MOMENTUM

JAN 31
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FEB 1

We wish to begin discussing the "Conservation" principles of momentum, energy and later on angular momentum. To be able to discuss conservation we must be able to look at an entire system, one in which all the interactions may be followed. There can be no external forces on the system nor can we be gaining or losing pieces of the system — such a system is an *isolated system*

MOMENTUM

We all have the conceptual understanding that a more massive body (e.g. Mack truck) in motion has more "punch" than a less massive body (e.g. VW) and that this punch is greater as one increases velocity (e.g. head on collision at 5mph vs 60mph)

This punch is *momentum*; for linear motion

$$\overset{\text{MOMENTUM}}{\vec{p}} = \overset{\text{ASS}}{m}\,\overset{\text{ELOCITY}}{\vec{v}}$$

Momentum is a vector with the direction of the velocity vector. Dimensions $[ML/T]$ - kg m/

Newton's 2nd Law

Generally

$$\vec{F} = m\vec{a}$$

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$\vec{F} = m\frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt} = \frac{d\vec{p}}{dt}$$

For const acceleration

$$a = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - t_0}$$

Since the mass does not change $\left(\frac{dm}{dt} = 0\right)$ we may include it in the derivative: This is the formulation of Newton's 2nd law as he gave it. In fact this formulation, in terms of momentum will be shown to be more general in that it still applies as $v \to c$ (special relativity) whereas $F = m\frac{dv}{dt}$ will break down (or even in problems when $m \neq$ const such as a rocket using up fuel)

$$F = m\frac{v - v_0}{t - t_0} = \frac{mv - mv_0}{t - t_0} = \frac{p - p_0}{t - t_0} = \frac{\Delta p}{\Delta t}$$

The change in momentum

$$\Delta\vec{p} = \vec{F}\Delta t$$ is called the impulse

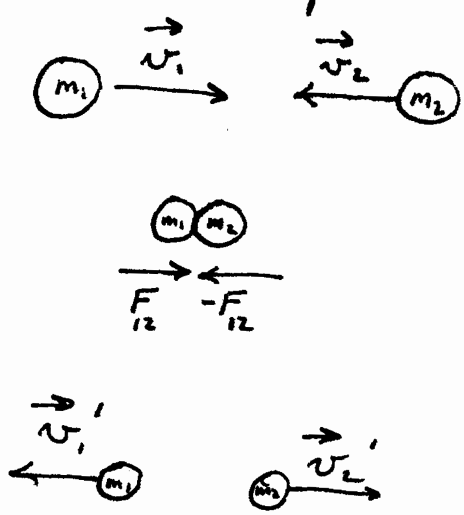
In a head on collision at 60 mph (27 m/s) a car with mass 1000 kg has received an impulse

$$\Delta\vec{p} = m\vec{v} - m\vec{v}_0 = 1000\,\text{kg}\cdot 0 - 1000\,\text{kg}\cdot 27\,\text{m/s}$$
$$= -27{,}000\ \text{kg m/s}$$

If from moment of impact, until the car stops is 1 sec, then the average force

$$F = \frac{\Delta p}{\Delta t} = \frac{2.7\times10^4\ kg\,m/s}{1\ sec} = 2.7\times10^4\ N$$

The concept of momentum is particularly important, because we may show by Newton's third law that the momentum of an isolated system is constant. If we consider a system of 2 masses



upon collision

By newton's third law force exerted by m_2 on m_1 must be equal and opposite to F_{12}!

$$\Delta p_1 = -F_{12}\,\Delta t \qquad \Delta p_2 = F_{12}\,\Delta t$$

$$\underline{\underline{\Delta p_{TOT}}} = \Delta p_1 + \Delta p_2 = -F_{12}\,\Delta t + F_{12}\,\Delta t = \underline{\underline{0}}$$

another way of writing this is

$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1{}' + m_2\vec{v}_2{}'$$

This is easily generalized to a system of any # of particles simply by noting that in each we do not have to examine the detailed interactions of the atoms + molecules that make them up

We have discussed the possible difference between inertial and gravitational mass

