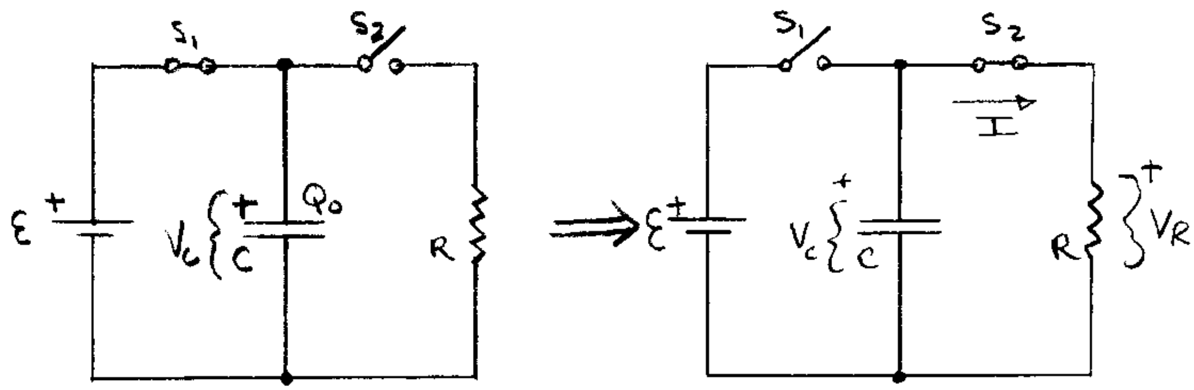
I. RC CIRCUITS



Capacitor C charged by battery to $V_c = \varepsilon$

$Q = Q_0$

Capacitor discharged through resistor R

Switch S_1 opened Then S_2 closed $\Rightarrow$ $V_C - V_R = 0$

$$V_C = V_R$$

$$V_C = Q/C \qquad V_R = IR$$

$$IR = Q/C$$

BUT $I = -dQ/dt$ ((−) because charge is <u>leaving</u> in direction shown)

$$R\frac{dQ}{dt} = -\frac{Q}{C}$$

$$\frac{dQ}{dt} = -\frac{1}{RC}Q$$

$$\frac{dQ}{Q} = -\frac{1}{RC}dt$$

$$\int \frac{dQ}{Q} = -\frac{1}{RC}\int dt$$

$$\ln|Q| = -t/RC + \text{const.}$$

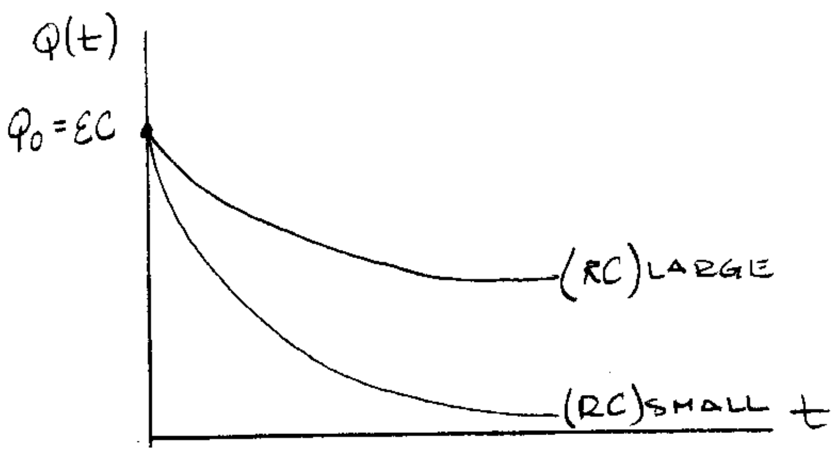
$$Q(t) = e^{-t/RC}\, e^{\text{const}}$$

$$Q(t) = A\, e^{-t/RC} \qquad \left(A \equiv e^{\text{const.}}\right)$$

$$Q(0) = Q_0 = \mathcal{E}C$$

$$\Rightarrow \quad A = \mathcal{E}C$$

$$\boxed{Q(t) = \mathcal{E}C\, e^{-t/RC}}$$



- "RC TIME CONSTANT"

NOTE THAT THE LARGER (RC) IS, THE LONGER IT TAKES FOR AN RC CIRCUIT TO CHARGE (OR DISCHARGE).

