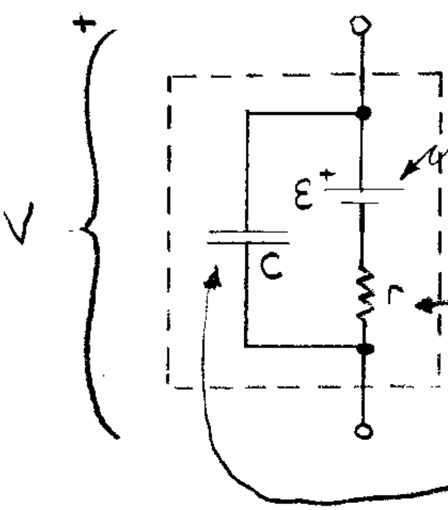
I. Real Batteries

Unlike the ideal batteries we've considered so far, real batteries have internal resistance and capacitance which greatly alters their behavior in actual circuits

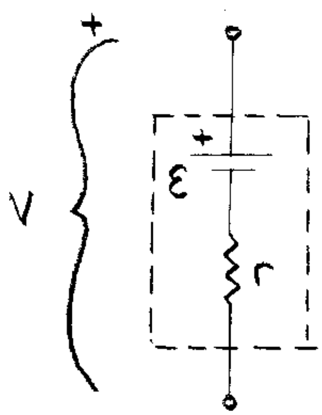


Ideal battery w/ potential $\mathcal{E}$
- Represents ideal chemical reactions

Internal Resistance
- Represents limitations to reaction rates, etc.

Internal Capacitance
- Represents charge stored on plates

We'll not consider C here, just r

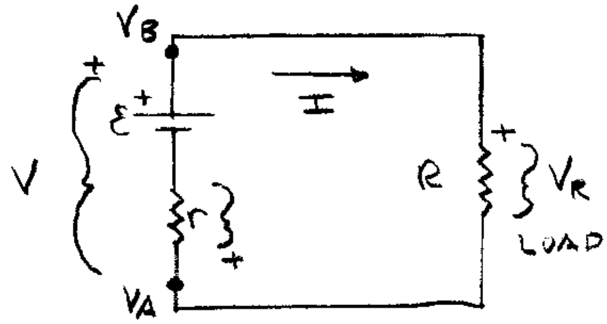


$\mathcal{E}$ = open-circuit voltage (no-load, ideal battery)

V = Terminal Voltage

r = Internal Resistance

- Under no-load conditions, no current is in r, so $V = \mathcal{E}$



$$V_A - V_r + \mathcal{E} - V_R = V_A$$

$$\mathcal{E} - V_r - V_R = 0$$

$$\mathcal{E} - Ir - IR = 0$$

$$\mathcal{E} = I(r+R)$$

$$\boxed{I = \mathcal{E}/(R+r)}$$

- actual current supplied by battery under load is less than the ideal case of $I = \mathcal{E}/R$

$$V = V_B - V_A$$

$$= (V_A - Ir + \mathcal{E}) - (V_A)$$

$$\boxed{V = \mathcal{E} - Ir}$$

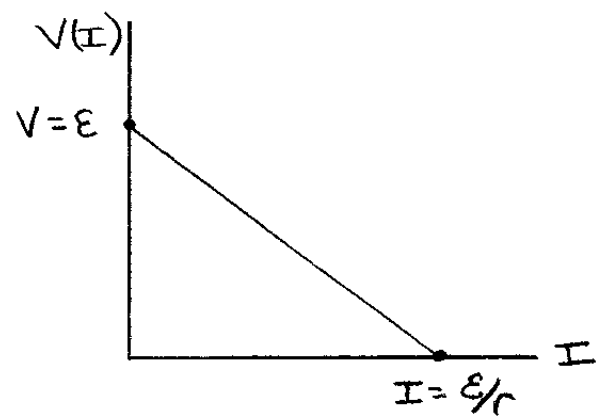
- actual voltage supplied by battery under load is less than ideal case of $V = \mathcal{E}$

$$V = \mathcal{E} - Ir$$

$$= \mathcal{E} - \frac{\mathcal{E}r}{(R+r)}$$

$$\boxed{V = \frac{\mathcal{E}R}{(R+r)}}$$

- Voltmeters are actually useless by themselves to test whether a battery is good.

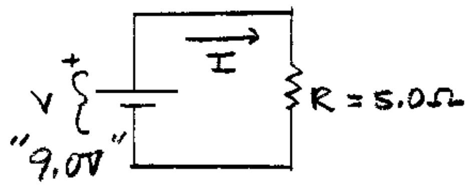


$\boxed{\text{Ex:}}$ PUT A VOLTMETER ON AN OLD 9.0V BATTERY & PERHAPS GET V = 9.0V READING.

