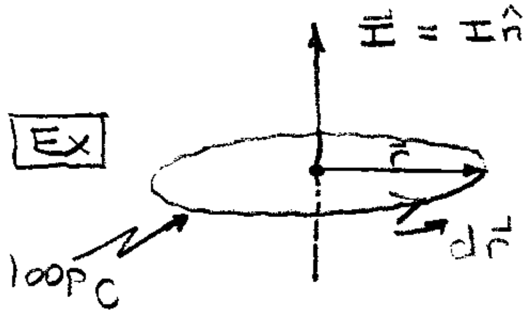



$$I = I_1 + I_2 - I_3$$

Choice of surface orientation ($\hat{n}$) defines sense of loop orientation

$d\vec{r}$ in direction of loop orientation.

$\boxed{\text{Ex}}$




$\vec{B}$- field around long current-carrying wire

$$\mu_0 I = \oint_C \vec{E}(\vec{r}) \cdot d\vec{r}$$

By symmetry of rotation around $\vec{I}$,

$$\mu_0 I = B(r) \oint_C dr$$

$$= B(r)(2\pi r)$$

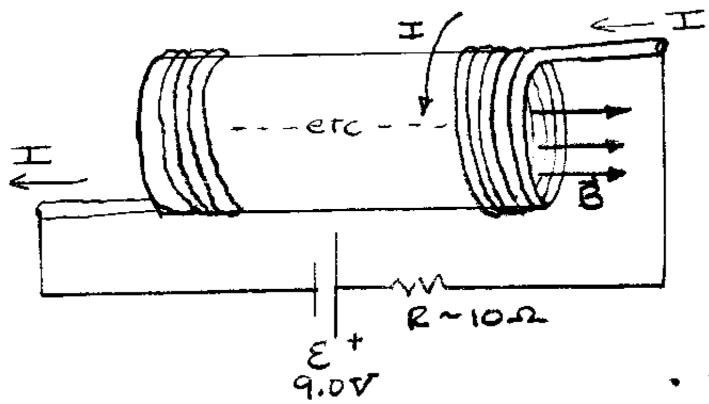
$$\boxed{B(r) = \frac{\mu_0 I}{2\pi r}}$$

Magnitude of magnetic field around long current-carrying straight wire, in vacuum ($\mu_0$), at perpendicular distance $r$ from the wire.

NOTE THAT MAGNETIC FIELDS ARE VECTOR FIELDS, SO THE ALSO HAVE A DIRECTION IN ADDITION TO THEIR MAGNITUDE. ABOVE, THE DIRECTION OF

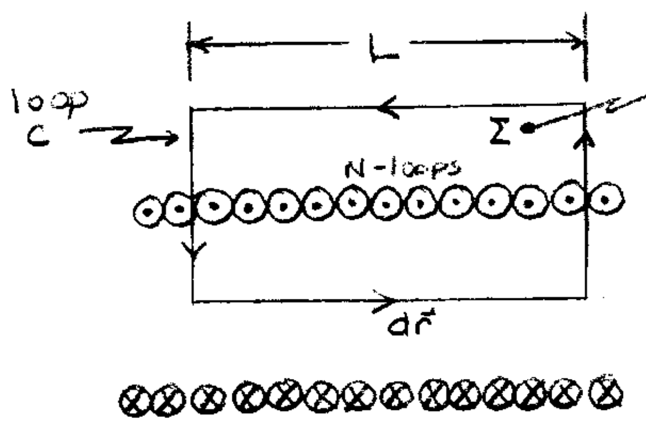
$\vec{B}(\vec{r})$ IS IN THE DIRECTION $d\vec{r}$ SHOWN.

