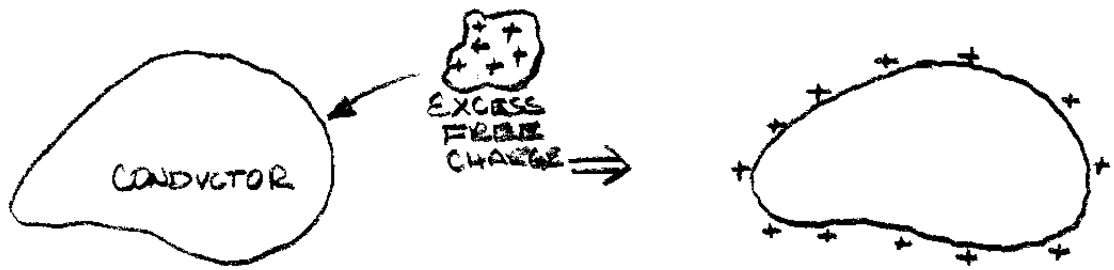
I. CAPACITANCE

(A) DEFINITION OF CAPACITANCE

- CAPACITANCE IS A MEASURE OF AN OBJECTS ABILITY TO STORE CHARGE

$\boxed{Ex!}$ PUT SOME FREE CHARGE ON A CONDUCTOR –



THE CHARGE ARRANGES ITSELF OVER THE SURFACE OF THE CONDUCTOR. THE SURFACE OF THE CONDUCTOR IS AN EQUIPOTENTIAL SURFACE, WITH SOME POTENTIAL V, RELATIVE TO A CONVENIENT REFERENCE (SUCH AS $r = \infty$, OR THE SURFACE OF THE EARTH, "GROUND".)

THE CAPACITANCE OF THIS SYSTEM IS DEFINED AS

$$\boxed{C \equiv \frac{Q}{V}} = \frac{(\text{TOTAL CHARGE STORED ON OBJECT})}{(\text{OBJECT'S POTENTIAL})}$$

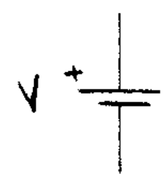
A TYPICAL UNIT OF CAPACITANCE IS THE FARAD

$$\boxed{1 \text{ FARAD} = \frac{1 \text{ Coulomb}}{1 \text{ Volt}} = \left(\frac{1\,C}{V}\right)}$$

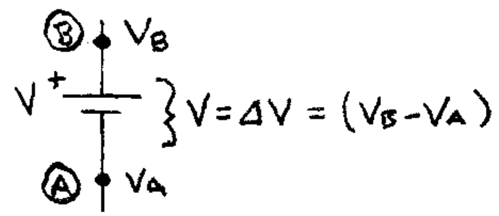
(B) IDEAL BATTERY

- HELPFUL TO HAVE SOMETHING THAT CAN MAINTAIN A CONSTANT POTENTIAL ACROSS SOME SYSTEM REGARDLESS OF QUANTITY OF CHARGE DELIVERED TO/FROM SYSTEM. CALL THIS AN IDEAL BATTERY

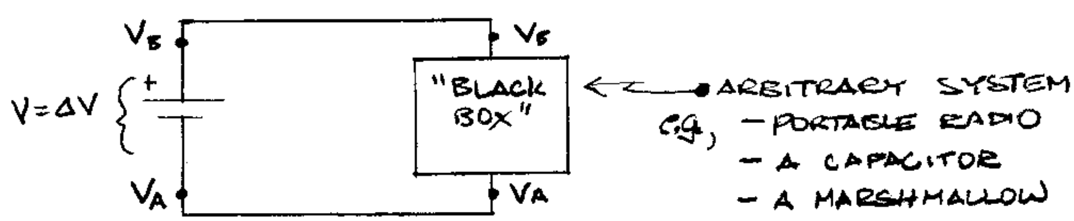
- SCHEMATIC SYMBOL FOR IDEAL BATTERY



MEANS:

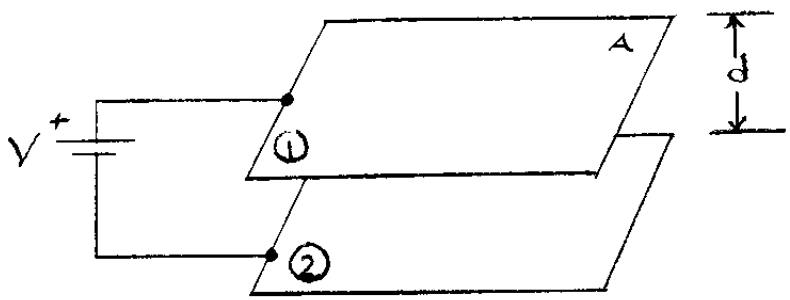


OBJECT WHICH MAINTAINS A CONSTANT POTENTIAL DIFFERENCE $V = \Delta V = (V_B - V_A)$ BETWEEN ITS TWO TERMINALS (A) AND (B) REGARDLESS OF WHAT THOSE TERMINALS ARE CONNECTED TO