BUT, INTERNAL RESISTANCE MAY BE AROUND 2.0Ω

HOOK THIS UP TO 5.0Ω LOAD —

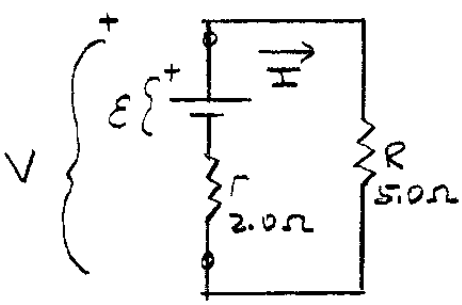
- THINK YOU'LL GET



$$V - IR = 0$$

$$I = \frac{V}{R} = \frac{(9.0V)}{(5.0\Omega)} = 1.8A$$

- REALLY GET



$$I = \frac{\mathcal{E}}{(R+r)} = \frac{(9.0V)}{(8.0\Omega)} = 1.2A$$

$$V = \frac{\mathcal{E}R}{(R+r)}$$

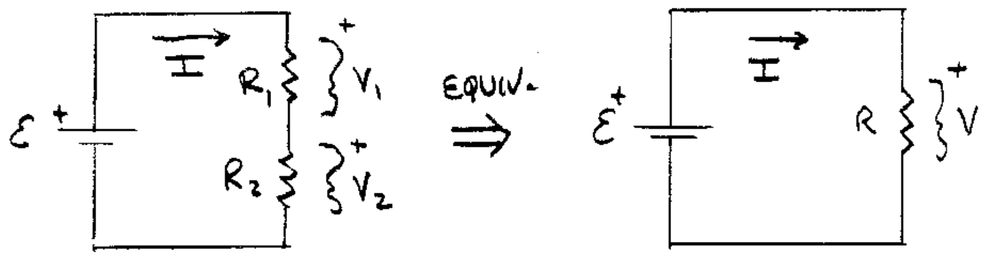
$$= \frac{(9.0V)(5.0\Omega)}{(5.0\Omega + 2.0\Omega)}$$

$$V = 5.6V$$

- This is what's really going-on in your portable radio when it starts sounding lousy and needs a new battery — The actual voltage its getting is dropping below the value it needs to work.

II. Resistors in Series and Parallel

(A) Series



$\mathcal{E} - V_1 - V_2 = 0$ $\qquad\qquad$ $\mathcal{E} - V_R = 0$

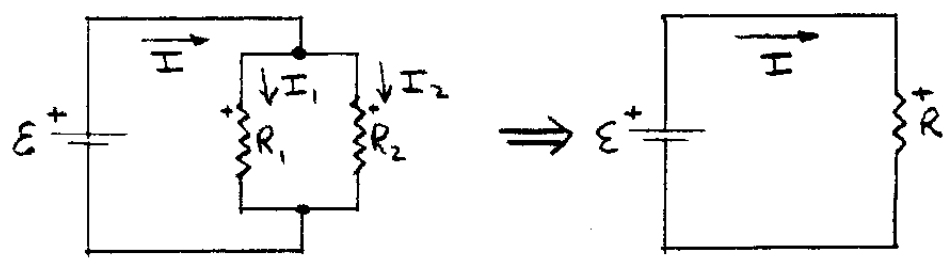
$\mathcal{E} = V_1 + V_2$ $\qquad\qquad$ $\mathcal{E} = V$

$$V = V_1 + V_2$$

$$IR = IR_1 + IR_2$$

$$\boxed{R = R_1 + R_2} \quad \text{SERIES}$$

(B) Parallel



$$I - I_1 - I_2 = 0$$

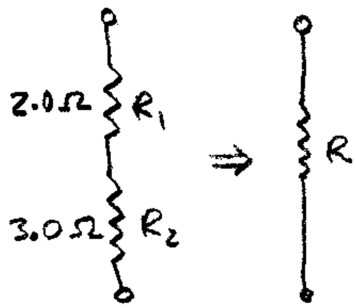
$$I = I_1 + I_2$$

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2}$$

$$\boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}} \quad \text{PARALLEL}$$

- Note: These relationships are just the "opposite" of those for capacitors

$\boxed{\text{Ex:}}$



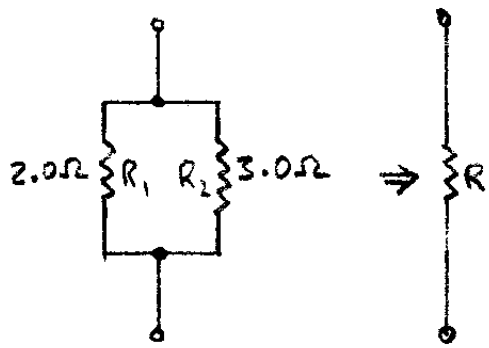
• SERIES

$$R = R_1 + R_2$$

$$= 2.0\Omega + 3.0\Omega$$

$$R = 5.0\Omega$$

• Equivalent Series resistance is always greater than greatest single resistance in series.



