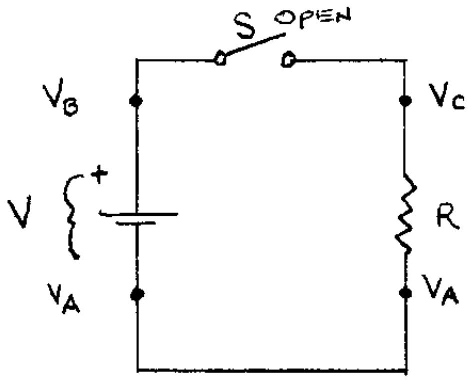
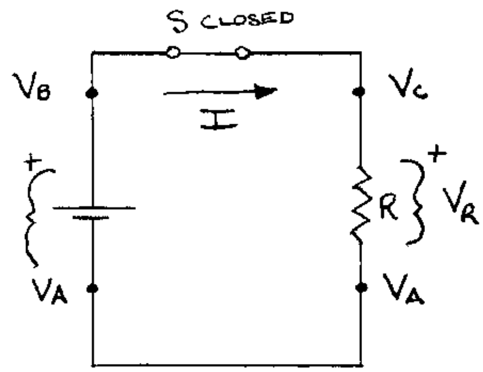
I. VOLTAGE DROPS AND RISES





$$V_A + V = V_B$$

$$V_C = V_A$$

$$V_A + V - V_R = V_A$$

$$V - V_R = 0$$

$$V - IR = 0$$

$$V = IR \quad \text{(JUST OHM'S LAW)}$$

$\boxed{\text{Ex:}}$ $V = 9.0\,V$ (ideal battery)

$$R = 5.0\,\Omega$$

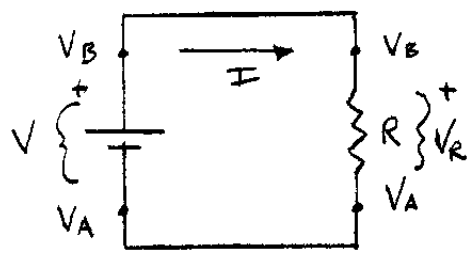
$$I = \frac{V}{R} = \frac{(9.0\,V)}{(5.0\,\Omega)} = 1.8\,A$$

$\boxed{\text{Ex:}}$ 9.0V ALKALINE CELL CAN DELIVER ABOUT 1000 mA·hr OF CHARGE BEFORE ITS ALL USED-UP (i.e. 1000 mA for 1 HR, OR 500 mA for 2 HRS, etc).

$$(1.8\,A)\left(\frac{1\ \text{BATTERY}}{1\,A \cdot \text{HR}}\right) = 1.8\ \text{BATTERIES/HR}$$

→ ABOVE BATTERY IS DEAD IN $1/1.3$ HRS = 33 min!

II. ENERGY & POWER



CHANGE IN POTENTIAL ENERGY OF CHARGE q MOVING THROUGH R FROM POTENTIAL V_B TO POTENTIAL V_A —

$$\Delta W = q \Delta V$$

$$(W_A - W_B) = q\,(V_A - V_B)$$

Choosing $W_A = V_A = 0$ (arbitrary - we can do this)

$$-W_B = q\,(-V_B)$$

OR SIMPLY $\quad W = qV$

$$[\text{POWER}] \equiv \left[\frac{\text{ENERGY}}{\text{UNIT TIME}}\right]$$

$$P = \frac{dW}{dt} \qquad \left(\text{e.g., } 1\,\text{Watt} = 1\,\text{J/sec}\right)$$

$$P = \frac{dW}{dt} = \frac{dq}{dt}V + q\frac{dV}{dt}$$

have $V = \text{const.}$ here, so $\frac{dV}{dt} = 0$

$$P = \frac{dq}{dt}V$$

$$\boxed{P = IV}$$

OR, SINCE $\quad V = IR$

$$\boxed{\begin{aligned} P &= I^2R \\ P &= V^2/R \end{aligned}}$$

all three forms are equivalent. Which one you use is up to you.

