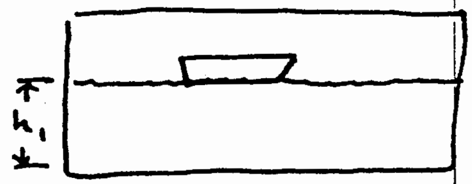
ARCHIMEDES (287-212 BC) Physicist, mathematician of Syracuse. Famous in his day for ingenious machines which captured the public's imagination. He is credited with prolonging the Roman siege of Syracuse for 3 years due to his inventions (huge parabolic mirrors to focus sunlight on Roman ships to set them afire, huge cranes that toppled ships, etc.)

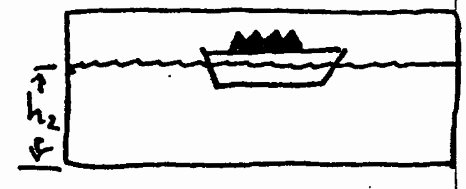
His written work for which he is remembered today include geometry, the basis for calculus, later improved + completed by Newton + Leibnitz, equilibrium + other subjects in mechanics - hence his machines - and hydrostatics.

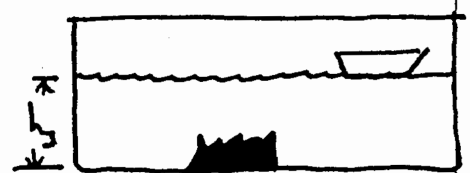
Archimedes was a favorite of King Hiero II of Syracuse + one day the king posed him the following problem:

Is this crown pure gold? Archimedes took the crown down to the Public Baths and devised the following experiment:

In a tank in a small vessel he carefully measured the level of water – h_1. Then he placed the crown in the vessel and again measured the level of the water – h_2. Finally, he placed the crown in the tank w/o the vessel and again measured the level of the water – h_3. Archimedes then did a simple calculation and informed Hiero that the crown was indeed impure – possibly with silver or lead. Hiero promptly called for the execution of the goldsmith and Archimedes – so pleased was he with his new discovery – ran home shouting "Eureka!" in the nude.







Let's follow Archimedes reasoning and calculation:

He reasoned that:

a) When the crown was in the vessel it must displace an amount (volume) of water equal in mass to that of the crown. He knew that buoyant forces are created when objects are placed in fluids - the heavier the object the more water is displaced. Hence the mass of the crown must be equal to the mass of water displaced

$$\underset{\text{mass}}{m} = \underset{\text{density}}{\rho}\ \underset{\text{Volume}}{V}$$

thus

$$m_{crown} = m_{H_2O\,displaced} = \rho_{H_2O} V_{H_2O\,displaced}$$

$$V_{H_2O} = (h_2 - h_1) \times l_{ength} \times w_{idth}$$

$$m_{crown} = \rho_{H_2O} \cdot (h_2 - h_1) \cdot l \cdot w$$

Now, when the crown is allowed to sink it displaces an amount of water equal to its Volume, so

$$V_{crown} = (h_3 - h_1) \cdot l \cdot w$$

and

$$\rho_{crown} = \frac{m_{crown}}{V_{crown}} = \rho_{H_2O} \frac{h_2 - h_1}{h_3 - h_1} \cdot \frac{\cancel{l \cdot w}}{\cancel{l \cdot w}}$$

Note Archimedes didn't even have to know the size of the tank. $\rho_{H_2O} = 1\ \text{gram/cm}^3$ and

$$\rho_{crown} = \frac{h_2 - h_1}{h_3 - h_1} \cdot 1\ g/cm^3$$

I happen to have here a copy of Archimedes lab notebook with the following measurements:

$$\left.\begin{aligned} h_1 &= 10 \text{ cm} \\ h_2 &= 17.5 \text{ cm} \\ h_3 &= 10.5 \text{ cm} \end{aligned}\right\} \text{ thus } \rho_{crown} = \frac{7.5}{0.5} \text{ g/cm}^3$$

