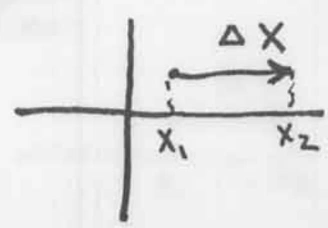
RECTILINEAR MOTION

Displacement, velocity, acceleration

We begin now the study of the motion of bodies called "<u>kinematics</u>." In kinematics we attempt to treat the motion of a body by describing its physical characteristics w/o appealing to a detailed treatment of the forces, etc. causing that movement. We begin with the simplest case – motion in a straight line (1 dimensional) = <u>rectilinear motion</u>

<u>Displacement</u>

When a body moves from one point to another we say it is <u>displaced</u>. The vector which connects the original position with the final position is called the <u>displacement vector</u>



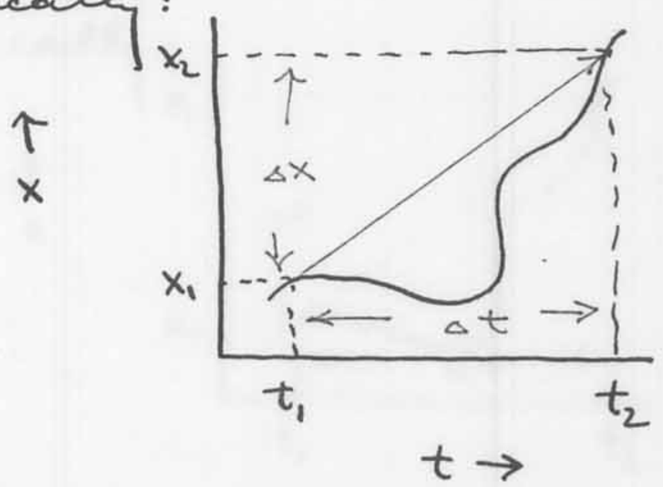
VELOCITY

If the displacement Δx occurs over a time interval $\Delta t = t_2 - t_1$ then we can <u>define</u> a quantity called the average velocity

$$\langle \vec{v} \rangle = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta \vec{x}}{\Delta t}$$

(n.b. Here velocity + displacement are vectors, time is a "scalar"

Graphically:



$\langle v \rangle$ is the slope of the line connecting point (x_1, t_1) with (x_2, t_2)

Note some interesting characteristics of the average velocity:

- Although $\langle v \rangle$ may represent the average velocity you may never actually have travelled at velocity v.

- <u>Velocity</u> is different from <u>speed</u>

Velocity is a vector, speed is a scalar. Suppose you drive from SD to LA at a very careful 69 mph and return at the same speed. Over that time

Speed = constant = 69 mph but your

Average Velocity $= \frac{\Delta x}{\Delta t} = \frac{0}{\sim 3\frac{1}{2}\text{hrs}} = \underline{\underline{0}}$

- Similarly, in this case the displacement (0) is not equal to the distance traveled ($\sim$ 250 miles).

Now, let's define our time coordinate to be zero at start of motion (this is OK, we can define $t=0$ to be *any* time as long as we're consistent

$t = 0 \quad x = x_0$ beginning of motion

Consider the displacement "x" at any time "t"

$$\langle v \rangle = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t - 0}$$

$$\underline{x - x_0 = \langle v \rangle t}$$ ← no surprises here

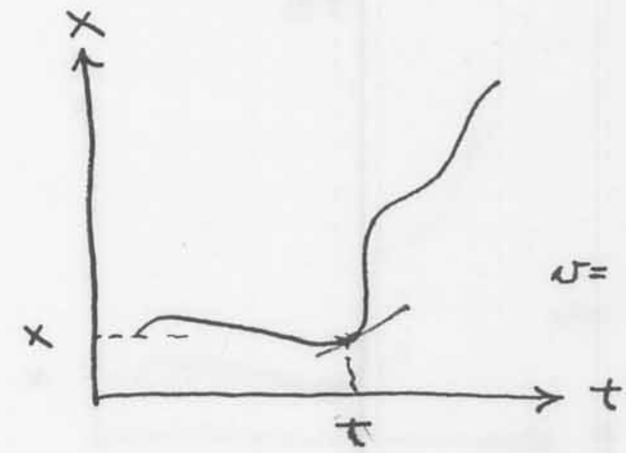
This equation is not very useful because every value of x + t will have a different value of $\langle v \rangle$