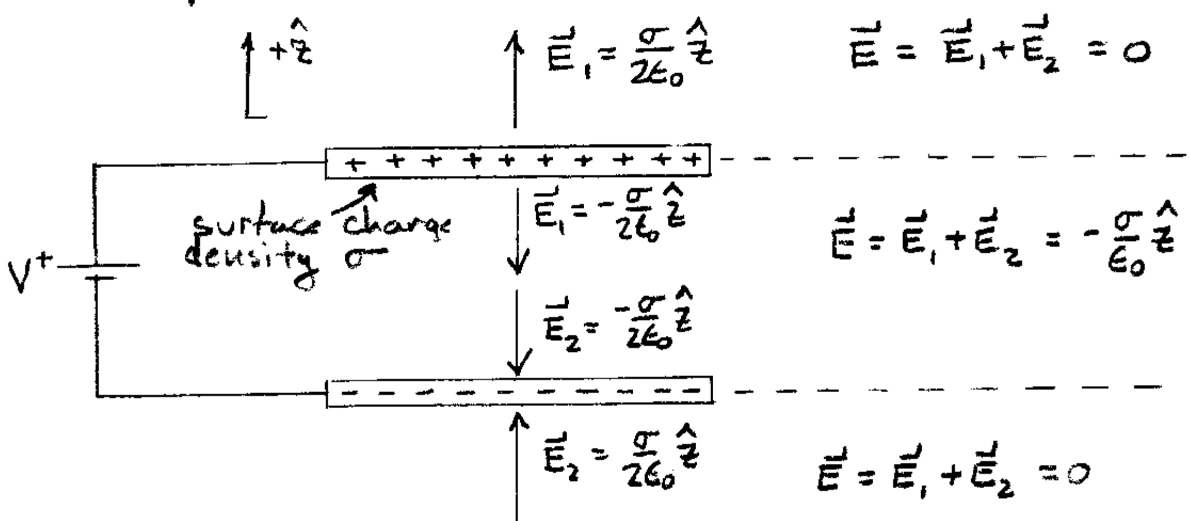


- NOTE: IT'S CUSTOMARY IN ELECTRONICS TO WRITE POTENTIAL DIFFERENCES WITHOUT THE Δ. I.E., $V = \Delta V = V_2 - V_1$ WHERE V_1 IS SOME CONVENIENT REFERENCE POTENTIAL, USUALLY CHOSEN TO BE $V_1 = 0$ AND CALLED "GROUND."

(C) Parallel Plate Capacitor



• Take two conducting sheets of surface area A separated by some distance d. This thing can store charge. Eg., by hooking plates up to a battery as shown above.



$$V = \Delta V = -\int_{z=0}^{z=d} \vec{E}\cdot d\vec{r} = -\int_{z=0}^{z=d} \left(-\frac{\sigma}{\epsilon_0}\hat{z}\right)(dz\,\hat{z})$$

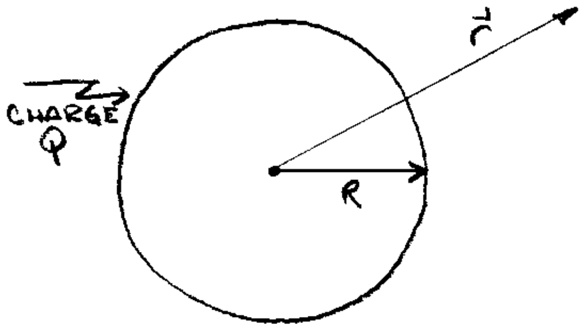
$$V = \frac{\sigma}{\epsilon_0}\int_0^d dz = \frac{\sigma}{\epsilon_0} z \Big|_0^d = \frac{\sigma d}{\epsilon_0}$$

$$Q = \sigma A \qquad C = Q/V = \frac{\sigma A}{\sigma d/\epsilon_0} = \frac{\epsilon_0 A}{d}$$

$$\boxed{C = \frac{\epsilon_0 A}{d}}$$

Capacitance of Parallel Plate Capacitor

(D) CAPACITANCE OF A SPHERICAL CONDUCTOR



$$C = \frac{Q}{V}$$

$$V = \Delta V = -\int_{\vec{r}_1}^{\vec{r}_2} \vec{E} \cdot d\vec{r}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r} \qquad d\vec{r} = dr\,\hat{r}$$

$$\vec{E} \cdot d\vec{r} = \left(\frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r}\right) \cdot \left(dr\,\hat{r}\right) = \frac{Q}{4\pi\epsilon_0}\frac{dr}{r^2}$$

