

Problem 15.83

Using Gauss's Law, determine the electric field in the air gap of a charged parallel-plate capacitor. [Hint: Use a cylindrical Gaussian surface with one endface embedded in the metal of one of the plates.]

Solution (from Hecht's solutions manual):

Following the hint given in the problem statement, imagine a cylindrical Gaussian surface with one endface embedded in the metal of one plate and the other in the air gap. Put both endfaces parallel to the plates. Now apply Gauss's Law: $\oint E_{\perp} dA = \sum q_{\bullet} / \epsilon_0$. Since the E -field is parallel to the curved portion of the Gaussian surface $E_{\perp} = 0$ there, and also $E = 0$ inside the metal. So the only non-vanishing contribution to the electric flux comes from the endface in the air gap:

$\oint E_{\perp} dA = EA$, where E is the field inside the gap and A is the area of that endface. Now look at the right-hand-side of Gauss's Law. The only charge enclosed by the Gaussian surface is the charge on a portion of area A of one plate, so $\sum q_{\bullet} = \sigma A$. Equate the two sides of Gauss's Law to obtain $EA = \sigma A / \epsilon_0$, whereupon

$$E = \frac{\sigma}{\epsilon_0}$$