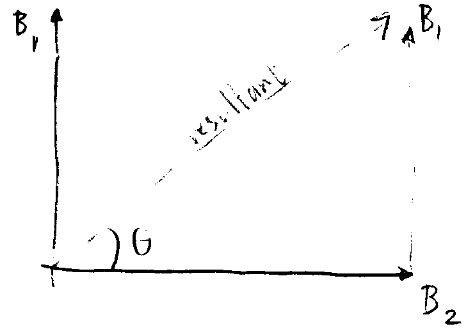
$\boxed{19.1}$

Compass needle registers net magnetic field:



Net field is the resultant vector:

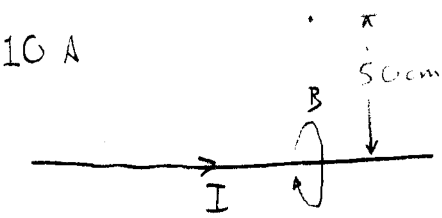
$$\tan\theta = \frac{B_1}{B_2}$$

WEEK 5 HW

19: ~~3~~, ~~5~~, ~~7~~, ~~11~~, ~~15~~, ~~35~~, ~~39~~, ~~61~~, ~~71~~, ~~89~~

20: ~~1~~, ~~5~~, ~~17~~, ~~21~~, ~~43~~, ~~71~~, ~~95~~, 107

19.3 B-field 50 cm from wire w/ I = 10 A



long straight wire

$$B = \frac{\mu_0 I}{2\pi r} \quad (19.2)$$

$$\mu_0 = 1.26 \times 10^{-6}\ \mathrm{T\cdot m/A}$$

$$B = \frac{(1.26\times10^{-6}\ \mathrm{T\cdot m/A})(10\ \mathrm{A})}{2\pi(0.5\ \mathrm{m})} = \boxed{4.01\times10^{-6}\ \mathrm{T} = B}$$

19.5 Lightning: $I = 20\ \mathrm{kA} = 20\times10^3\ \mathrm{A}$

What is B 1.0 m away?

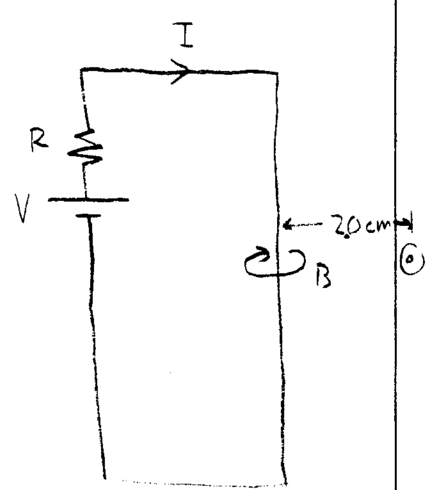
Assume bolt is a straight line,

$$B = \frac{\mu_0 I}{2\pi r} = \frac{(1.26\times10^{6}\ \mathrm{T\cdot m/A})(20\times10^3\ \mathrm{A})}{2\pi\ (1.0\mathrm{m})} = \boxed{4.0\times10^{-3}\ \mathrm{T} = B}$$

19.7 Wire 1.0m long, w/ $R = 1.2\ \Omega$

attached to 12 V battery.

What is B 20 cm away?



The resistance of the wire determines the current,

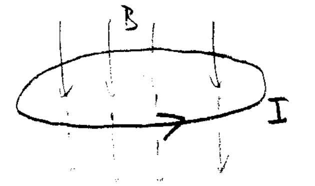
$$I = \frac{V}{R} = \frac{12\ \mathrm{V}}{1.2\ \Omega} = 10\ \mathrm{A}$$

B-field of a wire is

$$B = \frac{\mu_0 I}{2\pi r} = \frac{(1.26\times10^{-6}\ \mathrm{T\cdot m/A})(10\ \mathrm{A})}{2\pi\ (0.02\ \mathrm{m})} = \boxed{1.0\times10^{-4}\ \mathrm{T} = B}$$

$\boxed{19.11}$ Protons orbiting due to a perpendicular B-field.