LOOK AT THE UNITS OF RC —

$$RC = (\Omega)\cdot(F)$$
$$= \left(\frac{V}{A}\right)\cdot\left(\frac{q}{V}\right)$$
$$= \frac{q}{q/s}$$

$RC \Rightarrow$ sec. IT HAS DIMENSIONS OF TIME (AS IT SHOULD, SINCE $e^{-t/RC}$ MUST HAVE A DIMENSIONLESS, ie, TIME/TIME, exponent)

DEFINE

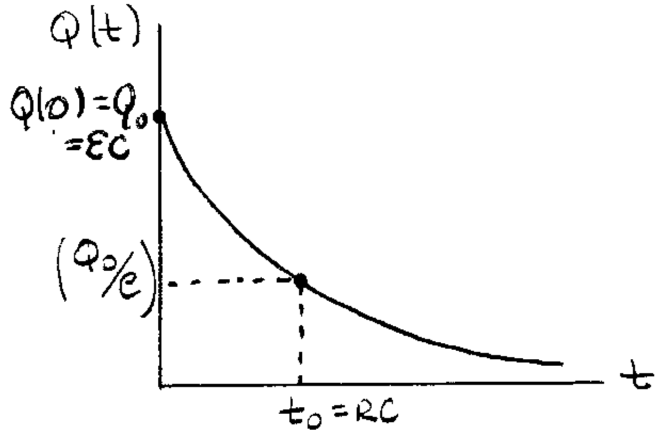
$$\boxed{t_0 \equiv RC}$$ "RC TIME CONSTANT"

WHAT HAPPENS IF $t = t_0$?

$$Q(t) = \mathcal{E}C e^{-t/RC}$$
$$= \mathcal{E}C e^{-t/t_0}$$
$$Q(t_0) = \mathcal{E}C e^{-1}$$
$$Q(t_0) = e^{-1} Q_0$$

t_0 SAYS HOW long IT will take for the capacitor to distcharge (charge) to $\frac{1}{e}$ of its original charge.

(* NOTE: DON'T CONFUSE THIS $e \simeq 2.718$
WITH THE OTHER $e = 1.602 \times 10^{-19}$ C !
This e is just $\ln(e) \equiv 1$)



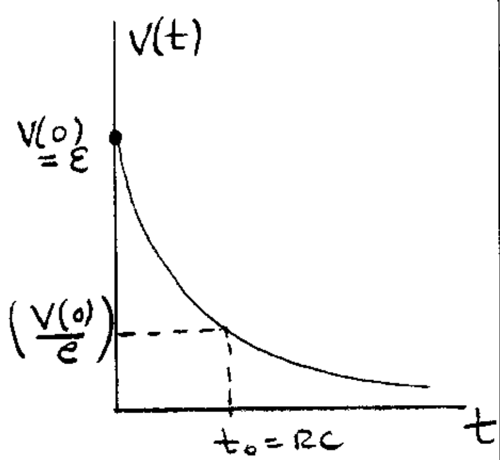
$\boxed{Ex:}$ FOR ABOVE CIRCUIT WITH $R = 2.5\Omega$, $C = 5.0\mu F$

$$t_0 = RC$$
$$= (2.5\Omega)(5.0\mu F)$$
$$t_0 = 13\mu s$$

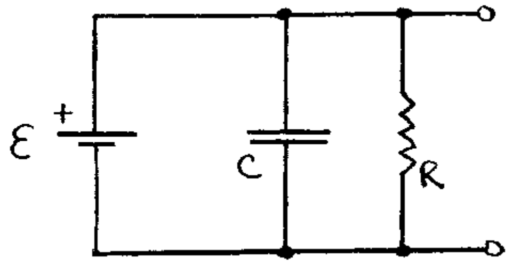
Takes $13\mu s$ for capacitor to discharge to $\frac{1}{e} \simeq .3679$ of its original charge.

- The Voltage: $V(t) = Q(t)/C$

$$V(t) = \frac{Q_0}{C} e^{-t/RC}$$
$$= \frac{\mathcal{E}C}{C} e^{-t/RC}$$
$$\boxed{V(t) = \mathcal{E} e^{-t/RC}}$$



$\boxed{\text{Ex:}}$ TYP. LASER POWER SUPPLY



⇒ TO LASER

$$\mathcal{E} = 5000\,V, \quad C = 1.0 \times 10^{3}\,\mu F, \quad R = 1.0\,M\Omega.$$

C IS MEANT TO SUPPLY THE HIGH-CURRENT BURSTS NEEDED BY LASER.