$\boxed{\text{Ex:}}$ POWER CONSUMPTION IN ABOVE EXAMPLE

KNOW: V & R ⇒ QUICKEST TO USE

$$P = V^2/R$$
$$= (9.0\,V)^2/(5.0\,\Omega)$$
$$= 16 \text{ Watts} \quad (J/s)$$

BUT, SINCE WE ALSO WENT TO THE TROUBLE OF CALCULATING I

$$P = IV$$
$$= (1.8A)(9.0V)$$
$$= 16 \text{ Watts}$$

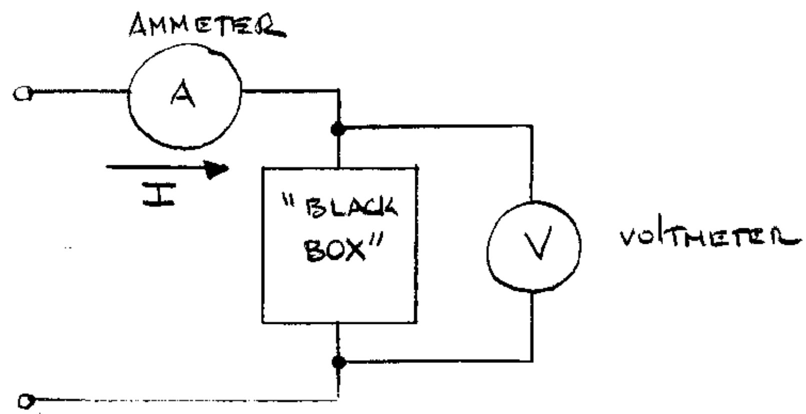
OR

$$P = I^2R$$
$$= (1.8A)^2(5.0\,\Omega)$$
$$= 16 \text{ Watts}$$

THIS ENERGY IS BEING SUPPLIED BY THE BATTERY. THE CHARGE-CARRIER COLLISIONS IN THE RESISTOR RESULT IN AN INCREASE OF KINETIC ENERGY OF THE MATERIAL, WHICH INCREASES ITS TEMPERATURE. THE ASSOCIATED HEAT IS DISSIPATED BY THE RESISTOR BY CONDUCTION/CONVECTION/RADIATION. THIS IS CALLED

$\boxed{\text{JOULE HEATING}}$

NOTE THAT THE $P = IV$ RELATIONSHIP ALLOWS ONE TO DETERMINE THE POWER CONSUMPTION OF ANY CIRCUIT SIMPLY BY MEASURING THE VOLTAGE ACROSS IT AND THE CURRENT THROUGH IT



$P = IV$, REGARDLESS OF WHAT'S IN THE "BLACK BOX".

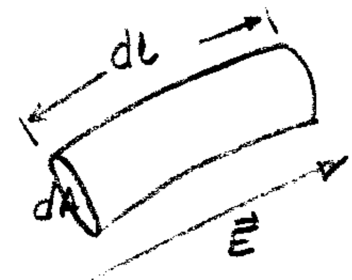
III. CURRENT DENSITY AND CONDUCTIVITY

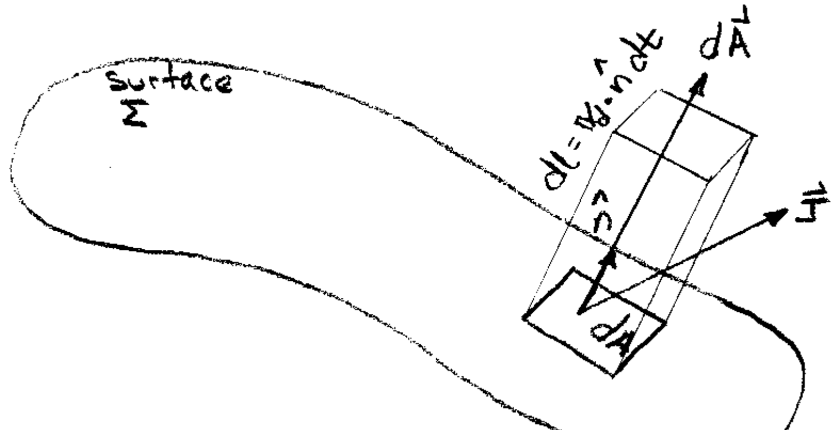
CHARGE IN AN ELECTRIC FIELD EXPERIENCES A FORCE

$$\vec{F} = q\,\vec{E}$$

AND THE CHARGE THEN MOVES ACCORDING TO THIS FORCE.