**Ex** A LONG SOLENOID




WRAP A BUNCH OF WIRE AROUND A PAPER-TOWEL TUBE, FOR EXAMPLE. HOOK-UP TO BATTERY & RESISTOR AS SHOWN (RESISTOR IS THERE ONLY TO HELP MAKE BATTERY LAST LONGER)

- SAFE TO DO AT HOME TRY IT. SEE WHAT HAPPENS TO NEARBY COMPASS




SURFACE ENCLOSED BY C

CROSS-SECTIONAL VIEW. SLICED-DOWN MIDDLE IN VERTICAL PLANE

Let $I_o$ be current in wire
Let $\eta$ be (# of current loops/unit length) = $N/L$

Total current passing through $\Sigma$

$$I = N I_o = \eta L I_o$$

$$\mu_o I = \oint_C \vec{B}(\vec{r}) \cdot d\vec{r} \qquad \text{AMPÈRE'S LAW}$$

$$\mu_o \eta L I_o = BL$$

$$\boxed{B = \mu_o \eta I_o}$$

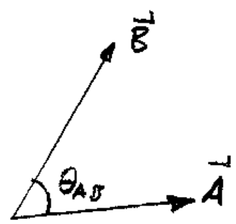
Magnetic field strength inside long solenoid carrying current $I_o$ with $\eta$ turns of wire per unit length.

## III. Biot-Savart Law

- Allows one to compute $\vec{B}$-field of arbitrary constant-current sources

(1) First, a little vector-refreshment

- Vectors have <u>two</u> types of multiplication




- Scalar-Product (Dot-Product)

$$\vec{A}\cdot\vec{B} = A^x B^x + A^y B^y + A^z B^z$$

$$= AB\cos\theta_{AB}$$

$$= \text{scalar}$$

- Vector-Product (Cross-Product)

$$\vec{A}\times\vec{B} = \vec{C}$$

$$= \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A^x & A^y & A^z \\ B^x & B^y & B^z \end{vmatrix}$$