• PARALLEL

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

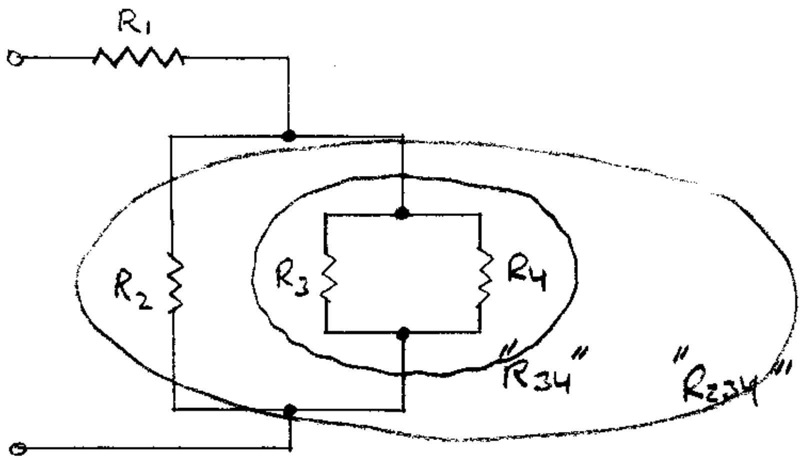
$$R = \frac{R_1 R_2}{R_1 + R_2}$$

$$= \frac{(2.0\Omega)(3.0\Omega)}{(2.0\Omega + 3.0\Omega)}$$

$$R = 1.2\Omega$$

• Equivalent parallel resistance is always smaller than smallest single resistance in parallel.

$\boxed{\text{Ex:}}$ SERIES AND PARALLEL COMBOS



$$\text{"}R_{34}\text{"} = R_3 \parallel R_4 = \frac{R_3 R_4}{R_3 + R_4}$$

$$\text{"}R_{234}\text{"} = R_2 \parallel (R_3 \parallel R_4)$$

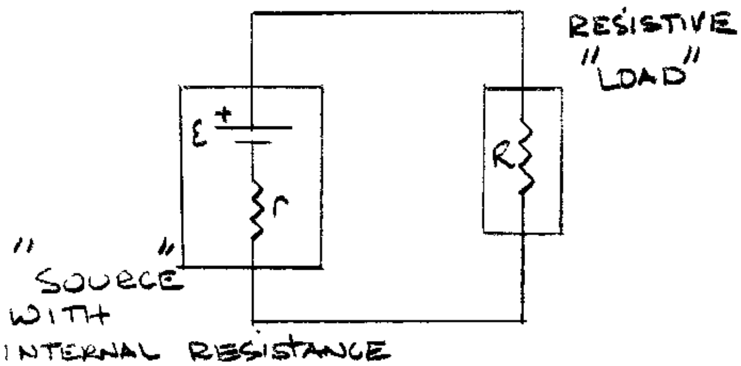
$$= \frac{R_2 (R_3 \parallel R_4)}{R_2 + (R_3 \parallel R_4)}$$

$$= \frac{R_2 \left(\frac{R_3 R_4}{R_3 + R_4} \right)}{R_2 + \left(\frac{R_3 R_4}{R_3 + R_4} \right)}$$

$$R_{eq} = R_{1234} = R_1 + R_{234}$$

$$R = R_1 + \frac{R_2 \left(\frac{R_3 R_4}{R_3 + R_4} \right)}{R_2 + \left(\frac{R_3 R_4}{R_3 + R_4} \right)}$$

III. POWER TRANSFER IN RESISTIVE CIRCUITS



POWER TRANSFER FROM SOURCE TO LOAD

$$P = I^2 R$$

$$I = \mathcal{E}/(R+r)$$

$$P = \frac{\mathcal{E}^2 R}{(R+r)^2}$$

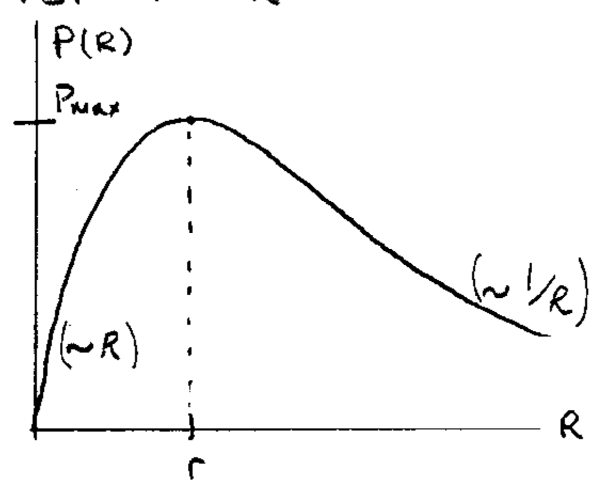
Maximum power transfer occurs at

$$\frac{\partial P}{\partial R} = 0$$

$$\frac{\partial P}{\partial R} = \mathcal{E}^2 \left[\frac{(R+r)^2(1) - (2)(R)(R+r)}{(R+r)^4} \right]$$

