ELASTIC COLLISIONS

- KINETIC ENERGY IS CONSERVED
- MOMENTUM IS CONSERVED

$$mv + MV = mv' + MV'$$

$$\frac{1}{2}mv^2 + \frac{1}{2}MV^2 = \frac{1}{2}mv'^2 + \frac{1}{2}MV'^2$$

Look at a specific case $V = 0$

$$\cancel{\tfrac{1}{2}}\, m v_0^2 = \cancel{\tfrac{1}{2}}\, m v'^2 + \cancel{\tfrac{1}{2}}\, M V'^2 \qquad \text{Cons of KE}$$

$$m v_0 = m v + M V \qquad \text{cons of mom}$$

$$m(v_0^2 - v^2) = MV^2 \qquad \text{KE}$$

factor

$$① \quad m(v_0 + v)(v_0 - v) = MV^2$$

$$② \quad m(v_0 - v) = MV \qquad \text{MOMENTUM}$$

Subst ② in ①

$$\cancel{MV}(v_0 + v) = \cancel{M}V^{\cancel{2}}$$

$$③ \quad V = v_0 + v$$

Subst in ② for V

$$m(v_0 - v) = M(v_0 + v)$$

Solve for v

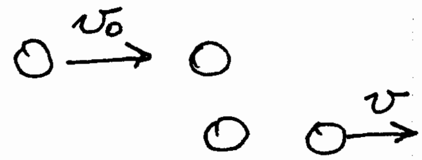
$$v = \frac{m - M}{m + M} v_0$$

Subst for v in ③

$$V = \frac{2m}{m + M} v_0$$

Billiard Balls

$$M = m$$

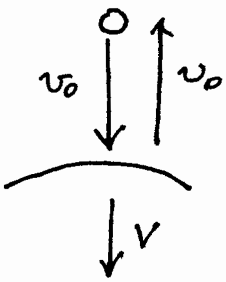


$$v = \frac{m-m}{2m} v_0 = 0$$

$$V = \; = \frac{2m}{m+m} v_0 = v_0$$

Ball + Earth $\quad M \gg m$

$$v = \frac{m-M}{\cancel{2m}\; m+M} v_0 \approx \frac{-M}{M} v_0$$



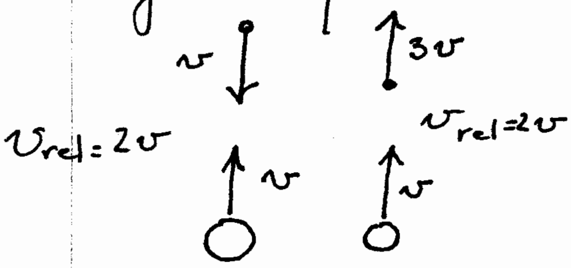
$$v \approx -v_0$$ Bounces off

$$V = \frac{2m}{m+M} v_0 \ll v_0$$ Earth doesn't move much

Finally look back at (3)

$$v = V - v_0$$

Generally the relative velocity will remain

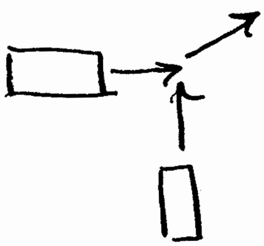


$$v_0 - V_0 = V - v$$

If cars are not in straight line must perform vector sum, e.g.

$v_c = 20\,m\,s^{-1}$ N $\qquad v_T = 10\,m\,s^{-1}$ E

(which has the greater momentum



$$\vec{P} = \vec{P}\,'$$

$$m_c \vec{v}_c + m_T \vec{v}_T = (m_c+m_T)\,\vec{v}\,'$$

Must solve by components:

x $\quad \cancelto{0}{m_c v_{cx}} + m_T v_{Tx} = m_c+m_T \ v_x'$

$$v_x' = \frac{m_T}{m_T+m_c} v_{Tx} = 7.5\,m\,s^{-1}$$

y $\quad m_c v_{cy} + \cancelto{0}{m_T v_{Ty}} = m_c+m_T \ v_y'$

$$v_y' = \frac{m_c}{m_T+m_c} v_{cx} = 5.0\,m\,s^{-1}$$

$$|v'| = \left[v_x'^2 + v_y'^2 \right]^{1/2} = 9.01\,m\,s^{-1}$$

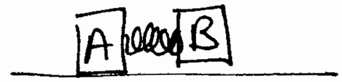
$$\tan\theta = \frac{v_y'}{v_x'} = \frac{\frac{m_c}{\cancel{m_T+m_c}} v_c}{\frac{m_T}{\cancel{m_T+m_c}} v_T}$$