R IS MEANT TO SLOWLY DISCHARGE C WHEN SUPPLY NOT IN-USE (TO MAKE IT "SAFE")

$$t_0 = RC$$
$$= (1.0 \times 10^{6}\,\Omega)(1.0 \times 10^{-3}\,F)$$
$$= 1.0 \times 10^{3}\,\text{sec} = 17 \text{ min to discharge to}$$
$$1/e \text{ of } 5000\,V = 1.8 \times 10^{3}\,V$$

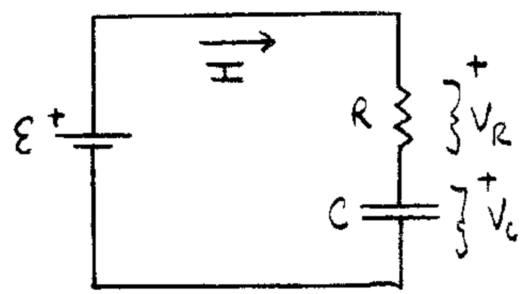
$\boxed{\text{Ex:}}$ PHYSICS GRAD STUDENT UNPLUGS LASER POWER SUPPLY, GETS COVER OFF WITHIN 1 MIN, & STICKS SCREWDRIVER IN

$$V(t) = V_0\, e^{-t/RC}$$
$$V(t{=}60\,\text{sec}) = (5000\,V)\, e^{-\left(\frac{60\,\text{sec}}{1.0 \times 10^{3}\,\text{sec}}\right)}$$
$$V(t{=}60\,\text{sec}) = 4700 \text{ Volts}$$

INSTANT GUEST-OF-HONOR AT DIOS DE LOS MUERTOS PARADE!

$\boxed{\text{LESSON: Just BECAUSE YOU SHUT-OFF A POWER SUPPLY DOESN'T MEAN IT WON'T KILL YOU!!}}$

• ANOTHER CIRCUIT



$$\mathcal{E} - V_R - V_C = 0$$
$$\mathcal{E} - IR - Q/C = 0$$
$$I = dQ/dt$$
$$\mathcal{E} - R\frac{dQ}{dt} - \frac{Q}{C} = 0$$

$$R\frac{dQ}{dt} + \frac{Q}{C} = \mathcal{E}$$
$$\frac{dQ}{dt} + \frac{1}{RC}Q = (\mathcal{E}/R)$$
$$\frac{dQ}{dt} = (\mathcal{E}/R) - \left(\frac{1}{RC}\right)Q$$
$$\frac{dQ}{(\mathcal{E}/R) - \left(\frac{1}{RC}\right)Q} = dt \quad \Rightarrow \quad \frac{dQ}{\left(\frac{1}{RC}\right) - (\mathcal{E}/R)} = -dt$$
$$\frac{dQ}{Q - (\mathcal{E}C)} = -\frac{1}{RC}dt \quad \Rightarrow \quad \int \frac{dQ}{Q - (\mathcal{E}C)} = -\frac{1}{RC}\int dt$$
$$\ln|Q - (\mathcal{E}C)| = -t/RC + \text{const.}$$
$$Q - (\mathcal{E}C) = A\,e^{-t/RC}$$
$$Q(t) = \mathcal{E}C + A\,e^{-t/RC}$$

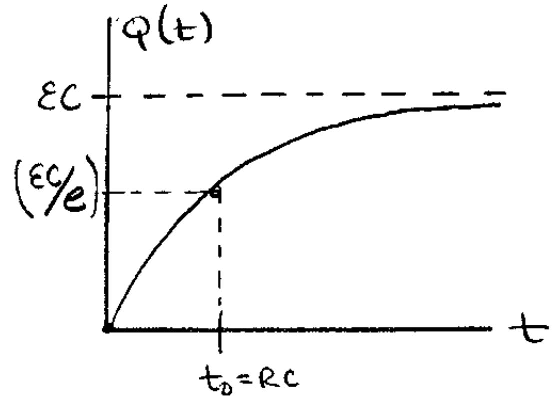
let $Q(0) = Q_0 \Rightarrow \mathcal{E}C + A = Q_0 \Rightarrow A = (Q_0 - \mathcal{E}C)$

