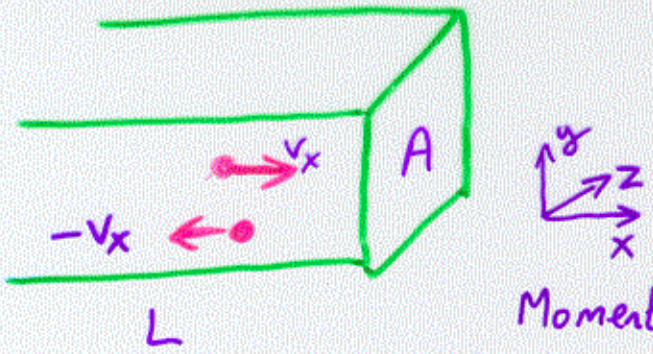
Pressure, Temp and Molecular Speeds

Gas container, side L, area of wall $= A$:



Molecule mass m
speed v
hits wall.

Momentum change $\Delta p = 2mv_x$ per collision

Time Δt between collisions with wall $= \frac{2L}{v_x}$

$\Rightarrow$ force on wall (Newton II) $F = \frac{\Delta p}{\Delta t} = 2mv_x \cdot \frac{v_x}{2L} = \frac{mv_x^2}{L}$

Total pressure on wall $P = \frac{\sum F}{A} = \frac{\sum mv_x^2}{A.L} = \frac{m\sum v_x^2}{V}$

Now in 3-d, $v^2 = v_x^2 + v_y^2 + v_z^2$ so on average $v_x^2 = \frac{1}{3}v^2$

Define RMS speed as $v_{rms}^2 = \frac{\sum v^2}{N}$

$\Rightarrow P = \frac{1}{V}\frac{Nm\,v_{rms}^2}{3} \Rightarrow PV = \frac{1}{3}Nmv_{rms}^2$

c.f. $PV = Nk_BT \Rightarrow v_{rms}^2 = \frac{3k_BT}{m}$

OR average k.e. of molecule $= \frac{3}{2}k_BT$.

$\frac{1}{2}mv_{rms}^2$

Details, Details .....

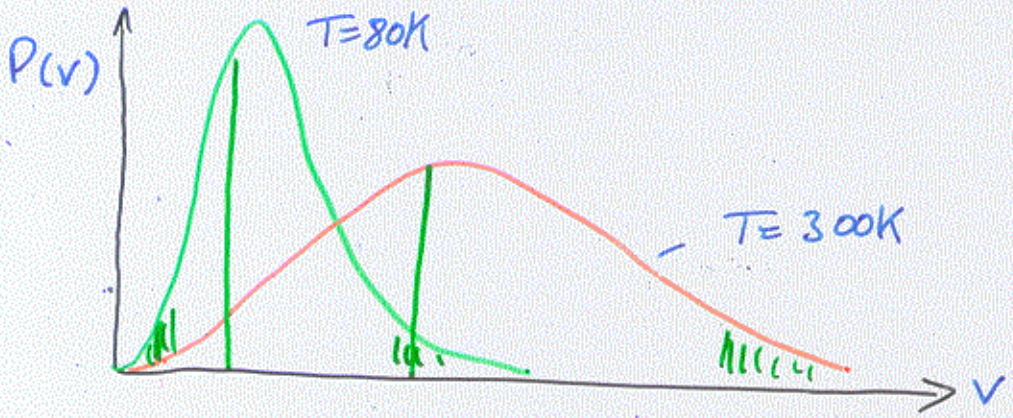
1. Mean free path (sec 20.6)

- In fact molecules collide frequently, exchanging K.E. with each other.

"Mean free path" = av. distance traveled between collisions. $\lambda \propto \frac{1}{\text{density}}$: ~ 0.1μm at sea level

≈ 16cm at 100km

2. Ideal gases have a distribution of molecular speeds: (sec 20.7, fig 20.7)



Maxwell speed distribution changes with T

$v_{rms}^2 = \frac{3k_BT}{m}$ but many molecules have $v >> v_{rms}$

From Kinetic Theory: Average k.e. of molecule $\frac{1}{2} m v_{rms}^2 = \frac{3}{2} k_B T$

e.g. At room temp ($T = 300K$):

k.e. of any molecule $= \frac{3}{2} k_B T = \frac{3}{2} \times 1.38 \times 10^{-23} \times 300$
$= 6.2 \times 10^{-21} J.$

So for helium ^4He, $\frac{1}{2} m v_{rms}^2 = 6.2 \times 10^{-21} J$

$\Rightarrow v_{rms} = \sqrt{\frac{3 k_B T}{m}} = 1366$ m/s, $\gtrsim$ escape speed from earth's gravity!

for O_2 (molecular mass = 32) $v_{rms} \propto \frac{1}{\sqrt{m}} = 483$ m/s

Internal Energy of a Monatomic Gas

E_{int} = total k.e. of molecules (no vibration, rotation)

$E_{int} = \frac{3}{2} N k_B T = \frac{3}{2} n (N_A k_B) T = \frac{3}{2} n R T$

So for 1 mole of any gas at room temp

$E_{int} = 3738.4 J$ (monatomic)

E_{int}, Heat and Work : Example

e.g. 2 moles of gas heated at constant pressure $P = 200\,kPa$ from 300 to 400 K.

$PV = nRT$ $\quad dE_{int} = Q - W$ $\quad E_{int} = \frac{3}{2} nRT = \frac{3}{2} PV$

a) Initial state: $P_i = 200\,kPa$, $T_i = 300\,K$, $n = 2 \Rightarrow V_i = \frac{2 \times 8.31 \times 300}{200 \times 10^3}$

$= 0.025\,m^3$

Final state: $V \propto T \Rightarrow V_f = 0.033\,m^3$

b) Work done $W = \int P.dV = P(V_f - V_i) = 1666.7\,J$

c) $\Delta E_{int} = \frac{3}{2} nR(T_f - T_i) = \frac{3}{2} P(V_f - V_i) = 2500\,J$

d) $\therefore$ Heat input required $Q = dE_{int} + W = 2500 + 1666.7$

$= 4167.7\,J$

So here, 2500 J of heat → internal energy (raises temp of gas)

1667.7 J → does actual work. $= mgh$.