$$= \frac{m_c v_c}{m_T v_T} = 0.667$$

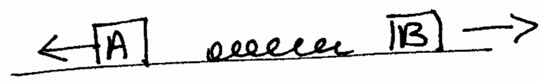
$$\theta = 33.7^\circ$$

Recoil

2 blocks A&B with spring between



let go



conserve mom

$$0 = m_A v_A + m_B v_B$$

$$\therefore \frac{v_A}{v_B} = \frac{-m_B}{m_A}$$

ratio of KE $\quad \dfrac{\frac{1}{2} m_A v_A^2}{\frac{1}{2} m_B v_B^2} = \dfrac{m_A}{m_B}\left(\dfrac{v_A}{v_B}\right)^2 = \dfrac{m_B}{m_A}$

ie $KE \propto 1/m$ $\quad$ small mass – more KE

momenta =l $\quad$ equal to impulse (Ft)

KE = work = F x distance

acce, vel, disp larger for smaller body

$\therefore$ KE larger

eg rifle recoil when bullet fired.

why hold rifle against shoulder?

Cons of energy

$$\cancel{\tfrac{1}{2}} kx^2 = \cancel{\tfrac{1}{2}} m_A v_A^2 + \cancel{\tfrac{1}{2}} m_B v_B^2$$

Subst

$$v_B = -\frac{m_A}{m_B} v_A$$

$$kx^2 = m_A v_A^2 + m_B \left(-\frac{m_A}{m_B} v_A\right)^2$$

$$= m_A v_A^2 + \frac{m_A^2}{m_B} v_A^2$$

$$= m_A \left(1 + \frac{m_A}{m_B}\right) v_A^2$$

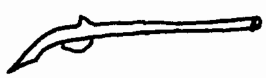
$$v_A = \left[\frac{k}{m_A}\left(1 + \frac{m_A}{m_B}\right)^{-1}\right]^{1/2} x$$

Recoil

Typical muzzle velocity $1000\ m\,s^{-1}$

Bullet $10g = 0.01\ kg$

Rifle $3\ kg$

 $\leftarrow * \longrightarrow$ D $m_b v_b$

$m_r v_r$

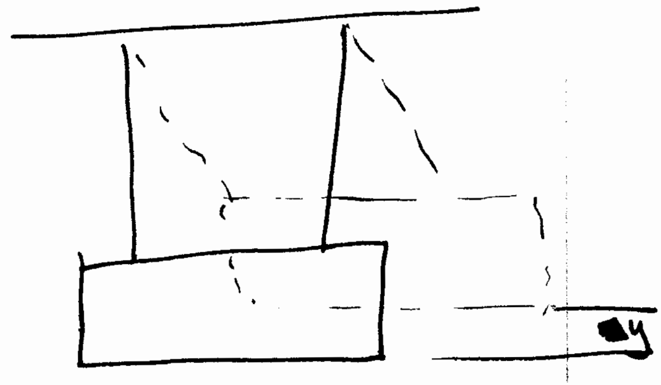
$10\ kg\ m\ s^{-1}$

$10\ kg\ m\ s^{-1}$

$v_r = 3\ m\,s^{-1}$ (~ 10 mph)

Ballistic pendulum. INELASTIC

bullet m, v before
pendulum M, $v=0$
after $(m+M)$ at V.



$$\therefore \quad mv = (m+M)\,V$$

conserv. horiz mom.
($\tau_{swing} \ll \tau_{impact}$)

KE after impact $= K = \frac{1}{2}(m+M)\,V^2$

Swings till KE converted to PE

$$\frac{1}{2}(m+M)V^2 = (m+M)\,gy$$

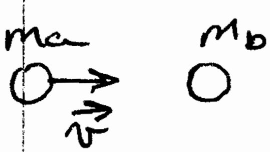
$$V = \sqrt{2gy}$$

$$\text{but } v = \frac{(m+M)}{m}V = \frac{m+M}{m}\sqrt{2gy}$$

what about KE $\quad \dfrac{K_2}{K_1} = \dfrac{\frac{1}{2}(m+M)V^2}{\frac{1}{2}mv^2} = \dfrac{m}{m+M}$

ie if $M = 1\,\text{kg}$ $\quad m = 1\,\text{g}$ $\quad \dfrac{K_2}{K_1} = 0.1\%$

~~ELASTIC~~ COLLISION



$$\vec{P}' = P \qquad m_a\vec{v}_a + m_b\vec{v}_b = m_a\vec{v}_a{}' + m_b\vec{v}_b{}'$$

$$KE' = KE \qquad \tfrac{1}{2}m_a v_a^2 + \tfrac{1}{2}m_b v_b^2 = \tfrac{1}{2}m_a v_a'^2 + \tfrac{1}{2}m_b v_b'^2$$

