WORK

In its common usage, the word <u>work</u> is a much maligned term, in man ways than one. In physics work is only performed <u>if</u>:

A force $\vec{F}$ causes displacement of an object $\vec{s}$ and it is only the component of $\vec{F}$ along the displacement that performs any work.

Mathematically $\left(W = \int \vec{F} \cdot d\vec{s} \right)$

$$W = \vec{F} \cdot \vec{s} \qquad \text{the dot product}$$

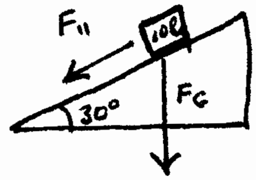
NOTE: <u>Work</u> is a scalar quantity, we do not speak of work up or down or east or west. The dot product of two vectors is a scalar quantity.

What this means is that the work is the product of the Force component // to the displacement and the displacement

$$W = F_{\parallel} \cdot s$$

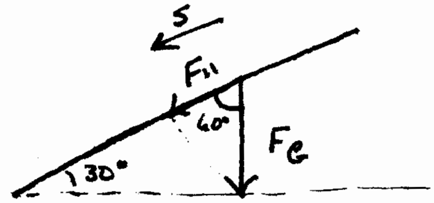
Which becomes $W = F \cos\theta \cdot s$

Consider an in clined plane (again!)



and the work performed moving the mass $m = 10$ kg down the plane 1 meter

$$W = F_{\parallel} \cdot s \Rightarrow F \cos\theta \cdot s$$



∡ between F_g + s is 60° $\Rightarrow F_{\parallel} = \cos 60°$

$$W = F_g \cos 60° \cdot s = 10\,\text{kg} \cdot 9.8\,\text{m/s}^2 \cdot 0.500 \cdot 1\,\text{m}$$
$$= 49\ \text{kg m}^2/\text{s}^2$$

NOTE
$$[W] = [F \cdot L] = [mL/T^2] = [mL^2/T^2]$$

in MKS we give the unit $1\ \text{kg m}^2/\text{s}^2$ the name the <u>joule</u>. One joule (J) is the work done by a force of 1 newton in moving a body 1 meter. In cgs the $\text{g cm}^2/\text{s}^2$ = 1 <u>erg</u>; the erg is about the amount of energy expended by a fly taking off from the floor.

By Newton's third law we can consider work done by an <u>applied force</u> (as in the case of the inclined plane above) or <u>against a force</u> (e.g. ...

of mass m by an amount h

$F = mg$

if the object is raised vertically $\cos\theta = 1$ $F_{\parallel} = F = mg$

$W = F \cdot s = mgh$)

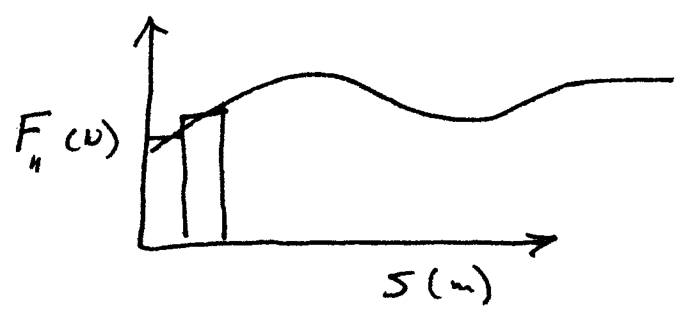
The two statements are equivalent since for every applied force there is a force acting against it. As above, it may help to consider one interpretation or the other in solving a problem

— o —

To remind you of the calculus nature of this subject, in the case when $F \neq$ const we must use the integral

$$W = \int \vec{F} \cdot d\vec{s}$$ where F is again the component $\parallel$ to the displacement