Grav mass

$$F_{grav} = \frac{G m_1 m_2}{r^2}$$

$$a_{grav} = \frac{G m}{r^2}$$

Inertial mass

$$F = \frac{d\vec{p}}{dt} = \frac{d m\vec{v}}{dt}$$

One means of measuring inertial mass is through cons. of momentum

$$m_1 \vec{v}_1 + m_2 \vec{v}_2 = m_1 \vec{v}_1{}' + m_2 \vec{v}_2{}'$$

$$m_1 (\vec{v}_1 - \vec{v}_1{}') = m_2 (\vec{v}_2{}' - \vec{v}_2)$$

$$\frac{m_1}{m_2} = \frac{(\vec{v}_2{}' - \vec{v}_2)}{(\vec{v}_1 - \vec{v}_1{}')}$$

if we take m_2 to be some std of mass (e.g. exactly 1 kg) then by measuring the initial and final velocities one can measure the mass of m_1

e.g. A car ($v = 20$ m/s) ~~and truck~~ collides with a stationary truck and the entangled wreck moves along with a velocity of 5m/s.

Cons. of momentum $\vec{P} = \vec{P}'$

$$m_c v_c + m_T v_T = m_c v_c' + m_T v_T'$$

Note since car + truck are entangled $v_c' = v_t' = v'$ but that's largely irrelevant

Rewriting

$$m_c (v_c - v') = m_T (v' - v_T)$$

$$\frac{m_c}{m_T} = \frac{v' - v_T}{v_c - v'} = \frac{5\,m/s - 0}{20\,m/s - 5\,m/s} = \frac{5}{15} = \frac{1}{3}$$

If e.g. the mass of the truck is 2000 kg then m_c = 667 kg.

"INELASTIC COLLISION"

Chapter 8 Impulse and momentum & collisions

If object has mass m, velocity v
it has momentum $p = mv$

If a force F acts from $t = t_1$ to $t = t_2$ later
then the impulse of the force is $F(t_2 - t_1)$

start with $v_1 = v_0 + at_1$

multiply by m $mv_1 = mv_0 + mat_1$

$$mv_1 = mv_0 + Ft_1$$

$$v_2 = v_0 + at_2$$

$$\therefore mv_2 = mv_0 + Ft_2$$

$$\therefore mv_2 - mv_1 = Ft_2 - Ft_1 = F(t_2 - t_1)$$

but $mv_2 - mv_1 = p_2 - p_1$ = change in momentum

$\therefore$ impulse of a force = change in momentum.

note similarity to work = ΔKE

Work = Force × dist = $K_2 - K_1$

Impulse = Force × time = $P_2 - P_1$

<u>Note</u> Newton's 2nd law

$$F = ma = m\frac{dv}{dt} = \frac{dp}{dt}$$

integrate to get

$$\int F\,dt = \int m\,dv = m\int dv = m(v_2 - v_1)$$

$$\therefore \int_{t_1}^{t_2} F\,dt = P_2 - P_1$$

if the force is constant from t_1 to t_2

then $$\int_{t_1}^{t_2} F\,dt = F(t_2 - t_1) = P_2 - P_1$$

Baseball example:

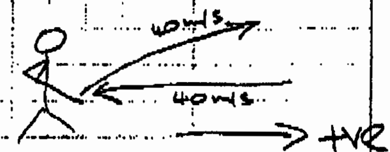
fastball travels at 40m/s (~90mph) towards batter & leaves bat with same speed in opposite dirn. Ball is in contact with bat for 1ms, ball has m = 0.16 kg what is average force of bat on ball?

$m_{ball} = .16\,kg \qquad t_{impact} = 1\times10^{-3}\,s$

$$\int F\,dt = \overline{F}\,\Delta t = P_2 - P_1 = mv_2 - mv_1$$

let $v_2 = 40\,m/s$

$v_1 = -40\,m/s$



$$\therefore\ P_2 - P_1 = 0.16\times40 - (-0.16\times40)$$

$$P_2 - P_1 = 0.16\times80 = 12.8\ kg\,m/s.$$

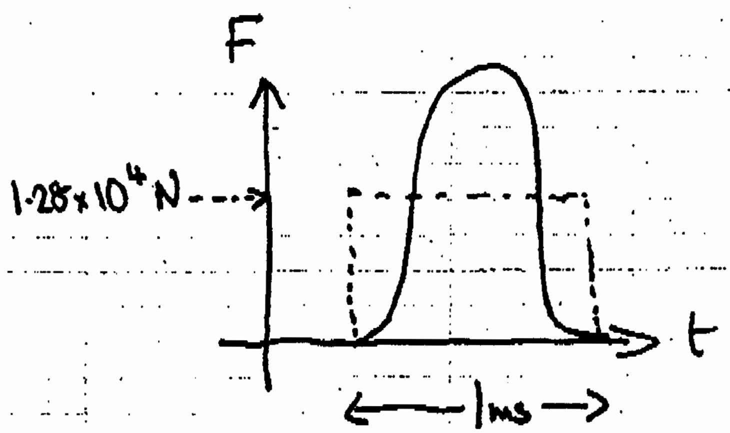
$$\therefore\ \overline{F}.\,\Delta t = 12.8$$

$$\therefore\ \overline{F} = \frac{12.8}{1\times10^{-3}} = 1.28\times10^{4}\left(kg\frac{m}{s^2} = N\right)$$