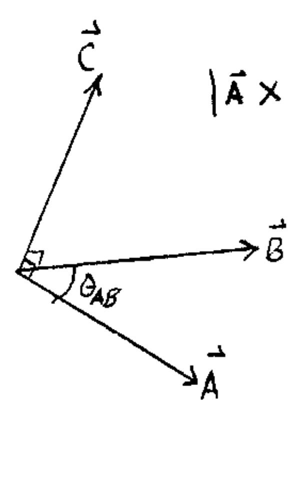
$$= \hat{x}(A^y B^z - A^z B^y)$$

$$+ \hat{y}(A^z B^x - A^x B^z)$$

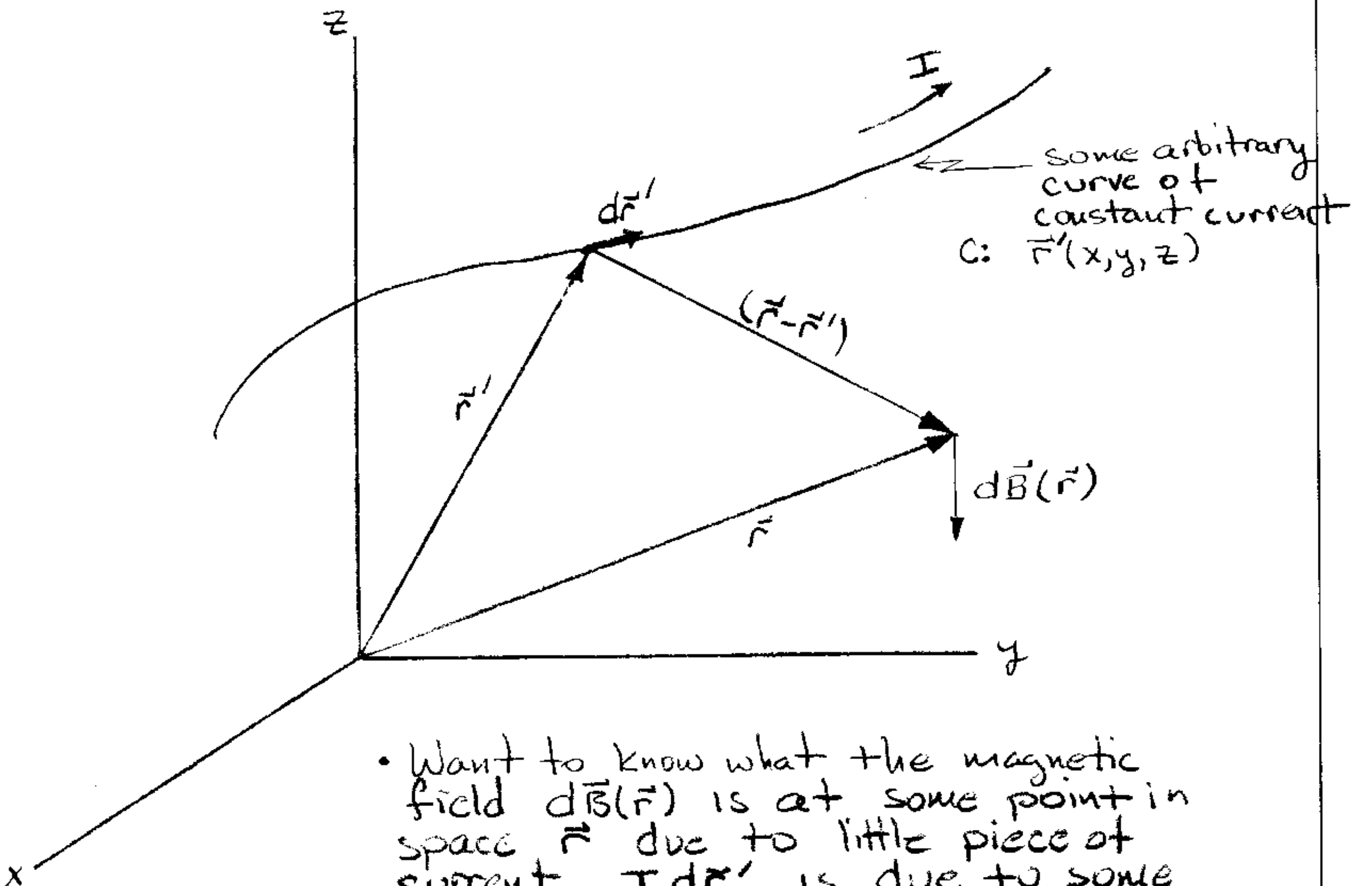
$$+ \hat{z}(A^x B^y - A^y B^x)$$

$$= \text{vector} = C^x\hat{x} + C^y\hat{y} + C^z\hat{z}$$

$$|\vec{A}\times\vec{B}| = |\vec{C}| = AB\sin\theta_{AB}$$



## (2) THE BIOT-SAVART LAW




- Want to know what the magnetic field $d\vec{B}(\vec{r})$ is at some point in space $\vec{r}$ due to little piece of current $I d\vec{r}'$ is due to some curve of current $\vec{r}'(x,y,z)$

$$\boxed{d\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{I\, d\vec{r}' \times (\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}}$$

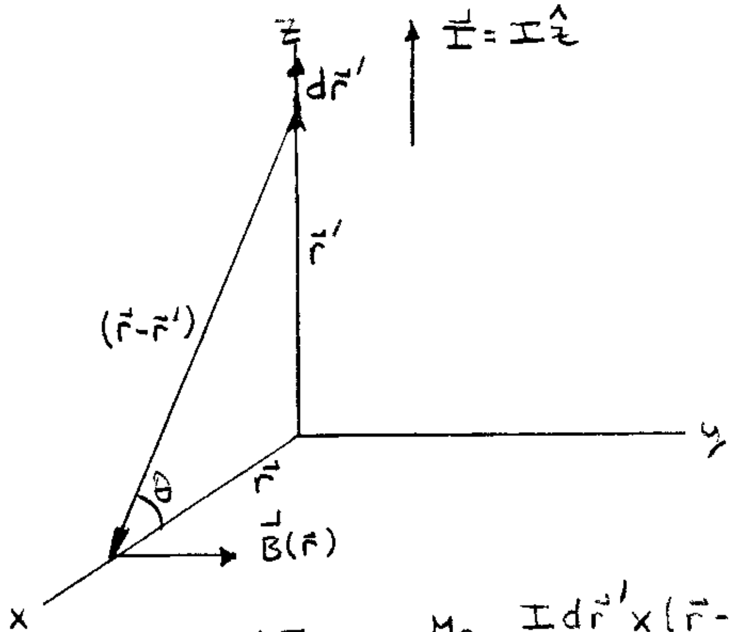
- To get total $\vec{B}$-field at $\vec{r}$ due to whole line of current, add-up all the little contributions of each piece