What is B @ center of 20-cm-diameter orbit from 0.10mA beam? (radius = 10 cm)



This is simply a ring of current.

The B-field in the center of a ring is

$$B = \frac{\mu_0 I}{2R} = \frac{(1.26\times10^{-6}\ \mathrm{T\cdot m/A})(0.1\times10^{-3}\ \mathrm{A})}{2\times(0.10\ \mathrm{m})} = \boxed{6.3\times10^{-10}\ \mathrm{T} = B}$$

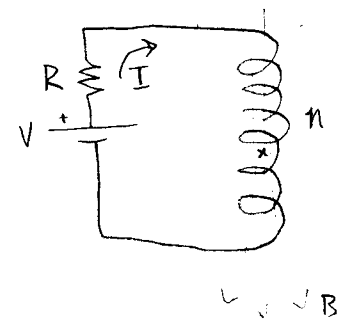
$\boxed{19.15}$ Solenoid w/ $\frac{100\ \text{turns}}{\text{cm}}$ and $R = 60\,\Omega$

Find B in the center when connected across 12 V.

Assume we want B-field in middle of long coil given by

$$B_z = \mu_0 n I \qquad (19.5)$$

where $n = \frac{100\ \text{turns}}{\text{cm}} \times \frac{100\ \text{cm}}{1\ \text{m}} = 1{,}000\ \text{turns}/\text{m}$



The current is

$$I = V/R = \frac{12\ \mathrm{V}}{60\,\Omega} = 0.2\ \mathrm{A}$$

so, the B-field is

$$B_z = \mu_0 n I = (1.26\times10^{-6}\ \mathrm{T\cdot m/A})(1000\ \text{turns}/\mathrm{m})(0.2\ \mathrm{A})$$

$$\boxed{B = 2.5\times10^{-4}\ \mathrm{T}}$$

Note that "turns" isn't really a proper unit and may be ignored or omitted (e.g., $n = 1000\ \mathrm{m}^{-1}$).

$\boxed{19.35}$ Long straight Cu pipe,

$D = 1''$, carries $50.0\,A$.

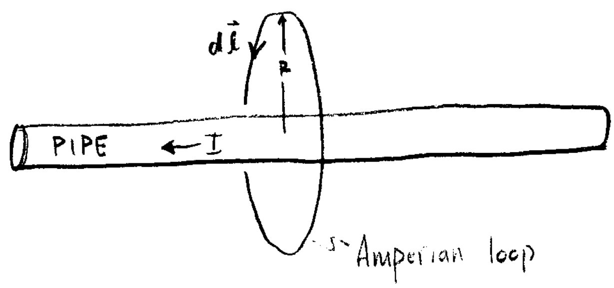
Find B-field at a distance 1m

from the pipe (in the middle).

Ampere's Law states

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \Sigma I \quad \text{OR} \quad \Sigma B_{\parallel} \Delta \ell = \mu_0 \Sigma I$$

We want to know the B-field at some radial distance from the pipe, so it makes sense to choose an Amperian loop around the pipe.



Then, by symmetry $|\vec{B}|$ is constant and $B_{\parallel} = B$ so

$$\Sigma B_{\parallel} \Delta \ell = B \Sigma \Delta \ell$$

but $\Sigma \Delta \ell$ is the length of the loop or the circumference

$\Sigma \Delta \ell = 2\pi R$

$\Rightarrow B \Sigma \Delta \ell$

$= B(2\pi R) = \mu_0 \Sigma I$

but $\Sigma I = I$, the current in the pipe,

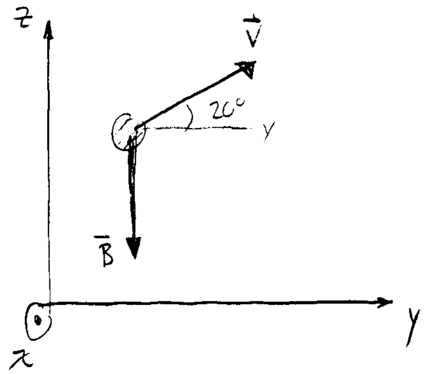
$\Rightarrow 2\pi R B = \mu_0 I$

$\Rightarrow B = \dfrac{\mu_0 I}{2\pi R}$ which is the result for B-field due to a straight wire.