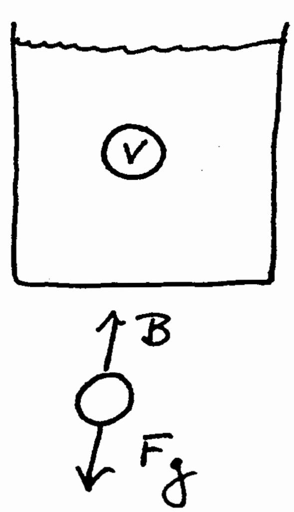
$$= 15 \text{ g/cm}^3$$

Well! The density of gold is 19.3 g/cm^3 so the crown must have been polluted with some less dense metal

We can state <u>Archimedes principal</u> as follows:

THE <u>BUOYANT</u> <u>FORCE</u> ON AN OBJECT IN A FLUID IS EQUAL TO THE <u>WEIGHT</u> OF THE DISPLACED FLUID

Let's think about a completely still fluid and consider a volume near the center of the fluid.



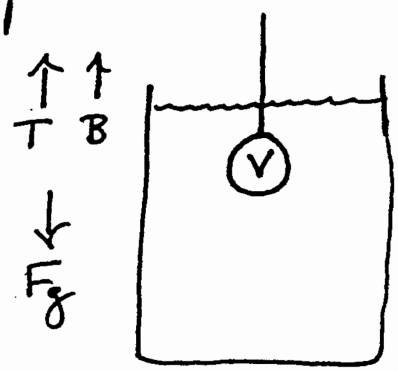
There is a gravitational force

$$F_g = mg = \rho_0 V g$$

acting downward on the volume. If it is not falling then there must be a force opposing and balancing the gravitational force (weight). We call this a <u>BUOYANT FORCE</u> and from the above we must have that

$$B = \rho_0 g V$$

Now suppose that we suspend an object of equal volume from a string in the fluid. The object will feel the following forces



$T =$ tension in the string

$w = F_g = \rho g V$

$B = \rho_0 g V$ - buoyant force will be the same as a volume of fluid

Thus, for the object to be stationary

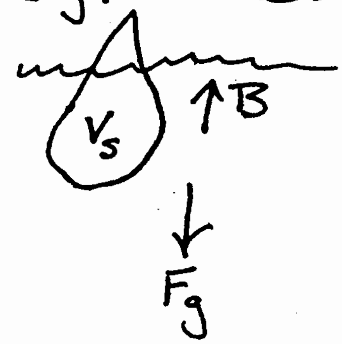
$$F_g = T + B$$

$$\rho g V = T + \rho_0 g V$$

$$\underline{T = (\rho - \rho_0) g V}$$

Thus, Archimedes might have performed his expt with the crown as follows. Holding the crown on a spring scale or other means of measuring tension float, partially submerged.

e.g. ICEBERG.



If V_s is the volume submerged then

$$B = \rho_0 V_s g$$

and

$$F_g = \rho V g$$

Equating the two

$$\rho_0 V_s g = \rho V g$$

$$\rho_0 V_s = \rho V$$

or $$\frac{\rho}{\rho_0} = \frac{V_s}{V}$$

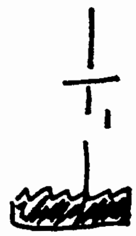
These are units we'll use

Ice has a density $0.92\ g/cm^3$ $(920\ kg/m^3)$
whereas seawater is $1.025\ g/cm^3$ $(1025\ kg/m^3)$

$$\frac{V_s}{V} = \frac{\rho}{\rho_0} = \frac{0.92}{1.025} = 0.898$$

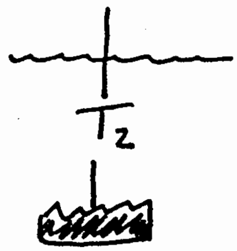
The iceberg is about 90% submerged
Alas Titanic!