$$V = -\frac{Q}{4\pi\epsilon_0}\int_{\infty}^{R}\frac{dr}{r^2} = \frac{Q}{4\pi\epsilon_0}\frac{1}{r}\Big|_{\infty}^{R} = \frac{Q}{4\pi\epsilon_0 R}$$

$$C = Q/V = Q/\left(Q/4\pi\epsilon_0 R\right)$$

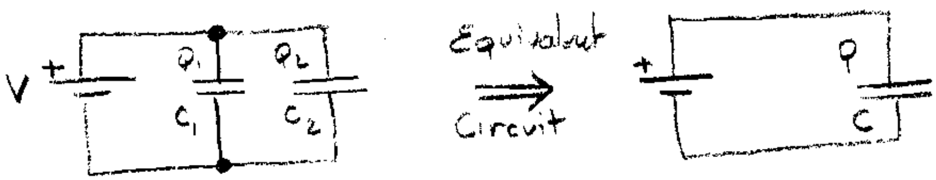
$$\boxed{C = 4\pi\epsilon_0 R}$$ CAPACITANCE OF SPHERICAL CONDUCTOR

II. Capacitors in Series and Parallel

(A) Schematic Symbol for a Capacitor

- Looks like a parallel plate capacitor, but just means <u>anything</u> with capacitance. Could be a spherical capacitor, etc.

(B) Capacitors in Parallel



$$Q = Q_1 + Q_2$$

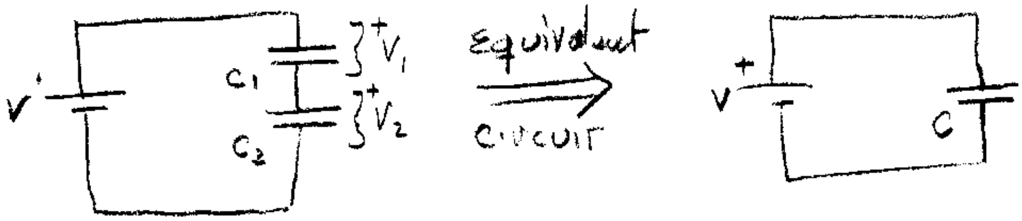
$$CV = C_1 V_1 + C_2 V_2$$

but $V = V_1 = V_2$

$$\Rightarrow \boxed{C = C_1 + C_2}$$

Equivalent Capacitance of Capacitors in parallel

(c) CAPACITORS IN SERIES



$$V = V_1 + V_2$$

$$\frac{Q}{C} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

but $\quad Q = Q_1 = Q_2$

$$\boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}}$$

Equivalent capacitance of capacitors in series

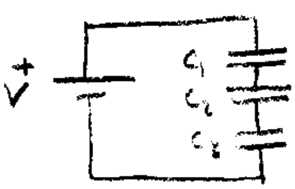
- Note: This can also be written

$$C = \frac{C_1 C_2}{C_1 + C_2}$$

and some people prefer to remember this form. However, it's best to just remember

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

Since, for example



$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

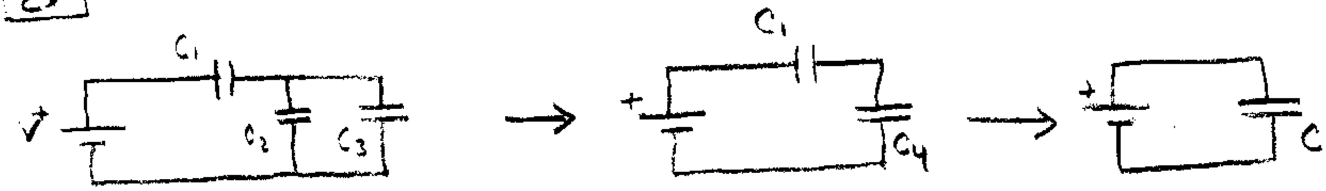
(easy to see generalization to more than two caps in series)

$$C = \frac{C_1 C_2 C_3}{C_1 C_2 + C_1 C_3 + C_2 C_3}$$

(not easy to see this as a generalization of $C = \frac{C_1 C_2}{C_1 + C_2}$

(D) Various Combinations

$\boxed{\text{Ex}}$



$$C_4 = C_2 \parallel C_3$$
$$= C_2 + C_3$$

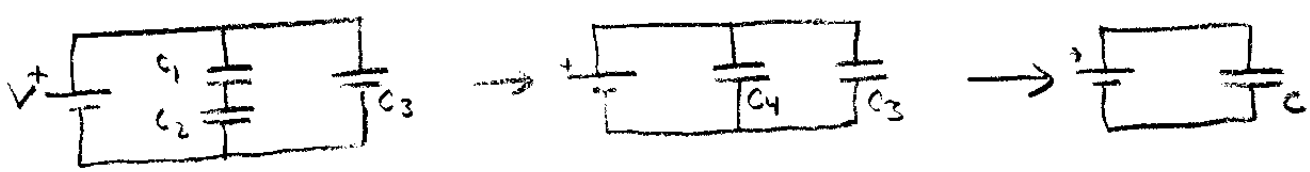
$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_4}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2 + C_3}$$