$$B = \frac{(1.26 \times 10^{-6}\ T \cdot m/A)(50.0\,A)}{2\pi\ (1.0\,m)} = \boxed{1.00 \times 10^{-5}\ T = B}$$

$\boxed{19.59}$

Negatively charged projectile:



(x-axis is out of the page)

Magnetic force:

$$\vec{F}_B = q(\vec{v} \times \vec{B})$$

The direction of $\vec{v} \times \vec{B}$ is given by the right-hand-rule:

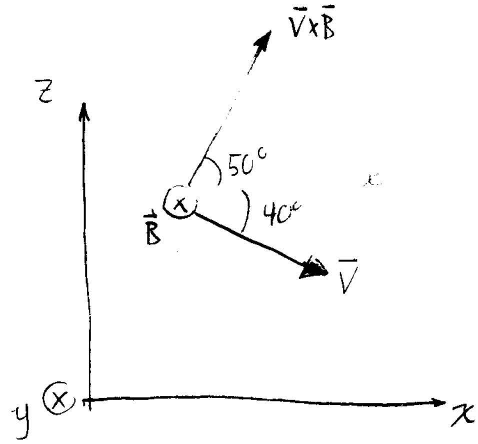
$(\vec{v} \times \vec{B})$ is in the $-x$-direction

The direction of the force is opposite to $\vec{v} \times \vec{B}$ if the charge, q, is negative

$\Rightarrow$ $\boxed{\vec{F}_B \text{ is in the } +x\text{-direction}}$

$\boxed{19.66}$ Proton

$\vec{F}_B = q(\vec{v}\times\vec{B})$



(y-axis is into page)

As in 19.54,

$\vec{v}\times\vec{B}$ is 50° above x-axis

by right-hand rule.

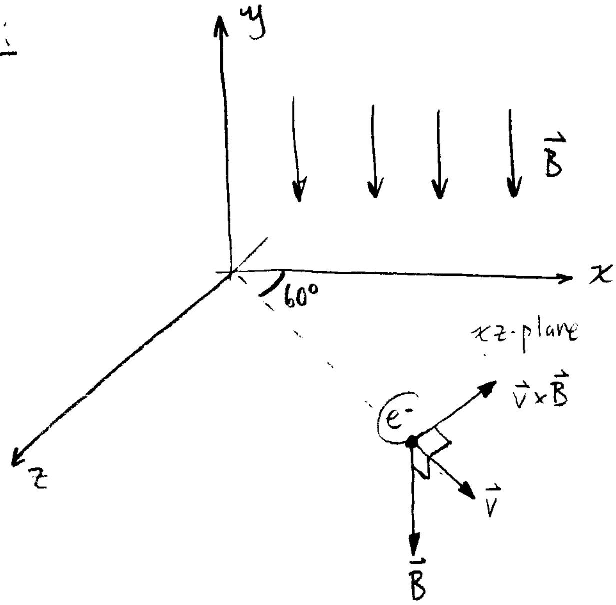
Since the particle is a proton,

the $\boxed{\text{magnetic force is also } 50^\circ \text{ above the x-axis}}$ (in the xz plane)

$\boxed{19.61}$ e^- in xz-plane @ $60°$ above x-axis

B-field in $-y$-direction

<u>Find the direction of force:</u>

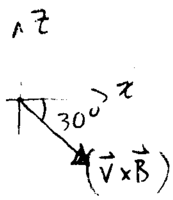


Magnetic force:

$$\vec{F}_B = q\,(\vec{v}\times\vec{B})$$

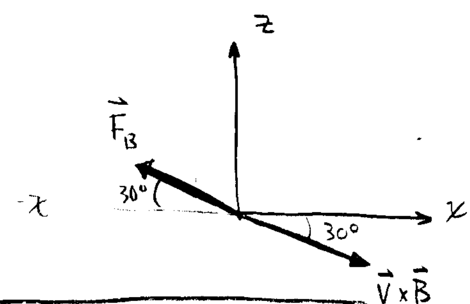
$(\vec{v}\times\vec{B})$ direction is given by the right-hand rule.