1) Before submersio



$$T_1 = F_g = \rho g V$$

2) After submersio



$$T_2 = (\rho - \rho_0) g V$$

$$\frac{T_2}{T_1} = \frac{(\rho - \rho_0) \cancel{g} \cancel{V}}{\rho \cancel{g} \cancel{V}} = \frac{\rho - \rho_0}{\rho}$$

or

$$\rho = \frac{\rho_0 T_1}{T_1 - T_2}$$

Suppose $T_1 = 15\,N$ $\quad T_2 = 14\,N$

$$\rho = \rho_0 \frac{15}{15-14} = 15\rho_0 = \underline{15\,g/cm^3}$$

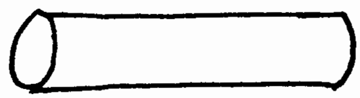
Same result.

What $\underline{\underline{if}}$ $\quad \rho < \rho_0$

(If we place this in the eqn we see T comes out negative. Actually there is a net buoyant force upwards. The object will rise to the surface and

Thus armed we continue with our discussion of fluid mechanics and a new principle embodied in BERNOULLI'S EQN which deals, in part, with fluid flow

Consider a pipe (or tube or ...)

$Q_1 \rightarrow$  $Q_2 \rightarrow$ into which we introduce a volume of fluid (incompressible in this case), Q_1 (m^3 per second). We call this the flow rate

If the fluid cannot be compressed, can't take a detour in the pipe, then it must come out the other end. That is to say that the flow rate out of the tube Q_2 must equal the flow rate into the tube (e.g. garden hose)

EQN OF CONTINUITY

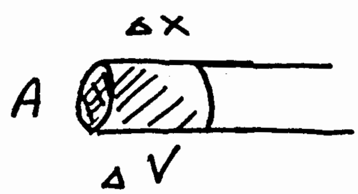
Flow rate in = flow rate out

$$Q_1 = Q_2$$

Nothing deep or earth-shaking about that!
It will turn out to be helpful + more general to write this in somewhat

different form:

Suppose in a time Δt we introduce a volume ΔV of fluid into our hose



so that it fills a length of hose ΔX as shown

Then $\Delta V = \Delta x\, A$

The velocity of the fluid will be

$$v = \frac{\Delta x}{\Delta t}$$

and the flow rate will be

$$Q = \frac{\Delta V}{\Delta t} = A\,\frac{\Delta x}{\Delta t} = Av$$

We can now write the eqn of continuity somewhat differently

$$\underline{A_1 v_1 = A_2 v_2}$$

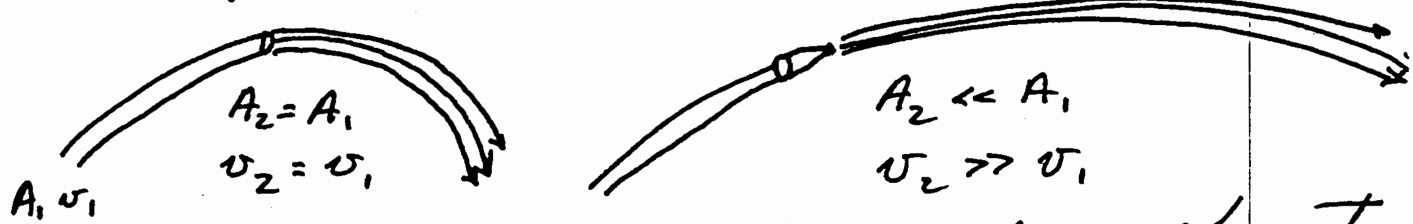
Consider now a tube with a constriction

If we introduce fluid into A_1 with velocity v_1, then the eqn of continuity requires the same volume to emerge from A_2 in the same time

$$v_2 = \frac{A_1}{A_2} v_1$$

If $A_1 > A_2$ then $v_2 > v_1$

e.g. Garden hose with nozzle



Same volume of water reaches the plants, but its moving w/ much higher velocity

Above is an example of STREAMLINE FLOW

We say that a flow is streamline if the fluid motion is uniform and streams of fluid do not cross. If motion is irregular + streamlines are not continuous + nonintersecting

we refer to the case as TURBULENT FLOW. Needless to say, streamline flow is much easier to treat quantitatively than turbulent flow. In particular, in streamline flow equ of continuity holds

Av is the same everywhere in the flow.