IT IS THE ELECTRIC FIELD $\vec{E}$ THAT DETERMINES WHAT'S GOING TO HAPPEN (AND OF COURSE THE CHARGE EXPERIENCING IT), SO IT SHOULDN'T BE SURPRISING THAT THE CURRENT AT SOME POINT IN SPACE DEPENDS ULTIMATELY ON THE ELECTRIC FIELD THERE





$$I = \frac{dq}{dt}$$ crossing some surface

$$dI = \frac{dq}{dt}$$ crossing some surface area element dA

$$dI = \left(\frac{dq}{dVol}\right)\left(\frac{dVol}{dt}\right)$$

$$= \left(\frac{dq}{dVol}\right)\left(\frac{dl}{dt}\right)dA$$

$$= \left(\frac{dq}{dVol}\right)\left(\mathbb{V}_d \cdot \hat{n}\right)dA$$

$$= \underbrace{\left[\left(\frac{dq}{dVol}\right)\mathbb{V}_d\right]}_{\text{call this } \vec{J}} \cdot \left(\hat{n}\, dA\right)$$

$$dI = \vec{J} \cdot d\vec{A}$$

Total CURRENT THROUGH ENTIRE SURFACE Σ IS

$$I = \int dI$$

$$\boxed{I = \int_{\Sigma} \vec{J} \cdot d\vec{A}}$$

FOR LINEAR MATERIALS AND $\vec{E}$-FIELDS NOT TOO LARGE, IT TURNS OUT THAT

$$\vec{J} \sim \vec{E}$$

i.e.,

$$\boxed{\vec{J} = \sigma \vec{E}}$$

σ IS CALLED THE CONDUCTIVITY OF THE MATERIAL.

Ex! FOR A SIMPLE RESISTOR

$$V = IR$$

$$V = I\frac{\rho L}{A}$$

$$EL = (\sigma E A)\frac{\rho L}{A}$$

$$1 = \sigma\rho$$

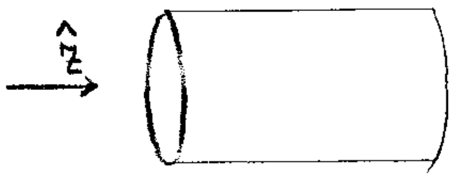
$$\Rightarrow \boxed{\sigma = 1/\rho}$$

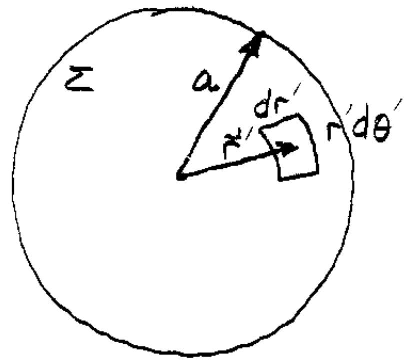
σ = conductivity

ρ = resistivity

Can be taken as a general definition.

$\boxed{\text{Ex:}}$ NONUNIFORM ELECTRIC FIELD IN RESISTOR
(MORE REALISTIC) $E(r) = Kr$





$$I(r) = \int_{\Sigma} \vec{J}(r) \cdot d\vec{A}(r)$$

$$\vec{J}(r) = \sigma \vec{E}(r)$$

$$= \sigma K E(r)\hat{z}$$

$$d\vec{A}(r) = dA\,\hat{z} = r\,dr\,d\theta\,\hat{z}$$

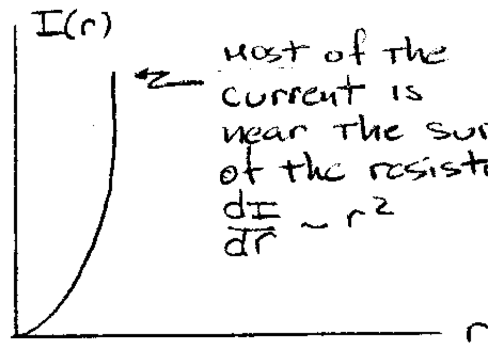
$$I(r) = \int_0^r dr' \int_0^{2\pi} d\theta' \left(\sigma k\, r'\ \hat{z}\right) \cdot \left(r'\hat{z}\right)$$

$$= \int_0^r dr' \int_0^{2\pi} d\theta'\ \sigma k\, r'^2$$

$$= 2\pi\sigma k \int_0^r dr'\, r'^2$$

$$I(r) = \frac{2\pi\sigma k}{3} r^3$$

$$\frac{dI(r)}{dr} = 2\pi\sigma k r^2$$



Most of the current is near the surface of the resistor
$\frac{dI}{dr} \sim r^2$