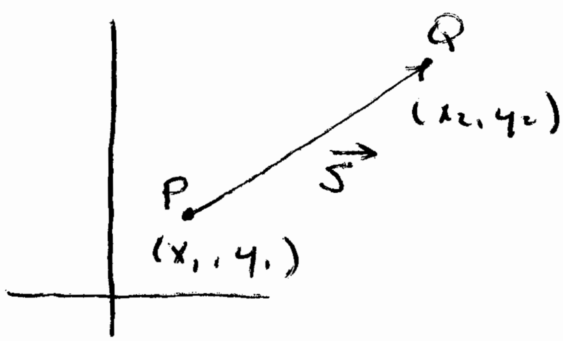
MOTION IN A PLANE

Today we are going to begin to expand our treatment of motion from the one dimensional case to two dimensions. Much of what we discuss will be strangely familiar – we've covered most of it before in talking about rectilinear motion. Basically we have the same quantities displacement, velocity, acceleration, force. In 2d motion we must deal with the vector nature of these quantities

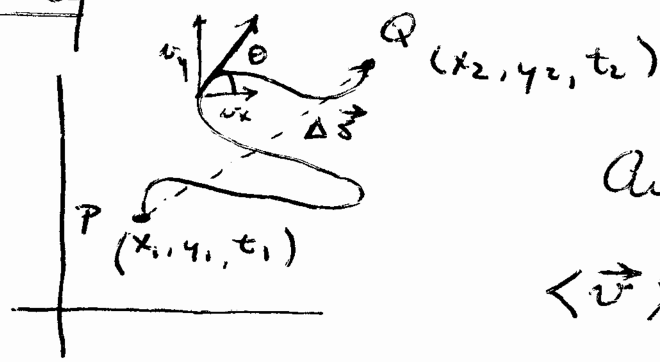
Displacement $\vec{s}$



$$|s| = \sqrt{(x_1-x_2)^2+(y_2-y_1)^2}$$

$\vec{s} = \vec{s}(t)$ Because s is a vector may treat components separately $x = x(t)$ $y = y(t)$

Velocity



Average velocity

$$\langle \vec{v} \rangle = \frac{\Delta \vec{s}}{\Delta t}$$

regardless of path from P – Q

Instantaneous velocity

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{s}}{\Delta t} = \frac{d\vec{s}}{dt}$$

Can treat components separately

$$\langle v_x \rangle = \frac{\Delta x}{\Delta t} \qquad \langle v_y \rangle = \frac{\Delta y}{\Delta t}$$

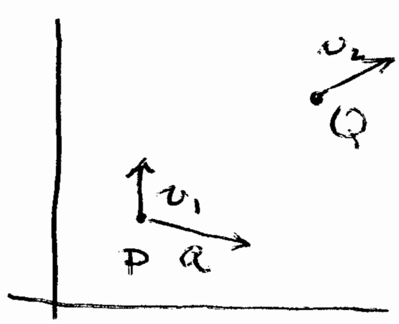
$$v_x = \frac{dx}{dt} \qquad v_y = \frac{dy}{dt}$$

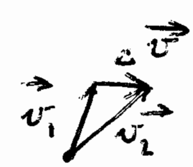
with

$$|v| = \sqrt{v_x^2 + v_y^2}$$

$$\tan\theta = \frac{v_y}{v_x}$$

Acceleration





$$\langle \vec{a} \rangle = \frac{\Delta \vec{v}}{t}$$

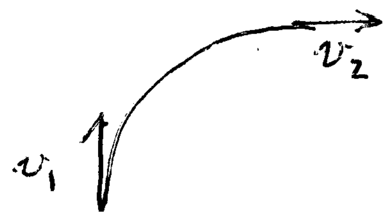
Instantaneous acceleration

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

Again can divide into components

$$a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2} \quad a_y = \frac{d^2y}{dt^2}$$

Consider a car which is traveling $10\ m\,s^{-1}$ around a curve in $10\,s$

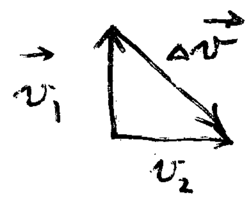


$v_1 = 10\,m\,s^{-1}$ north

$v_2 = 10\,m\,s^{-1}$ east

$v_{1x} = 0 \quad v_{1y} = 10\,m\,s^{-1}$

$v_{2x} = 10\,m\,s^{-1} \quad v_{2y} = 0$



$\Delta v_x = 10\,m\,s^{-1} \quad \Delta v_y = -10\,m\,s^{-1}$

$$\Delta v = \sqrt{v_x^2 + v_y^2} = \sqrt{(10)^2 + (-10)^2}$$

$$\Delta v = 14\ m\,s^{-1} \qquad \Delta t = 10\,s$$

$$\langle a \rangle = \frac{\Delta v}{\Delta t} = 1.4\,m\,s^{-2}$$

Is this a real acceleration – the speed of the car remains the same $10\,m\,s^{-1}$ and the direction has

Projectile Motion

FREELY FALLING "PARTICLE" acting under the influence of GRAVITY (ignore wind/air resistance) follows a path called a TRAJECTORY

@ $t = 0$ $\vec{v} = v_0$ at angle θ from ground

We separate motion into 2 components

(x)

$$x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$$

$$[\, x = v_{0x} t \,]$$

$$v_x = v_{0x} + a_x t$$

(y)

$$y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2$$

$$[\, y = \frac{1}{2} g t^2 + v_{0y} t \,]$$

$$v_y = v_{0y} + a_y t$$

The eqns above are completely general
[Eqns in brackets assume $x_0, y_0, a_x = 0$]
We separate motion into components and follow the motion independently

