ANOTHER FORCE LAW + A NEW KIND OF POTENTIAL ENERGY

Work done by (on) a spring

$$F = -kx$$ (– indicates force in opposite direction

Note here F varies as spring is stretched or compressed. Note force <u>opposes</u> displacement

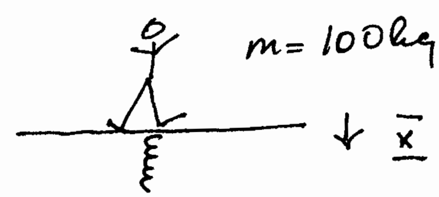
$$W = \int F_{\parallel} \cdot d_s = \int F\,dx$$

$$= \int kx\,dx = k\int x\,dx$$

$$= \frac{1}{2}kx^2$$

This is a conservative force $\quad PE = \frac{1}{2}kx^2$

e.g. Spring Scale



$$\text{Weight} = F_g = mg = kx$$

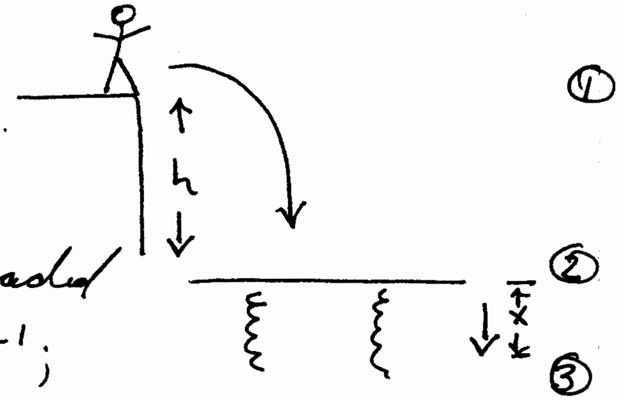
Typically $k = 5\times10^{5}\ N\,m^{-1}$

$$x = \frac{mg}{k} = \frac{9.8\,ms^{-2}\cdot 100\,kg}{5\times10^{5}\,Nm^{-1}}$$

$$x = 1.96\times10^{-3}\,m \approx 2mm$$

$(m = 100\,kg)$

Dockworker jumps from $h = 2m$ onto spring loaded platform $k = 10^5\ N\,m^{-1}$; how far down does he depress the platform



At ① Energy is all grav PE

$$E = PE = mgh = 9.8\ m\,s^{-2} \cdot 100\,kg \cdot 2m$$
$$= 1.96 \times 10^{3}\ J$$

This is the "ENERGY BUDGET"

At ② Energy is all KE

$$E = \frac{1}{2} m v^2 = 1.96 \times 10\ J$$

At ③ Energy is Spring PE + (negative) grav PE

$$E = \frac{1}{2} k x^2 + mg(-x) = 1.96 \times 10^{3}\ J$$

Rewrite

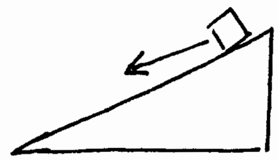
$$\frac{1}{2} \cdot 1 \times 10^5 x^2 - 100\,kg \cdot 9.8\,m\,s^{-2} x - 1.96 \times 10^3\ J = 0$$

$$5 \times 10^4\ N\,m^{-1}\ x^2 - 9.8 \times 10^2\ N \cdot x - 1.96 \times 10^5\ J = 0$$

$$x = \frac{9.8 \times 10^2 \pm \sqrt{(9.8 \times 10^2 N)^2 - 4 \cdot (5 \times 10^4\ N\,m^{-1})(-1.96 \times 10^3)}}{2 \cdot (5 \times 10^4\ N\,m^{-1})}$$

$$x = 0.208\,m = \underline{2\,cm}$$

Quiz Problem



$\Delta PE = mg\Delta h$

$\Delta KE = \frac{1}{2}mv^2$

$$\Delta PE = mg\Delta h = 5kg \cdot 9.8ms^{-2} \cdot (-0.3m)$$
$$= -14.7J$$

$$\Delta KE = \frac{1}{2}m(v^2 - \cancel{v_0^2}^{\,0}) = \frac{1}{2}mv^2$$

$$W_{nc} = f_k \cdot s = \mu_k N s \quad = -9.8N \cdot s$$
$$= \cancel{9.8}\ -4.9J$$

$$-4.9J = \Delta KE - 14.7J$$
$$\Delta KE = \frac{1}{2}mv^2 = 14.7J - 4.9J$$
$$v^2 = \frac{2 \cdot 9.8J}{5kg} = 3.92m^2s^{-2}$$
$$v = 1.98ms^{-1}$$