But suppose we define our time intervals to be smaller and smaller and smaller ... So that the interval $\Delta t \to 0$. Under these circumstances we can define a more generally useful quantity, the instantaneous velocity

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

Most of you will recognize that this is the <u>derivative</u> of displacement wrt time

$$v = \frac{dx}{dt}$$



v = slope of tangent at point (x, t)

The magnitude of the instantaneous velocity will be the reading on your speedometer

Acceleration

If there is a change in the motion of a body we say that the body is accelerated. Similar to the case for velocity

$$\langle a \rangle = \frac{\Delta v}{\Delta t}$$

defines the average acceleration over time interval Δt. A car which accelerates from 0 – 60 (mph) in 10 s has an average acceleration

$$\langle a \rangle = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{60\,\text{mph} - 0}{10\,\text{s} - 0}$$

$$\langle a \rangle = 6\ \text{mph}/\text{s} \quad \text{(note these are awkward units}$$

or

$$\langle a \rangle = \frac{6\ \text{mph}/\text{s}}{3600\ \text{s}/\text{h}} = \frac{6\ \text{mi}/\cancel{\text{h}}/\text{s}}{3600\ \text{s}/\cancel{\text{h}}}$$

$$= \frac{1}{600}\ \text{mi}\ \text{s}^{-2}$$

Once again we can reduce our time intervals smaller and smaller so that as $\Delta t \to 0$ we define the <u>instantaneous</u> <u>acceleration</u>

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

and, since $v = \frac{dx}{dt}$

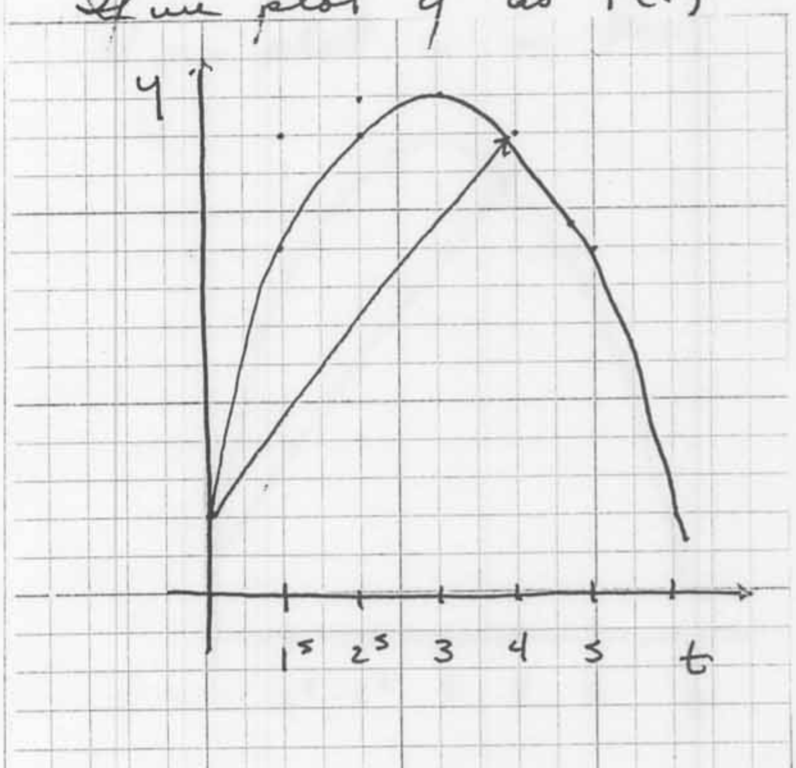
$$a = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} \quad \left(\begin{array}{c}\text{2ND.} \\ \text{Derivative}\end{array}\right)$$