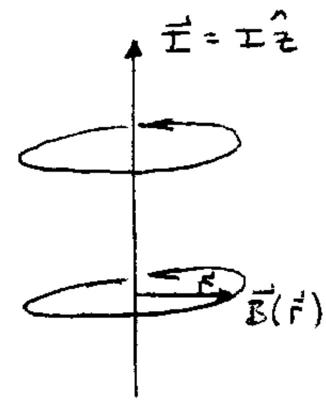
$$\vec{B}(\vec{r}) = \int d\vec{B}(\vec{r})$$

$$\boxed{\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int_{\substack{\text{current}\\ \text{line}}} \frac{I\, d\vec{r}' \times (\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}}$$

Biot-SAVART LAW

$\boxed{\text{EX:}}$ MAGNETIC FIELD AROUND LONG ("INFINITE") STRAIGHT LINE OF CURRENT




$$d\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\,\frac{I\,d\vec{r}\,' \times (\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}$$

• Start with Biot-Savart Law

$$dB(r) = \frac{\mu_0 I}{4\pi}\,\frac{dr'\,|\vec{r}-\vec{r}\,'|\sin\theta}{|\vec{r}-\vec{r}\,'|^3}$$

• Consider magnitude first

$$= \frac{\mu_0 I}{4\pi}\,\frac{dr'\sin\theta}{(r^2+r'^2)}$$

$$\sin\theta = \frac{r'}{(r^2+r'^2)^{1/2}}$$

$$dB(r) = \frac{\mu_0 I}{4\pi}\,\frac{dr'\,r'}{(r^2+r'^2)^{3/2}}$$

$$= \frac{\mu_0 I}{4\pi}\,\frac{r\,d(r'/r)\cdot r\,(r'/r)}{r^3\left(1+(r'/r)^2\right)^{3/2}}$$

$$= \frac{\mu_0 I}{4\pi r}\,\frac{d(r'/r)}{\left(1+(r'/r)^2\right)^{3/2}}$$

$$(r'/r) = \tan\theta$$

$$d(r'/r) = \sec^2\theta\, d\theta$$

$$dB(r) = \frac{\mu_0 I}{4\pi r} \frac{\sec^2\theta\, d\theta}{(1+\tan^2\theta)^{3/2}}$$