$v_{0x} = v_0 \cos\theta_0$

$a_x = 0$

$x_0 = 0$

$v_x = v_0 \cos\theta_0$

$x = v_0 \cos\theta_0 \, t$

$v_{0y} = v_0 \sin\theta_0$ $\qquad v_y = v_0 \sin\theta_0 - g\,t$

$a_y = +g = -9.8\,ms^{-2}$ $\qquad y = v_0 \sin\theta_0 t + \frac{1}{2} g t^2$

$y_0 = 0$

At any given time

$$v = \sqrt{v_x^2 + v_y^2} \quad @ \text{ angle } \theta$$

$$\ni \quad \tan\theta = \frac{v_y}{v_x}$$

If we wish to describe the motion in terms of a relationship between x and y (i.e. plot the trajectory in the above graph)

$$x = v_0 \cos\theta_0 t \Rightarrow t = \frac{x}{v_0 \cos\theta_0}$$

Subst in eqns for y

$$y = \frac{\cancel{v_0} \sin\theta_0 \cdot x}{\cancel{v_0} \cos\theta_0} + \frac{1}{2} g \left(\frac{x}{v_0 \cos\theta_0}\right)^2$$

$$y = \tan\theta_0 \, x + \frac{g}{2 v_0^2 \cos^2\theta_0} x^2$$

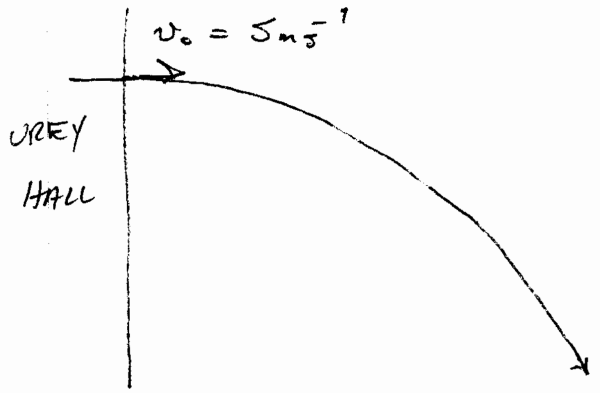
This eqn has form

$$y = ax + bx^2$$

where $a = \tan\theta_0$ $\qquad b = \frac{g}{2 v_0^2 \cos\theta_0^2}$

which is eqn for a parabola

WATERMELON



Remember question posed at end of last week — how do we treat watermelon tossed horizontally

How long to fall ; what is final $\vec{v}$?

2D — separate vectors

X

Y

INITIAL

$X_0 = 0$

$v_{0x} = 5 m s^{-1}$

$a_x = 0$

$y_0 = 15 m$

$v_{0y} = 0$

$a_y = g = -9.8 m s^{-2}$

FINAL

Since $a_x = 0$
$v_x = $ const

$x = ?$

$v_x = 5 m s^{-1}$

$t = ?$ ← same value of t → $t = ?$

$y = 0$

$v_y = ?$

$x = \cancel{x_0}^{0} + v_{0x} t + \frac{1}{2} \cancel{a_x}^{0} t^2$

$x = v_{0x} t = 5 m s^{-1} t$

$v_x = v_{0x} + \cancel{a_x t}^{0}$

$v_x = 5 m s^{-1}$

$y = y_0 + \cancel{v_{0y}}^{0} t + \frac{1}{2} a_y t^2$

$y = 15 m + \frac{1}{2}(-9.8 m s^{-2}) t^2$

$= 15 m - 4.9 m s^{-2} t^2$

$v_y = v_{0y} + a_y t$

How long does it take to fall. Eqn for y

$$0 = 15\,m - 4.9\,ms^{-2}t^2$$

$$t^2 = \frac{15m}{4.9\,ms^{-2}}$$

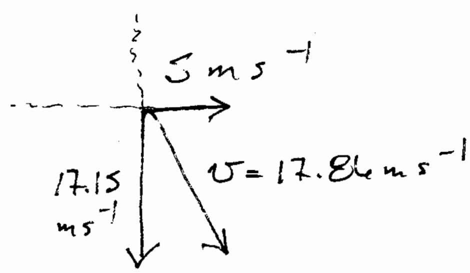
$$\underline{t = 1.75\,s}$$

How fast is it going?

$$v_y = -9.8\,ms^{-2} . t = -9.8\,ms^{-2} . 1.75\,s$$

$$= -17.15\,ms^{-1}$$

$$v_x = 5\,ms^{-1}$$

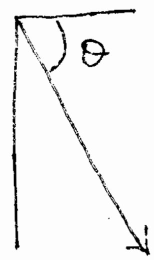


$$|v| = \sqrt{v_x^2 + v_y^2} = \sqrt{(5ms^{-1})^2 + (17.15ms^{-1})^2}$$

$$= 17.86\ ms^{-1}$$

(Note this is the same magnitude as when the melon was tossed st-up - Could there be some conservation principal here?)

Angle of impact



$$\tan\theta = -\frac{17.15}{5} = -3.43$$

$$\theta = -74^\circ$$

How far from foot of the wall

$$x = x_0 + v_{0x}\,t$$
$$= 5\,\mathrm{m\,s^{-1}} \cdot 1.75\,\mathrm{s}$$
$$= 8.75\,\mathrm{m}$$ called the range