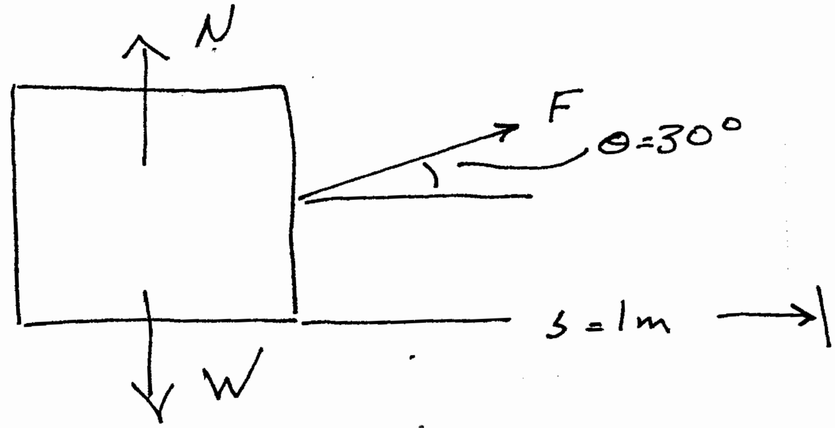
Reminding ourselves of the graphical interpretation if we plot F vs S



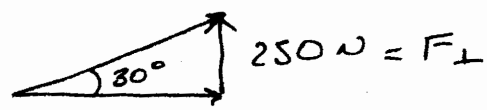
W is the area under the F-S curve

(just as S = area under the v-t curve

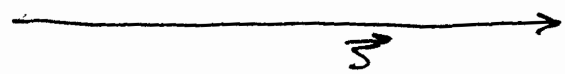
100 kg block



Let $F = 500\ N$



$F_{\parallel} = 433\ N = F \cos 30°$



Work done by F

$W_F = F_{\parallel} s = 433 N \cdot 1 m$

$= 433\ Nm = 1\ J$

Case 1: Block moves at constant speed

$\Rightarrow$ net $F = 0$; must be opposing force

friction $f_k = 433\ N \leftarrow$

$W_{frict} = f_k s = -F_{\parallel} s = -W_F = -433 N$

Case 2 $f_k = 0$ $F = 433\ N$ $a = 4.33\ m s^{-2} = \frac{F}{m}$ (remember)

$v = \sqrt{2as}$

$= 2.94\ m\ s^{-2}$

In this case $F \Rightarrow$ acceleration $\Rightarrow$ velocity
$W \Rightarrow$ energy stored in the motion of the block

Recall
$$v^2 - v_0^2 = 2a(s - s_0)$$
for simplicity let $v_0 = 0$ at $s_0 = 0$
$$v^2 = 2as$$

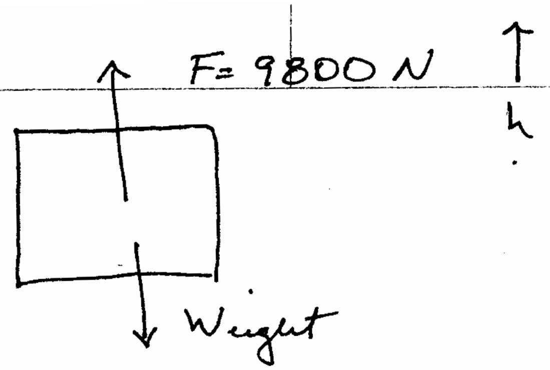
Note v, a, s are vectors + all in same direction.
$$a = \frac{F}{m} \quad (\text{this is } F_{\parallel} \text{ to } s)$$

$$v^2 = 2\frac{F}{m}s$$
or, rearranging
$$\frac{1}{2}mv^2 = Fs = W$$
We define <u>KINETIC ENERGY</u> $\boxed{KE = \frac{1}{2}mv^2}$

$W_F \Rightarrow KE$ of block. We have stored work in the energy of motion of the block

Of course $KE = \frac{1}{2}mv^2 = \frac{1}{2} \cdot 100\,kg \cdot (2.94\,ms^{-1})^2$
$$= 433\ J$$

CASE 3



We move block 1m up

$$W_F = F_{\parallel} s = 9800N \cdot 1m = 9800 J$$

Gravity is also doing work:

$$W_g = F_g \cdot s = m g s = -9800 J$$

In this case we have created <u>POTENTIAL ENERGY</u>

grav. energy can be used to do work

<u>Near</u> <u>earth's</u> <u>surface</u>

$$\boxed{(P.E.)_{grav} = mgh}$$

Work against <u>CONSERVATIVE FORCES</u> is stored in P.E. Gravity is a conservative force