$$\frac{1}{C} = \frac{C_1 + C_2 + C_3}{C_1(C_2 + C_3)}$$

$$C = \frac{C_1(C_2 + C_3)}{C_1 + C_2 + C_3}$$

$\boxed{\text{Ex}}$



$$\frac{1}{C_4} = \frac{1}{C_1} + \frac{1}{C_2}$$

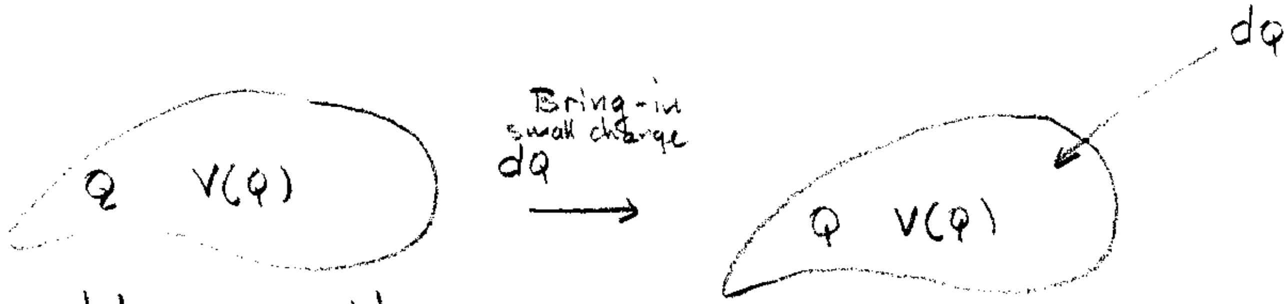
$$C = C_3 + C_4$$

$$C_4 = \frac{C_1 C_2}{C_1 + C_2}$$

$$C = C_3 + \frac{C_1 C_2}{C_1 + C_2}$$

$$C = \frac{C_3(C_1 + C_2) + C_1 C_2}{C_1 + C_2}$$

III. Energy Stored in Capacitors



arbitrary capacitor
with charge Q
and potential V(Q)

- Work done on system by bringing in charge dQ to system is

$$dW = V(Q)\,dQ$$

but $\quad C = \frac{dQ}{dV}$

$$dQ = C\,dV$$

$$dW = CV\,dV$$

$$\Delta W = \int dW = C\int V\,dV = \frac{1}{2}CV^2$$

$$\boxed{\mathcal{E}_c = \frac{1}{2}CV^2}$$

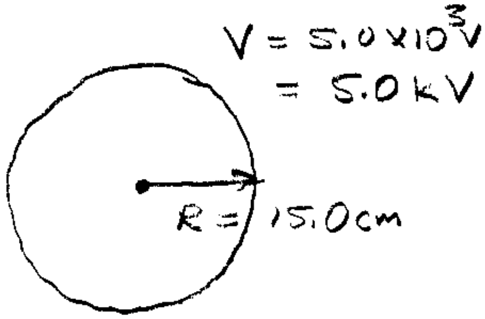
Energy stored in capacitor C at voltage V

or, since $C = Q/V$

$$\mathcal{E}_c = \frac{1}{2}QV$$

$$\mathcal{E}_c = \frac{1}{2}\frac{Q^2}{C}$$

Ex. ENERGY STORED ON VAN DE GRAAFF GENERATOR



$$\mathcal{E} = \frac{1}{2} C V^2$$

$$C = 4\pi \epsilon_0 R$$

$$\mathcal{E} = 2\pi\epsilon_0 R V^2$$

$$= (2)(3.1415)\left(8.85\times10^{-12}\,\frac{C^2}{N\cdot m^2}\right)\left(15.0\times10^{-2}\,m\right)\left(5.0\times10^{3}\,\frac{J}{C}\right)^2$$

$$\boxed{\mathcal{E} = 2.1\times10^{-4}\,J}$$

Not much energy. This is why you didn't die when the Van de Graaff discharged through your body, even though its potential was up at 5.0 kV.

- Total Charge on sphere —

$$C = \frac{Q}{V} \Rightarrow Q = CV$$

$$= 4\pi\epsilon_0 R V$$

$$= (4)(3.1415)\left(8.85\times10^{-12}\,\frac{C^2}{N\cdot m^2}\right)\left(15.0\times10^{-2}\,m\right) \times \left(5.0\times10^{3}\,\frac{J}{C}\right)$$

$$Q = 4.2\times10^{-8}\,C$$

$$\boxed{Q = .42\,nC}$$

Really small amount of charge.