$$Q(t) = \mathcal{E}C + (Q_0 - \mathcal{E}C)\,e^{-t/RC}$$

FOR $Q(0) = Q_0 = 0$,

$$\boxed{Q(t) = \mathcal{E}C\left(1 - e^{-t/RC}\right)}$$

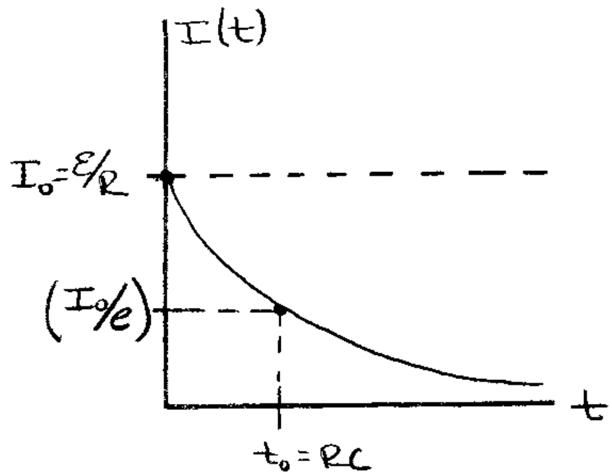
- FOR ABOVE CIRCUIT (WITH $Q(0) = Q_0 = 0$)
  - $\boxed{Q(t) = \mathcal{E}C\left(1 - e^{-t/RC}\right)}$



- $I(t) = \dfrac{dQ(t)}{dt}$

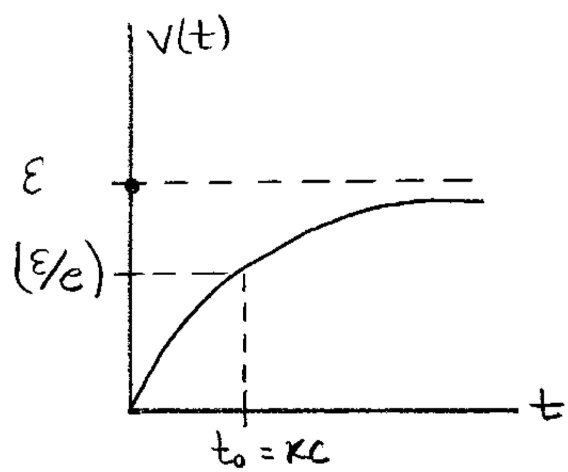
  $= \mathcal{E}C\left(\dfrac{1}{RC} e^{-t/RC}\right)$

$$\boxed{I(t) = \frac{\mathcal{E}}{R} e^{-t/RC}}$$



- $V(t) = Q(t)/C$

$$\boxed{V(t) = \mathcal{E}\left(1 - e^{-t/RC}\right)}$$



II. NETWORK ANALYSIS / KIRCHOFF'S RULES

(A) KIRCHOFF'S RULES

(1) LOOP RULE ("KLR")

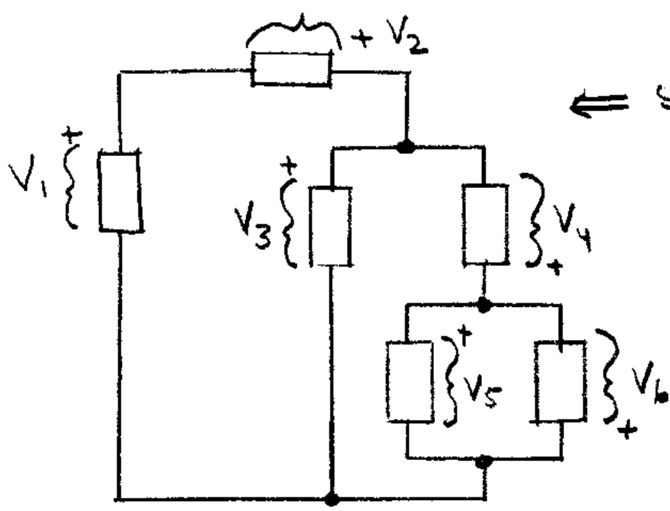
$$\left(\sum_{\substack{\text{closed} \\ \text{loop}}} V_i = 0 \right)$$

- SUM OF ALL VOLTAGE DROPS/RISES THROUGH ANY CLOSED LOOP IS ZERO.