Suppose we take a ball and hurl it upward with a velocity $v_0 = 65\text{mph} \approx 96\ \text{ft/s}$ (good arm!) from the roof of WLH $h = 64\ \text{ft}$

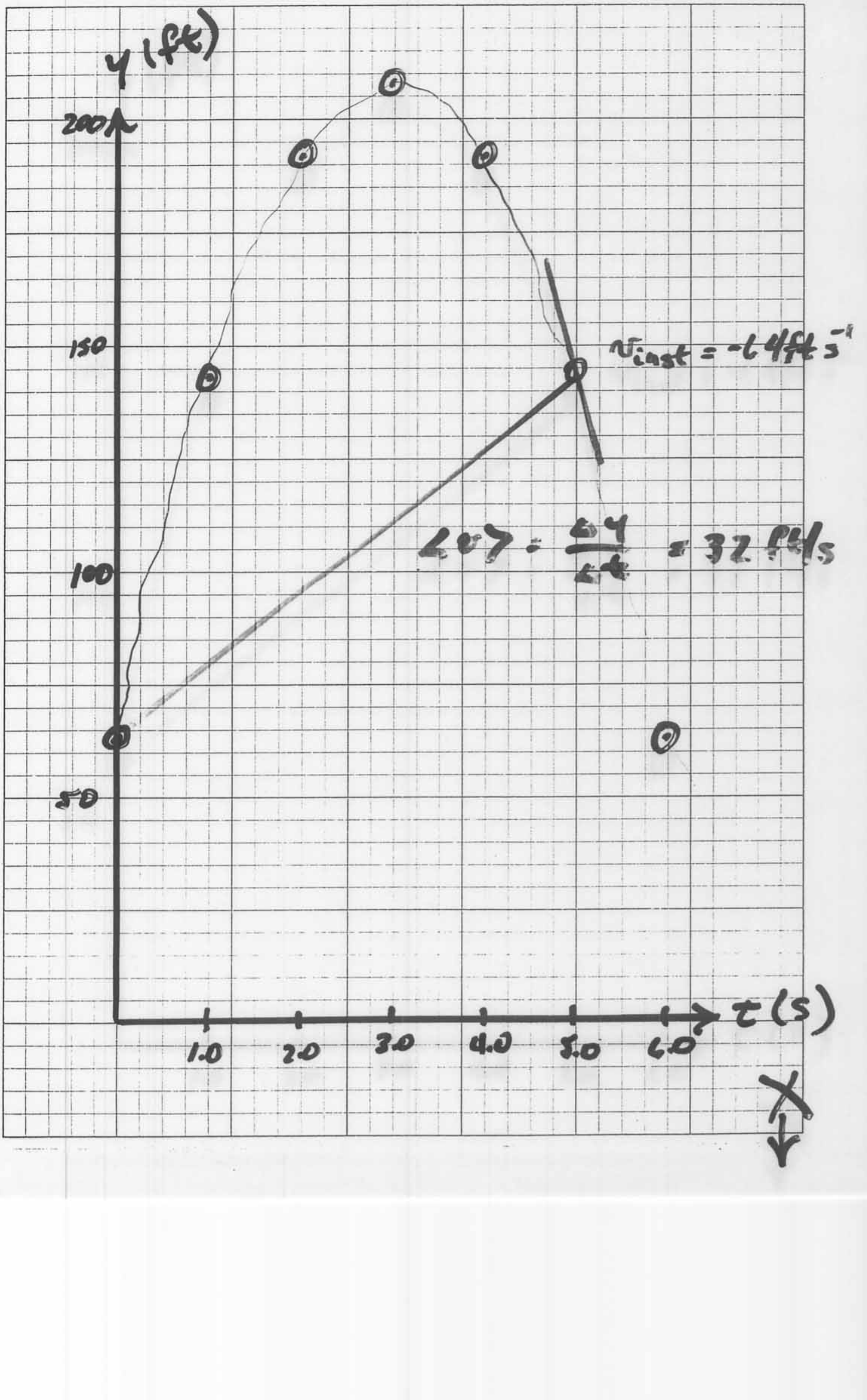
I claim that the ball will have a displacement (use y rather than x 'cause its vertical)

