VECTORS

The concept of vectors is of particular importance to physicists. A vector is a quantity that has both magnitude and direction. This is as opposed to scalar quantities which have only a magnitude.

e.g.:

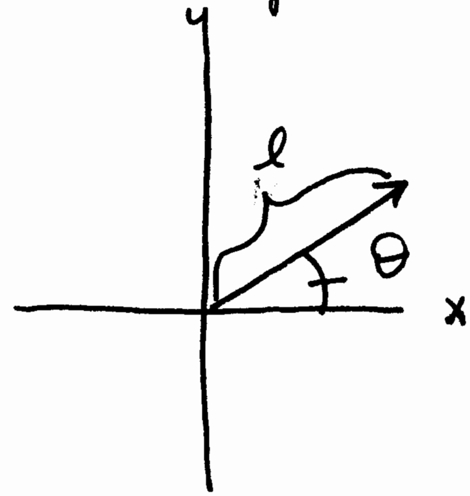
Vectors	Scalars
- velocity = 60 mph south	- Speed = 60 mph (the cop doesn't care which direction)
- displacement = distance moved (it makes a difference if a punter kicks a football 100 yds st. up or 100 yds down field)	- Temperature (°)
	- Time (h m s)
	- Relative Humidity (%)
- force (may it be with you - in your direction - rather than against you)	- Interest rate
	- energy

We will encounter a large no. of vector quantities in this course and it will be impt. to know how to treat them.

We denote vectors by **BOLDFACE** type (as the text does) or by an arrow (as we'll do in.

e.g. **V** or $\vec{\ }$

In two dimensions we require two numbers to specify a vector. Intuitively these are a magnitude and a direction = <u>components</u> (length = ℓ) (angle = θ)



Walk 45 paces NE to the treasure

ℓ = 45 paces $\approx$ 45 m
$\theta = 45^{\circ}$

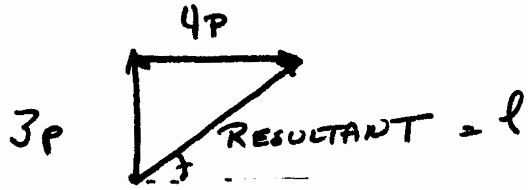
(We note parenthetically that only for displacement [distance] is this graphical representation absolutely true, everything else is a scaled representation (e.g. 1 cm on the paper represents 10 km s^{-1}, etc)

Given a ruler and a protractor one may add vectors graphically:

Long John Silver walks 3 paces north

then 4 paces east

to find the final displacement (resultant) of Long John Silver



If we measure we will find

$l = 5$ paces $\quad \theta = 37°$

We can take any arbitrary no of vectors (e.g. 4 paces NNW, 3 paces N, 2 paces ENE) and add them together graphically by placing the "foot" of one vector at the "head" of the previous vector to find the resultant or "head tail distance"

But this is a cumbersome process.

We may, however, specify vectors by different components in Cartesian Coordinate System:

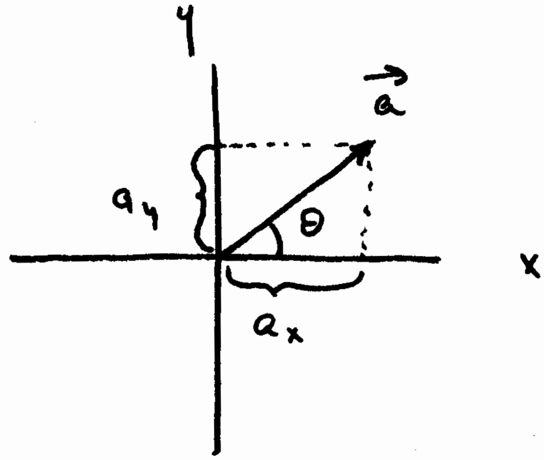
$$\text{Instead of } \vec{a} = (l, \theta)$$

$$\text{we use } \vec{a} = (a_x, a_y)$$

Trig def'ns:

$\sin\theta = \frac{y}{r}$ $\tan\theta = \frac{y}{x}$

$\cos\theta = \frac{x}{r}$



$a_x = |a| \cos\theta$

$a_y = |a| \sin\theta$

In the above, simple example we have

$\vec{a}_1 = (0, 3 \text{ paces})$

$\vec{a}_2 = (4 \text{ paces}, 0)$

To add these vectors we simply add the components separately

$a_x = a_{1x} + a_{2x} \quad (= 0 + 4)$

$a_y = a_{1y} + a_{2y} \quad (= 3 + 0)$

Thus $\vec{a} = (4 \text{ paces}, 3 \text{ paces})$

In general

$a_x = \sum_i a_{ix}$

$a_y = \sum_i a_{iy}$

for any number of vectors

Recalling our geometry

This denotes the magnitude $\rightarrow |a|^2 = a_x^2 + a_y^2 = 4^2 + 3^2 = 25$

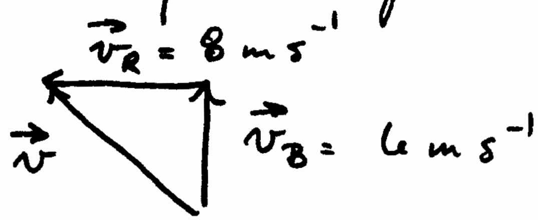
$|a| = 5$ paces

Relating to the angle

$\tan\theta = \frac{a_y}{} = 0.75 \qquad \theta = \tan^{-1} 0.75 = 37°$

For most applications in this course we'll find the cartesian notation most useful. Now that we have the basics let's try a few games:

(1) A boat which can travel $6\ \mathrm{m\,s^{-1}}$ heads north across a rapid section of the Colorado which is traveling west at $8\ \mathrm{m\,s^{-1}}$; what is the resultant velocity (magnitude + direction)



$$|v|^2 = 6^2 + 8^2 = 36 + 64 = 100$$

$$|v| = 10\ \mathrm{m\,s^{-1}}$$

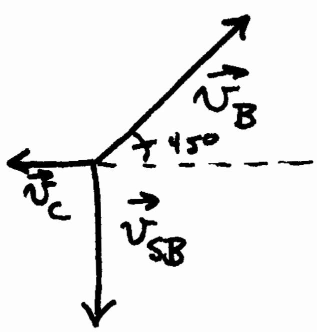
$$\vec{v} = \vec{v}_B + \vec{v}_R$$

$$= (0, 6\ \mathrm{m\,s^{-1}}) + (-8\ \mathrm{m\,s^{-1}}, 0)$$

$$\vec{v} = (v_x, v_y) = (-8\ \mathrm{m\,s^{-1}}, 6\ \mathrm{m\,s^{-1}})$$

But armed with these tools we can tackle more difficult problems:

Consider a boat headed NE at 10 m s^{-1} with a current of 2 m s^{-1} W. On the boat is a skateboarder who is moving 4 m s^{-1} S (wrt the boat). What is his velocity relative to the shore. We have then



$$\vec{v}_B = (10\,\mathrm{m\,s^{-1}}\cos 45°,\ 10\,\mathrm{m\,s^{-1}}\sin 45°)$$
$$= (7.07\,\mathrm{m\,s^{-1}},\ 7.07\,\mathrm{m\,s^{-1}})$$

$$\vec{v}_C = (-2\,\mathrm{m\,s^{-1}},\ 0\,\mathrm{m\,s^{-1}})$$

$$\vec{v}_{SB} = (0\,\mathrm{m\,s^{-1}},\ -4\,\mathrm{m\,s^{-1}})$$

$$v_x = 7.07\,\mathrm{m\,s^{-1}} + (-2\,\mathrm{m\,s^{-1}}) + 0\,\mathrm{m\,s^{-1}}$$
$$= 5.07\,\mathrm{m\,s^{-1}}$$

$$v_y = 7.07\,\mathrm{m\,s^{-1}} + 0\,\mathrm{m\,s^{-1}} + (-4\,\mathrm{m\,s^{-1}})$$
$$= 3.07\,\mathrm{m\,s^{-1}}$$

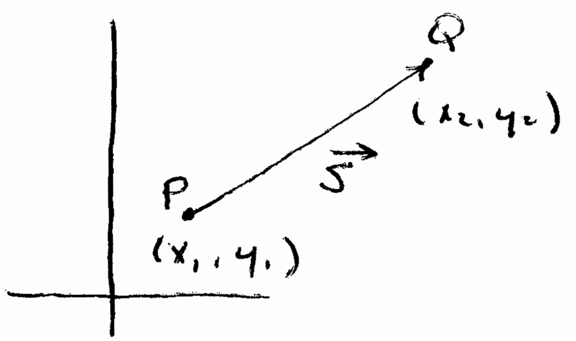
$$\vec{v} = (5.07\,\mathrm{m\,s^{-1}},\ 3.07\,\mathrm{m\,s^{-1}})$$

$$|v| = 6\,\mathrm{m\,s^{-1}} \qquad \theta = \tan^{-1}\frac{v_y}{v_x} = 31°$$

MOTION IN A PLANE

Today we are going to begin to expand our treatment of motion from the one dimensional case to two dimensions. Much of what we discuss will be strangely familiar – we've covered most of it before in talking about rectilinear motion. Basically we have the same quantities displacement, velocity, acceleration, force. In 2d motion we must deal with the vector nature of these quantities

Displacement $\vec{s}$



$|s| = \sqrt{(x_1-x_2)^2+(y_2-y_1)^2}$

$\vec{s} = \vec{s}(t)$ Because s is a vector may treat components separately $x = x(t)$ $y = y(t)$