

I. Systems of Measures

British system

mile - 1000 standard paces (l-r-l) of a Roman legion (milia)

yard - new to antitethed fingers

foot - size 12 shoe - Did the old English have big feet?

inch - last joint of thumb

Metric system developed in France, imposed on much of world by Napoleon

meter $\approx$ 1 yard subdivided into $\frac{1}{100}$ (centi) etc.

gram = mass of 1 cm^3 of H_2O

second - time difficult orig based on rot'n of earth (day) hard to subdivide naturally.

meter - $\frac{1}{10^7}$ polar circumference of earth (Barcelona - Dunkirk)

Pt-Ir bar in Int Bn. Wt Measures since France 1889

$1,650,763.73 \times \pi$ (K_r^{86} orange red) 196

Now defined in terms of speed of light

gram - $1 \text{ cm}^3 \text{ H}_2\text{O} @ 4^\circ\text{C}$
mass of Pt-Ir bar

sec - Cs^{133} 9,192,631,770 cycles of hyperfine transition

	LENGTH	MASS	TIME	FORCE	ENERGY	CHARGE
SI (mks)	meter	kilogram	second	Newton 1 kg m s^{-2}	Joule	Coulomb
cgs	centimeter	gram	second	dyne 1 g cm s^{-2}	erg	
British	foot	pound (slug)	second	(poundal) pound	BTU	

Conversions:

$$1 \text{ m} = 100 \text{ cm} = 3.281 \text{ ft} = 39.37 \text{ in}$$

$$1 \text{ kg} = 1000 \text{ g} = 2.205 \text{ lb}$$

$$1 \text{ lb} = 454 \text{ g} = 0.454 \text{ kg}$$

II Units / Dimensional Analysis

Keep track of units! dimensions often prove invaluable in keeping track of complex problems (and sometimes making order of magnitude estimates)

How many seconds in a year?

$$\begin{aligned}
 1 \text{ year} &= 1 \text{ year} \times \frac{365 \text{ days}}{\text{year}} \times \frac{24 \text{ hours}}{\text{day}} \times \frac{60 \text{ min}}{\text{hour}} \times \frac{60 \text{ sec}}{\text{min}} \\
 &= 3.15 \times 10^7 \text{ sec} \quad (\approx \pi \times 10^7 \text{ easy to remember})
 \end{aligned}$$

Always carry dimensions through

Ex: Driving from S.D. to Pasadena, distance 100 miles is traversed in 90 min (hr!)

What is average speed? Should I get a ticket

$$\bar{v} = \frac{\Delta d}{\Delta t} = \frac{100 \text{ mi}}{90 \text{ min} \times \frac{\text{hr}}{60 \text{ min}}}$$

$$\bar{v} = \frac{100 (60) \text{ mi}}{90 \frac{\text{min}}{\text{hr}}} = 66.7 \text{ miles/hr}$$

$$\text{nb. } 100 \text{ mi} / (90 \text{ min} \cdot 60 \text{ min/hr}) = .0185 \frac{\text{mi hr}}{\text{min}^2} \quad \text{MAKES NO SENSE}$$

III Order of Magnitude Estimates

A. What is the mass of the Galaxy?

The Milky Way galaxy consists of approx ¹⁰⁰ billion stars

The sun, a typical star, has a mass of
 2×10^{33} grams = 2×10^{30} kg

$$m_{\text{gal}} \approx 1 \times 10^{11} \text{ stars} \times 2 \times 10^{30} \text{ kg/star} \\ \approx 2 \times 10^{41} \text{ kg}$$

Each H atom has a mass $m_H = 1.67 \times 10^{-27}$ kg
so that there are approx (considering that most
of universe is hydrogen)

$$n = \frac{2 \times 10^{41} \text{ kg}}{1.67 \times 10^{-27} \text{ kg/atom}} \approx 10^{68} \text{ atoms in MW}$$

B. What is the angular size of the space shuttle in
the sky

$$l \sim 20 \text{ people lengths} \times 2 \text{ m/person} \approx 40 \text{ m}$$

$$d \sim 100 \text{ miles} \times 1.6 \text{ km/mile} \approx 1.6 \times 10^2 \times 10^3 \\ (\text{low earth orbit})$$

$$\theta \approx \frac{l}{d} = \frac{4 \times 10^1 \text{ m}}{1.6 \times 10^5 \text{ m}} = 2.5 \times 10^{-4} \text{ radians } (2 \times 10^5 \frac{\text{arc}}{\text{rad}}) \\ [\text{recall } 57^\circ \text{ per radian}]$$

$$= 50 \text{ arcsec}$$

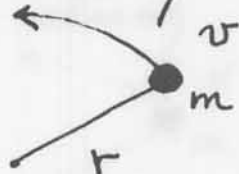
$$\approx 1 \text{ arc min}$$

IV Dimensional Analysis

This technique can prove very powerful in simple physical arguments and has varying degrees of sophistication. Simply consider

Force units $1 \text{ N (newton)} = 1 \text{ kg m s}^{-2}$
 $\vec{F} = m \vec{a}$ we can write MLT^{-2}

There are various kinds of forces consider the force on a mass rotating ~~at~~ at the end of a string. What can the force depend upon



- 1) mass - larger mass has more force M
- 2) velocity - larger $v \Rightarrow$ greater force LT^{-1}
- 3) radius - larger $r \Rightarrow$ smaller force L

we want dimensions MLT^{-2} . What quantities can we construct w/ available quantities

$$F_{\text{cent}} = \frac{m^\alpha v^\beta}{r^\gamma}$$

try $\alpha = 1$ want $\frac{v^\beta}{r^\gamma} \Rightarrow LT^{-2}$; v is the only quantity which has time that yields $\beta = 2$. In order to get L^{+1} need $\gamma = 1$

$$F_{\text{cent}} = \frac{mv^2}{r}$$

We cannot rule out some constant, but in fact (as we shall see) this is the correct law.