$$y = -16\ \text{ft/s}^2 \cdot t^2 + 96\ \text{ft/s} \cdot t + 64\ \text{ft}$$

If we plot y as $f(t)$



t=0	y = 64 ft
t=1s	144
2s	192
3s	208
4s	192
5s	144
6s	64
7s	~~-48~~ oops


y (ft)
200
150
100
50
t (s)
1.0
2.0
3.0
4.0
5.0
6.0
$v_{inst} = -64 ft s^{-1}$
$\langle v \rangle = \frac{\Delta y}{\Delta t} = 32$ ft/s

Between $t=0 \rightarrow 4s$

$$\langle v \rangle = \frac{\Delta x}{\Delta t} = \frac{192\,ft - 64\,ft}{4s - 0s} = \frac{128\,ft}{4\,s}$$

$$\underline{\langle v \rangle = 32\ ft/s}$$

Now the instantaneous velocity

$$v = \frac{dx}{dt} = \frac{d}{dt}\left(-16\,ft/s^2 \cdot t^2 + 96\,ft/s \cdot t + 64\,ft\right)$$

which is like

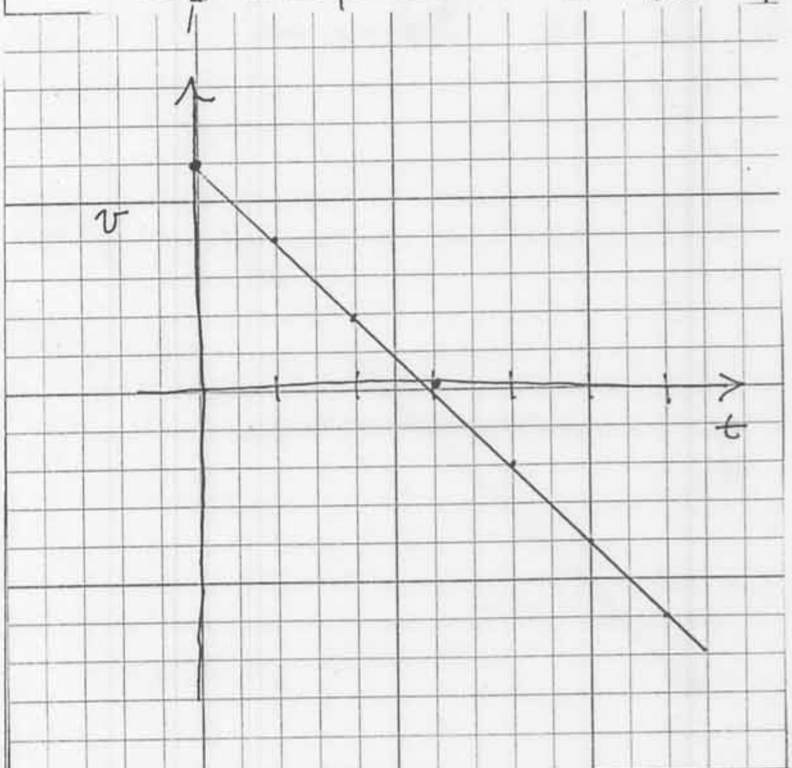
$$\frac{d}{dt}\left(at^2 + bt + c\right) = 2at + b$$