$$1+\tan^2\theta = \sec^2\theta$$

$$(1+\tan^2\theta)^{3/2} = \sec^3\theta$$

$$dB(r) = \frac{\mu_0 I}{4\pi r} \frac{\sec^2\theta}{\sec^3\theta} d\theta$$

$$= \frac{\mu_0 I}{4\pi r} \frac{d\theta}{\sec\theta}$$

$$= \frac{\mu_0 I}{4\pi r} \cos\theta\, d\theta$$

$$B(r) = \int dB(r)$$

$$= \frac{\mu_0 I}{4\pi r} \int_{-\pi/2}^{\pi/2} \cos\theta\, d\theta$$

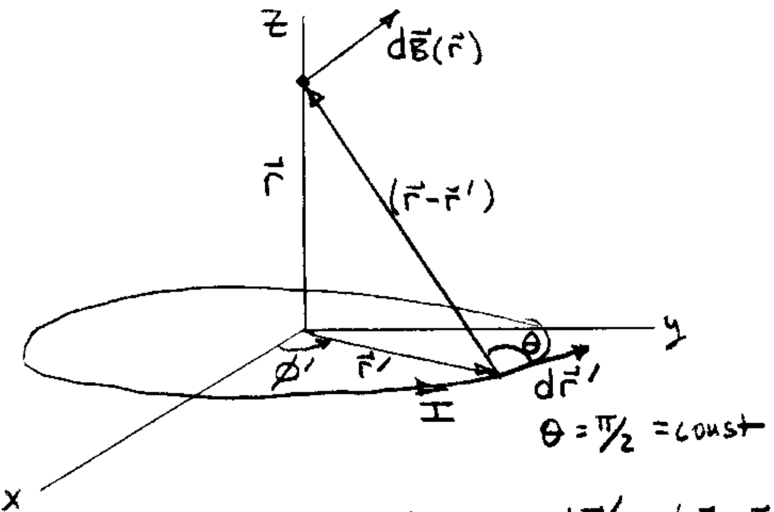
$$= \frac{\mu_0 I}{4\pi r} (\sin\theta)\Big|_{-\pi/2}^{\pi/2}$$

$$\boxed{B(r) = \frac{\mu_0 I}{2\pi r}}$$

* Overkill - we already got this much more simply from ampère's Law.

Lesson: Try to use Ampère's Law first. Resort to Biot-Savart if not useful.

$\boxed{\text{Ex}}$ "AMPÈRIAN" CURRENT-LOOP (AMPÈRE'S LAW NOT USEFUL HERE DUE TO LACK OF AN OBVIOUS SYMMETRY CAPABLE OF SIMPLIFYING INTEGRAL.)




$$d\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\,\frac{I\,d\vec{r}\,' \times (\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}$$

$$|d\vec{r} \times (\vec{r}-\vec{r}\,')| = dr'\,|\vec{r}-\vec{r}\,'|\sin\theta$$

$$= r'd\phi'\,(r^2+r'^2)^{1/2} \qquad (dr' = r'd\phi')$$

$$dB(r) = \frac{\mu_0 I}{4\pi}\,\frac{r'(r^2+r'^2)^{1/2}d\phi'}{(r^2+r'^2)^{3/2}}$$

$$= \frac{\mu_0 I}{4\pi}\,\frac{r'd\phi'}{(r^2+r'^2)}$$

But $r = \text{const}$, $r' = \text{const}$ as $\phi'$ changes,

$$B(r) = \int dB(r) = \frac{\mu_0 I}{4\pi}\,\frac{r'}{r^2+r'^2}\int_0^{2\pi} d\phi'$$

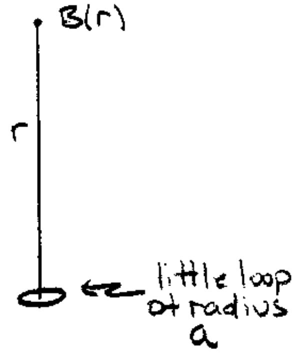
$$\boxed{B(r) = \frac{\mu_0 I\, r'}{2(r^2+r'^2)}}$$

magnetic field strength along symmetry axis of current loop of radius $r'$ at a distance $r$ from its center.

For convenience, let's call $r' = a$ = radius of current loop.

$$B(r) = \frac{\mu_0 I}{2} \frac{a}{(a^2+r^2)}$$

There are a couple of cases worth looking at




(1) $r \gg a$

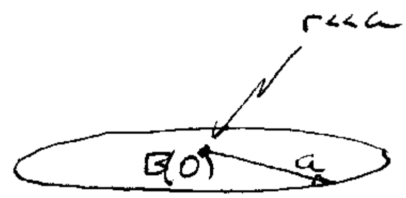
$$B(r) = \frac{\mu_0 I}{2} \frac{a}{r^2(1+(a/r)^2)}$$

$$\boxed{B(r) \simeq \frac{\mu_0 I}{2} \frac{a}{r^2}}$$

Field-strength falling-off like $1/r^2$ as we get farther away from loop (or as loop becomes very tiny — like atomic-size).