(2) NODE RULE ("KNR")

$$\left(\sum_{\text{node}} I_i = 0 \right)$$

- SUM OF ALL CURRENTS ENTERING/LEAVING A NODE IS ZERO.



⟸ SOME WHACKY CIRCUIT

"KLR": ① $V_1 + V_2 - V_3 = 0$

② $V_1 + V_2 + V_4 - V_5 = 0$

③ $V_1 + V_2 + V_4 + V_6 = 0$

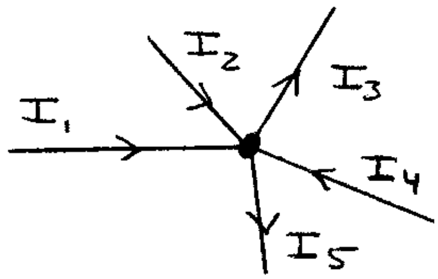
④ $V_3 + V_4 - V_5 = 0$

⑤ $V_3 + V_4 + V_6 = 0$

⑥ $V_5 + V_6 = 0$

- Any closed loop you go-around <u>must</u> return you to the same potential you started at!

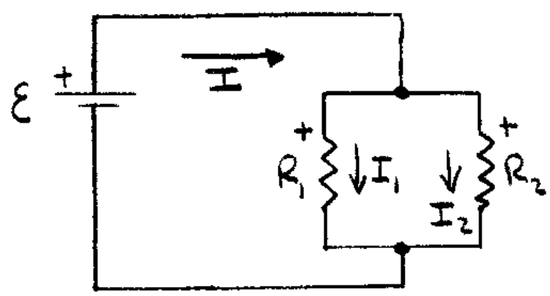
"KNR"



⇐ SOME WHACKY NODE WITH FIVE CURRENTS ENTERING / LEAVING

$$I_1 + I_2 - I_3 + I_4 - I_5 = 0 \qquad \text{(etc).}$$

$\boxed{\text{Ex:}}$ SEEN BEFORE. NOW THROUGH KIRCHOFF'S EYES—



① "KLR": $\mathcal{E} - I_1 R_1 = 0$

$\mathcal{E} - I_2 R_2 = 0$

"KNR": $I - I_1 - I_2 = 0$

3 eqns.

3 unks (I, I_1, I_2)

⇒ guaranteed a unique solution

② SOLVE: $I_1 = \mathcal{E}/R_1$

$I_2 = \mathcal{E}/R_2$

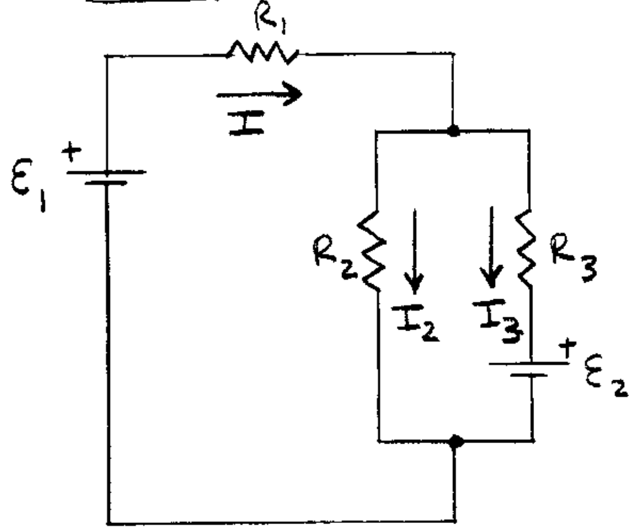
$$I - \frac{\mathcal{E}}{R_1} - \frac{\mathcal{E}}{R_2} = 0$$

Just $\dfrac{\mathcal{E}}{R_{eq}} = \dfrac{\mathcal{E}}{R_1} + \dfrac{\mathcal{E}}{R_2}$

* OVERKILL: COULD HAVE JUST THOUGHT ABOUT IT FIRST, AND SOLVED AS BEFORE.