with $a = -16\,ft/s^2$ $b = 96\,ft/s$ $c = 64\,ft$

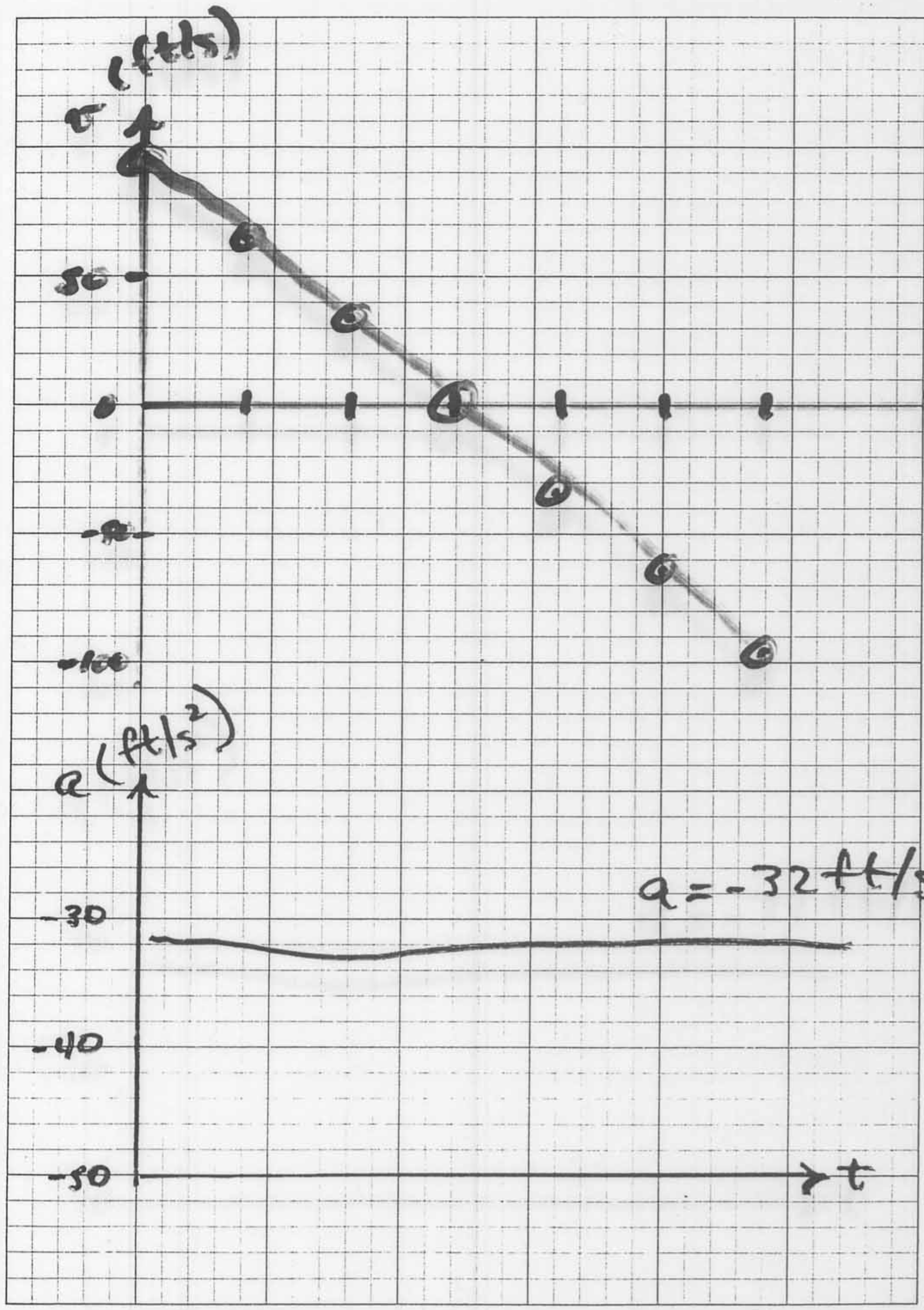
Thus $v = -32\,ft/s^2 \cdot t + 96\,ft/s$ (NOTE THIS IS GENERAL FOR ALL t)

at $t = 4s$ $\underline{v = -32\ ft/s}$

if we plot v as $f(t)$



$t=0$	$v = 96\ ft/s$
1	64
2	32
3	0
4	-32
5	-64
6	-96



v (ft/s)
50
0
-50
-100
t
a (ft/s^2)
a = -32 ft/s
-30
-40
-50
t

In this case between $t = 0 + 4s$

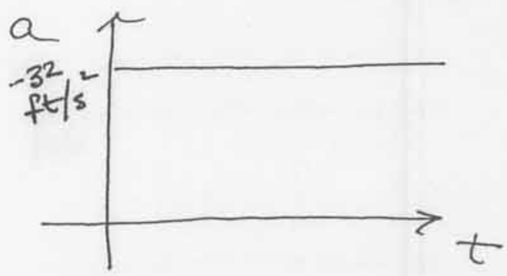
$$\langle a \rangle = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{(-32\ ft/s) - 96\ ft/s}{4s - 0s}$$