(2) for $r \ll a$

$$B(r) = \frac{\mu_0 I}{2} \frac{a}{a^2(1+(r/a)^2)}$$




$$B(r) \simeq \frac{\mu_0 I}{2a}$$

Right near center of loop, We have the expression given in Hecht

$$\boxed{B(0) = \frac{\mu_0 I}{2a}}$$

Both expressions are correct. They apply as approximations to the exact expression we derived, depending on the cases we are considering:

$r \ll a$ or $r \gg a$

## IV. How did we get here?

- We started-off several millenia ago by noticing that material matter (e.g., magnetite) possessed an interesting property that could be transferred to certain other materials. Later, we would come to call this magnetism.

- Oersted made a boo-boo, but was keen-enough to notice that currents influenced things in the same way as magnetism.

- Oersted, Ampère, Biot and Savart studied this, and they were able to quantify the relationship between currents (moving-charge) and magnetism.

- The hands-down conclusion is

  | |
  |---|
  | MAGNETIC FIELDS ARE THE RESULT OF MOVING-CHARGE |

- But wait — does that mean that magnetite is magnetic because it has charge moving-around?

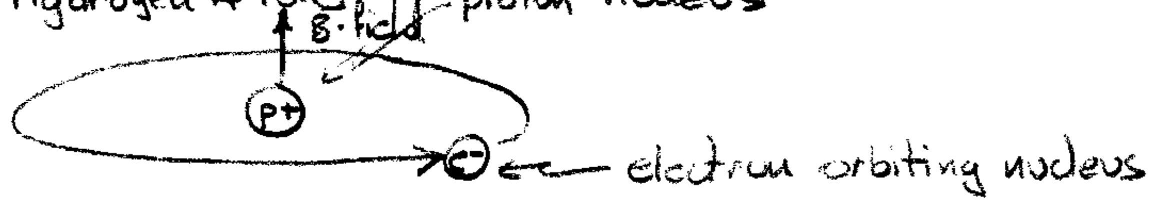
  Yes! But it's even more subtle than that.

## V. MAGNETIC MATERIALS AND THEIR ATOMIC NATURE.

- THE REAL UNDERSTANDING OF THE NATURE OF THE CURRENTS IN MAGNETIC MATERIALS WOULD HAVE TO WAIT UNTIL THE BIRTH OF QUANTUM THEORY IN THE EARLY 1900s (AND WE WILL ULTIMATELY HAVE TO WAIT UNTIL PHYSICS 1C IN THE EARLY 2000s BEFORE WE CAN REALLY UNDERSTAND IT!)

BUT, WE CAN GET A FLAVOR FOR IT BY CONSIDERING A "PLANETARY MODEL" OF THE ATOM (TAKE NOTE THAT THE EXACT NOTION OF AN ATOM WAS NOT YET KNOWN BEFORE 1900.)

$^1_1H$ Hydrogen Atom




$e^-$ electron orbiting nucleus in circle
⇒ looks like a small current loop
⇒ Produces a tiny $\vec{B}$-field.

- LATER, DURING DEVELOPMENT OF QUANTUM THEORY, IT WOULD BE DISCOVERED THAT protons, electrons, etc possessed something called "Spin" (which behaved as if the little $p^+$ and $e^-$ charges themselves were spinning and producing little magnetic fields of their own. DON'T take this idea too seriously, though. Electrons are truly point-like, so this can't be the same mechanism. Turns out that "Spin" is a completely Quantum phenomenon, and can't be explained classically, i.e, as a "current loop".)