$\boxed{\text{* LESSON: THINK FIRST}}$

EX:



- NOT SO EASY TO SEE WHAT WILL HAPPEN. JUST PICK <u>ANY</u> DIRECTION FOR I, I_1 & I_2
- ALL DEPENDS ON $\mathcal{E}_1$ & $\mathcal{E}_2$.

① "KLR": $\mathcal{E}_1 - IR - I_2R_2 = 0$ ①

$\mathcal{E}_1 - IR_1 - I_3R_3 - \mathcal{E}_2 = 0$ ②

② "KNR": $I - I_2 - I_3 = 0$ ③

Rearrange —

$IR_1 + I_2R_2 = \mathcal{E}_1$ ①

$IR_1 + I_3R_3 = (\mathcal{E}_1 - \mathcal{E}_2)$ ②

$I_3 = I - I_2$ ③

SOLVE — $IR_1 + (I - I_2)R_3 = (\mathcal{E}_1 - \mathcal{E}_2)$ ④ (SUBST ③ INTO ②)

$$\begin{pmatrix} I(R_1+R_3) - I_2R_3 = (\mathcal{E}_1 - \mathcal{E}_2) \\ IR_1 + I_2R_2 = \mathcal{E}_1 \end{pmatrix}$$ (JUST ①)

same

$$\begin{pmatrix} I\left(\frac{R_1+R_3}{R_3}\right) - I_2 = \frac{(\mathcal{E}_1 - \mathcal{E}_2)}{R_3} \\ I\,R_1/R_2 + I_2 = \mathcal{E}_1/R_2 \end{pmatrix}$$

ADD EQNS TO ELIMINATE I_2 —

$$I\left(\frac{R_1}{R_2}+\left(\frac{R_1+R_3}{R_3}\right)\right)=\frac{(\mathcal{E}_1-\mathcal{E}_2)}{R_3}+\frac{\mathcal{E}_1}{R_2}$$

$$I\left[\frac{R_1R_3+(R_1+R_3)R_2}{R_2R_3}\right]=\frac{R_2(\mathcal{E}_1-\mathcal{E}_2)+\mathcal{E}_1R_3}{R_2R_3}$$

$$I=\frac{R_2(\mathcal{E}_1-\mathcal{E}_2)+\mathcal{E}_1R_3}{R_2R_3}\cdot\frac{R_2R_3}{R_1R_3+(R_1+R_3)R_2}$$

$$I=\frac{R_2(\mathcal{E}_1-\mathcal{E}_2)+\mathcal{E}_1R_3}{R_1R_3+R_2(R_1+R_3)}$$

NOTICE THAT $I<0$ FOR

$$R_2(\mathcal{E}_1-\mathcal{E}_2)+\mathcal{E}_1R_3<0$$

OR

$$\mathcal{E}_2>\mathcal{E}_1\left(1+R_3/R_2\right)$$

THAT'S OKAY — IT JUST MEANS THAT THE CURRENT IS <u>REALLY</u> GOING IN A DIRECTION OPPOSITE TO WHAT YOU PICKED BEFORE YOU REALLY KNEW WHAT WAS GOING TO HAPPEN.

<u>DON'T</u> CHANGE YOUR ARROWS — IT'S UNDERSTOOD THAT A NEGATIVE VALUE FOR THEM JUST MEANS "THE OTHER WAY."

STRATEGY

① USE PHYSICAL REASONING FIRST. SIMPLE PROBLEMS CAN BE SOLVED BY SIMPLE MEANS, WITHOUT RESORT TO KIRCHOFF'S RULES.

② DEPENDING ON WHAT YOU WANT TO KNOW, SIMPLIFY CIRCUITS TO EQUIVALENT CIRCUITS (NO NEED TO ANSWER A QUESTION THAT IS NOT OF CONCERN.)

③ WHEN YOU CAN GO NO FARTHER WITH ① AND ②, RESORT TO THE "BRUTE-FORCE" OF KIRCHOFF'S RULES.

(a) Arbitrarily assign current directions.

(b) LABEL ⊕ SIDE OF VOLTAGE SOURCES, ETC.

(c) Write-down Loop Equs, "KLR".

(d) Write-down Node Equs, "KNR".

(e) SOLVE YOUR N-simultaneous Eqns for the N-unknowns.

(f) Remember that negative values for currents only mean that the current is really in a direction opposite to that which you originally chose.