$(\overline{F} \simeq 1.5\ \text{tons}!)$ no wonder it hurts

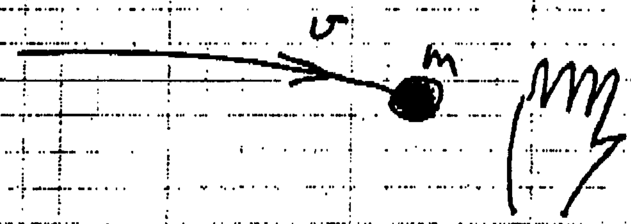
area undercurve $= \int F\,dt$

area under rectangle $= \overline{F}\,\Delta t$



$$\int F\,dt = \overline{F}.\,\Delta t = P_2 - P_1$$

Catching a ball:



Initial mom ball mv

final mom 0

$\therefore \Delta$ momentum $= mv = F(t_2 - t_1) = F\Delta t$

if hand held still Δt small $\therefore$ F large which hurts

if hand moved along with ball Δt large $\therefore$ F much smaller $\therefore$ hurts less

Other examples – landing on hard ground from a jump – bend legs increases Δt, decreases F

running shoes

– car crash w seatbelt well secured

Δt is same as for crumpling of front end

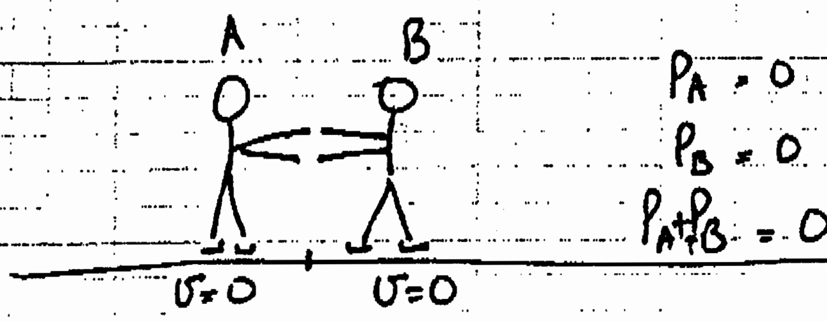
(earlier ex – 50 km/h to 0, 0.4 m = crumple front end

$\Delta t = 60\,\text{ms}$

$\Delta mv = 50\,\text{kg} \times 14\,\text{m/s} = 700\,\text{kg m/s} = F\Delta t$

$\therefore F = 700/60\times10^{-3} = 1.2\times10^{4}\,\text{N}$

Suppose 2 skaters on ice each $m = 40\,kg$

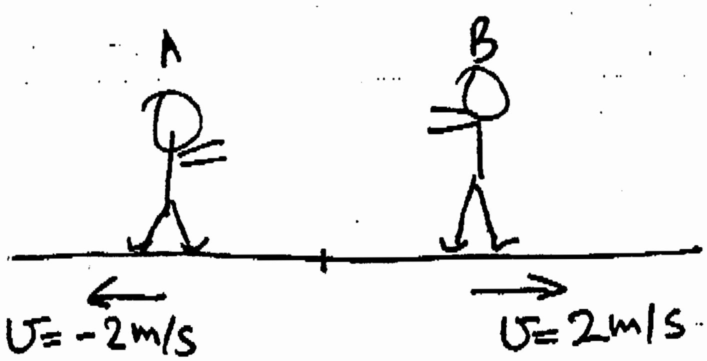


$P_A = 0$

$P_B = 0$

$P_A + P_B = 0$

start at rest & then push off from each other



Newtons third law (action & reaction equal & opposite) means that the impulse given by A to B is equal & opposite that given by B to A

∴ change in momentum of each skater is equal & opposite

$P_A = -40\,kg \times 2\,m/s$ $P_B = 40\,kg \times 2\,m/s$, $P_A + P_B = 0$

ie change of total momentum of both A & B is zero