$$= \frac{2\mathcal{E}^2(r-R)}{(R+r)^3} = 0 \text{ for } r = R$$

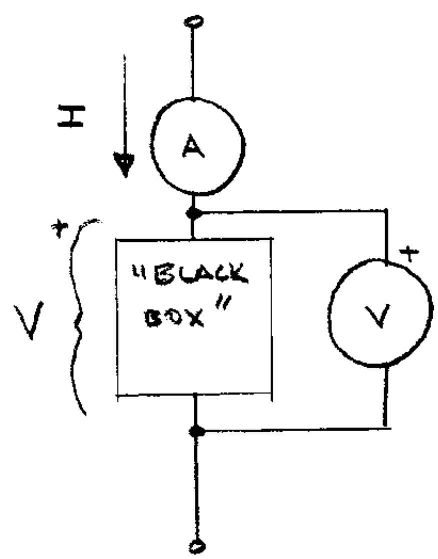
$$P_{max} = \frac{\mathcal{E}^2 R}{(R+R)^2} = \frac{1}{4} \mathcal{E}^2/R$$



IV. AMMETERS AND VOLTMETERS

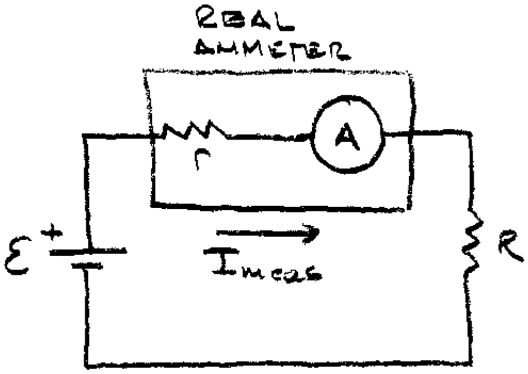
- AN IDEAL AMMETER WOULD TELL YOU EXACTLY WHAT CURRENT YOU HAVE THROUGH SOME CIRCUIT WITHOUT CAUSING ANY CHANGE TO THE CURRENT MEASURED.

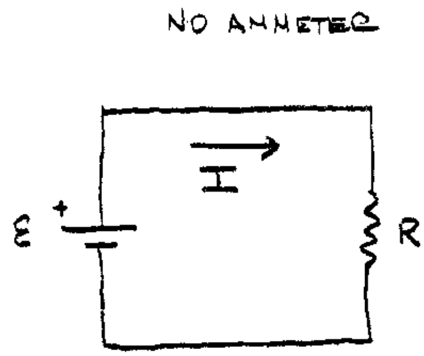
- AN IDEAL VOLTMETER WOULD TELL YOU EXACTLY WHAT VOLTAGE YOU HAVE ACROSS SOME CIRCUIT WITHOUT CAUSING ANY CHANGE TO THE VOLTAGE MEASURED.



- IDEAL AMMETER READS EXACTLY I
- IDEAL VOLTMETER READS EXACTLY V
- THEY HAVE NO AFFECT ON THE CIRCUIT.

- OF COURSE, REAL VOLTMETERS AND AMMETERS ARE THEMSELVES ELECTRONIC DEVICES, AND THEY HAVE INTERNAL RESISTANCES ALL THEIR OWN. HOOKING THEM UP TO A CIRCUIT ACTUALLY CHANGES THE CIRCUIT, SO VOLTAGES AND CURRENTS BEING MEASURED ARE NOT THE SAME AS WITHOUT THE INSTRUMENTS ATTACHED.





$$\mathcal{E} - I_{meas} r - I_{meas} R = 0$$

$$\mathcal{E} = I_{meas}(R + r)$$

$$I_{meas} = \frac{\mathcal{E}}{(R+r)}$$

$$= \frac{(\mathcal{E}/R)}{1 + (r/R)}$$

$$\boxed{I_{meas} = \frac{I}{1 + (r/R)}}$$

$$\mathcal{E} - IR = 0$$

$$\mathcal{E} = IR$$

$$I = \frac{\mathcal{E}}{R}$$

- Want to have $I_{meas} = I$, but this requires $r/R = 0$ (or, $r = 0$, which is not possible). Best one can do is to try to get r/R as small as possible (Takes very expensive ammeters to accurately measure small resistances!)