$$= -32\ ft/s^2$$

And

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(-32\ ft/s^2 \cdot t + 96\ ft/s\right)$$

$$= -32\ ft/s^2$$



This is the impt + interesting case of uniform acceleration ($a = $ const) We'll see that this corresponds to gravity near the earth's surface.

Let's work this through going the other direction for $a = $ const

$$v = \int dv = \int \left(\frac{dv}{dt}\right) dt = \int a\, dt$$

In general a will be some complicated function which we may or may not be able to treat analytically.

In the special case where $a = \text{const}$ however

$$v = \int a\,dt = at + \text{const}$$

If we know that $v = v_0$ at $t = 0$ then we have

$$\boxed{v = at + v_0} \quad (1)$$

Now $x = \int dx = \int \left(\frac{dx}{dt}\right) dt = \int v\,dt$

Substituting from above

$$x = \int (at + v_0)\,dt$$

$$x = \frac{1}{2}at^2 + v_0 t + \text{const}$$

Again, if $x = x_0$ at $t = 0$ we have

$$\boxed{x = \frac{1}{2}at^2 + v_0 t + x_0} \quad (2)$$

UNIFORM ACCELERATION

"Will He Stop in Time?"

AN EXAMPLE OF APPLYING CONSTANT ACCELERATION

A motorist travelling with velocity $v = 20\ ms^{-1}$ (~ 45 mph) is 50 m from an intersection when the traffic light turns yellow. He has 3^s before the light turns red and can decelerate with $a = -4.2\ ms^{-2}$. Will he stop before the intersection if he hits the brake immediately?

Here's the difficult part - what is this problem asking?

AN APPROACH TO PROBLEM SOLVING

(1) Write down a list of the relevant quantities. Note that we have some latitude in defining our system of coordinates (x, t)

	"initial"	"final"
Choose	$t_0 = 0$	$t = ?$
	$x_0 = 0$	$x = ?$
	$v_0 = 20\ m s^{-1}$	$v = 0$ when he stops

$$a = -4.2\ m s^{-2}$$

(2) Write down the relevant eqns

$$v = v_0 + at \qquad (1)$$

$$x = x_0 + v_0 t + \tfrac{1}{2} a t^2 \qquad (2)$$

Now how do we go about solving?

– nb $t \neq 3^s$; it doesn't matter *when* he stops, what matters is that he stops ($v=0$) before he reaches the intersection (at $x = 50m$)

From inspection of our initial + final conditions and the equations – we need $x + t$ when $v=0$

$$(1) \quad v = v_0 + at$$
$$0 = 20ms^{-1} - 4.2\,ms^{-2} \cdot t$$
$$t = \frac{20ms^{-1}}{4.2ms^{-2}} = 4.76\,s$$

Substitute in

$$(2) \quad x = x_0 + v_0 t + \tfrac{1}{2} a t^2$$
$$= 0 + 20ms^{-1} \cdot 4.76s + \tfrac{1}{2}(-4.2ms^{-2})(4.76$$
$$= 47.6m$$

OK he stops "in time"

What if he had put pedal to metal?

Assume $a = 4\,ms^{-2}$

What is question now? What is position at $t = 3s$

$$x = x_0 + v_0 t + \tfrac{1}{2} a t^2$$
$$= 0 + 20ms^{-1} \cdot 3s + \tfrac{1}{2} \cdot 4ms^{-2} \cdot (3s)^2$$
$$= 60m + 18m = 78m$$

(prob through intersection)

Car starts from rest accelerates at uniform $5\ m/s^{-2}$ (11.2 mph/s) for 5^s then cruises at uniform speed at $25\ m\,s^{-1}$ (56 mph)