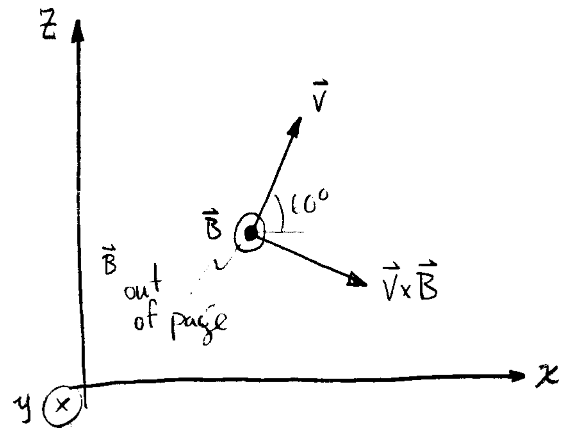
$\vec{v}\times\vec{B}$ is this direction,



same diagram

but $q<0$ for an electron, so the force is actually opposite to $\vec{v}\times\vec{B}$ or

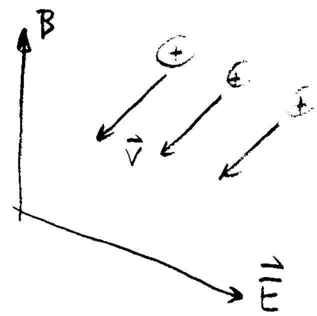




so $\boxed{\vec{F}_B \text{ is } 30° \text{ above } -x \text{ axis in the } xz\text{-plane.}}$

y-axis is into page

$\boxed{19.71}$ Beam of protons at various speeds enter $\vec{E}$- and $\vec{B}$-fields perpendicular to $\vec{v}$. When $v = E/B$ particles are undeflected.



Particles are undeflected when force on them equals zero.

The force on a charged particle in the presence of both electric and magnetic fields is

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

but this is zero for "undeflection"

$$0 = q(\vec{E} + \vec{v} \times \vec{B}) \qquad \text{but } q \text{ is constant}$$

$$\Rightarrow \quad 0 = \vec{E} + \vec{v} \times \vec{B} \qquad \text{but } \vec{v} \text{ and } \vec{B} \text{ are perpendicular}$$

so, looking at magnitudes

$$0 = E \pm vB$$

where vB is either + or − depending on if B is up or down.

Let's assume that B is such that $|\vec{v} \times \vec{B}| = -vB$

$$\Rightarrow \quad 0 = E - vB$$

$$\Rightarrow \quad E = vB \quad \Rightarrow \quad \boxed{v = \frac{E}{B} \text{ to not be deflected}}$$

Note that the magnitude of the cross product of two perpendicular vectors is just the product of their individual magnitudes.

(i.e., $|\vec{A} \times \vec{B}| = AB$ when $\vec{A} \perp \vec{B}$)

$\boxed{19.89}$ 1.5 m long straight wire

max force of 2.00 N

when 1.333 T B-field is present.

What is I?

Force on current carrying wire is

$$F_M = I \ell B \sin\theta \qquad (19.13)$$

F_M is a max when $\sin\theta$ is a max.

$\sin\theta$ is a max @ $\theta = 90^\circ \Rightarrow \sin 90^\circ = 1$

$$\Rightarrow F_{M,max} = I \ell B$$

$$\Rightarrow I = \frac{F_M}{\ell B} = \frac{(2.00\text{ N})}{(1.5\text{ m})(1.333\text{ T})} = \boxed{1.0\text{ A} = I}$$

$$\text{Tesla} = \frac{\text{kg}}{\text{meter} \cdot \text{Coulomb}}$$