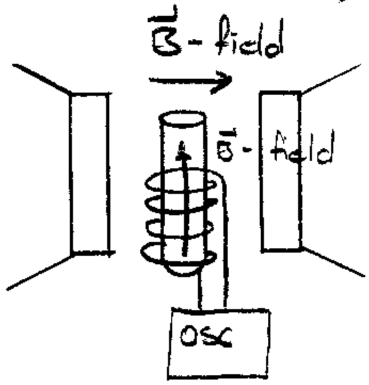
- Unfortunately, without more understanding of quantum theory (coming-up in physics 1C), we will only be able to catalogue here some of what's been learned:
  - There are four basic types of magnetic behavior at the atomic level
    - Diamagnetism
      - Due to "orbital-angular-momentum" of atoms.
      - Acts to oppose an externally-applied $\vec{B}$-field that produces it. Decreases total $\vec{B}$-field.
    - Paramagnetism
      - Due to interaction of elementary particle "spin" with externally-applied $\vec{B}$-fields. Enhances total $\vec{B}$-field
      - Strongest in atoms with odd number of electrons (electrons in atoms tend to pair their spins oppositely, causing a net spin-zero, in atoms with even numbers of electrons.)
    - Ferromagnetism ($Fe_3O_4$ in this category)
      - Spin-interactions over long ranges developing alignment of atoms and long-range order in materials (all the little magnets tend to point mostly in one direction, enhancing overall $\vec{B}$-field.)
    - Antiferromagnetism
      - Material likes to have "anti-pairing" of spins, resulting in no net $\vec{B}$-field.

- OF SPECIAL IMPORTANCE TO LIFE-SCIENCES IS PARAMAGNETISM —

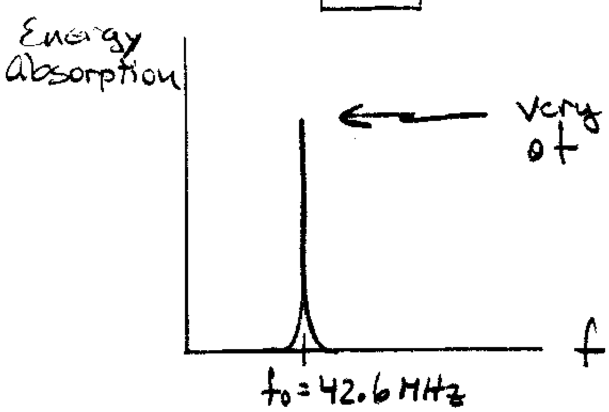
"NMR" - Nuclear MAGNETIC RESONANCE

APPLIED PARAMAGNETISM → "MRI" - MAGNETIC RESONANCE IMAGING

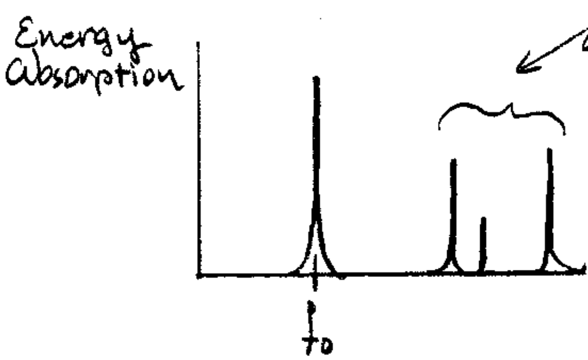
(USED TO BE CALLED "NMRI", THE "N" FOR "NUCLEAR", BUT SOME PEOPLE THOUGHT THAT WAS TO "SCARY", SO THE "N" WAS DROPPED.)




- Sample placed in test tube inside of large electromagnet. "Tickler" coil wound around sample to "tickle" nuclei with time-varying magnetic field




very sharp peak for sample of (e.g.) $^1_1H$ all by itself.




Very sharp peaks of resonances due to interaction with surrounding atoms of different species

MRI translates these peaks into colors, so you can "see" actual atomic interactions/material in, e.g., your patients head.