We will look here at a specific case $v_b = 0$ (but the results will turn out to have wider applicability

$$m_a \underset{\equiv v}{v_a} = m_a \underset{\equiv v_a'}{v_a'} + m_b \underset{\equiv v_b'}{v_b'} \qquad (1)$$

$$\tfrac{1}{2} m_a v_a^2 = \tfrac{1}{2} m_a v_a'^2 + \tfrac{1}{2} m_b v_b'^2 \qquad (2)$$

(2) $$\cancel{\tfrac{1}{2}} m_a v^2 = \cancel{\tfrac{1}{2}} m_a v_a'^2 + \tfrac{1}{2} m_b v_b'^2$$

$$m_a(v^2 - v_a'^2) = m_b v_b'^2$$

$$m_a(v - v_a')(v + v_a') = m_b v_b'^2$$

(1) $$m_b v_b' = m_a(v - v_a')$$

Subst (1) above

$$\cancel{m_b}\,\cancel{v_b}\,(v + v_a) = \cancel{m_b}\, v_b^{\cancel{2}}$$

(3) $$v_b' = v + v_a'$$

Subst in ①

$$m_b(v + v_a') = m_a(v - v_a')$$

Solving for v_a' $\Rightarrow$

$$v_a' = \frac{m_a - m_b}{m_a + m_b} v$$

Then subst this for v_a

$$v_b' = \frac{2m_a}{m_a + m_b} v$$

Suppose $m_a = m_b \equiv m$ (Billiard Balls)

$\circ \rightarrow \circ$

$$v_a' = \frac{m - m}{2m} v = 0$$

$\circ \quad \underset{v}{\circ \rightarrow}$

$$v_b' = \frac{2m}{m + m} v = v$$

Suppose $m_b \gg m_a$

$\circ \rightarrow \bigcirc$

$$v_a' = \frac{m_a - m_b}{m_a + m_b} v \approx \frac{-m_b}{m_b} v = -v$$

$\leftarrow \circ \quad \bigcirc$ __Bounces off__

Finally consider ⑧

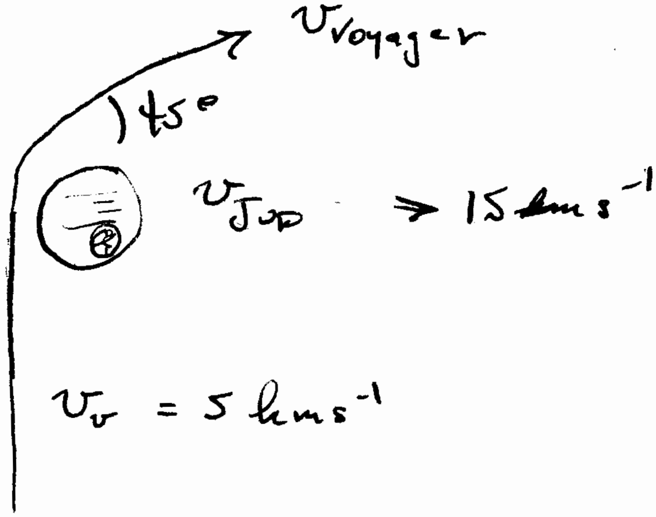
$$v = v_b - v_a$$

Go back to old system More generally

RELATIVE VELOCITY $\quad v = v_a - v_b = v_b' - v_a'$

[illegible]

GRAVITATIONAL "SLINGSHOT"



This is an elastic collision (gravity is the mediator)

$$\vec{v}_{rel} = \vec{v}_J - \vec{v}_V = (15 kms^{-1}, 0) - (0, 5 kms^{-1})$$
$$= (15\ kms^{-1}, -5\ kms^{-1})$$

$$|v_{rel}| = 15.81\ kms^{-1}$$

after encounter

$$|v'_{rel}| = \left| (v'_{vx} - v_J)^2 + v'^2_{vy} \right|^{1/2} = 15.81\ kms^{-1}$$

$$\left[(v'_v \cos 45° - 15 kms^{-1})^2 + (v'_v \sin 45°)^2 \right]^{1/2} = 15.81\ kms^{-1}$$

$$v'^2_v \cos^2 45° - 30\, v'_v \cos 45° + 225 + v'^2_v \sin^2 45° = 250$$

$KE'/KE = 3$ when frame

$$v'^2_v - 21.21\, v'_v - 25 = 0$$

$$v = 8.65\ km\ s^{-1}$$