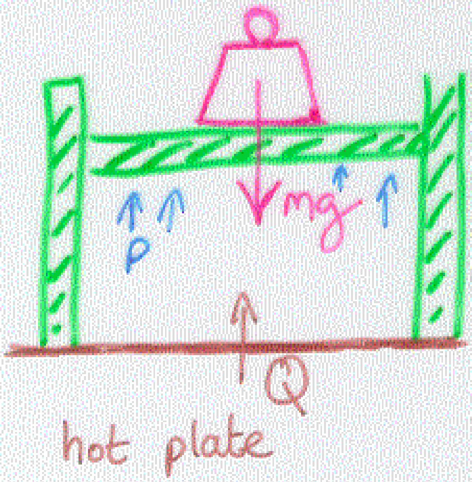
Heat and Work energy (19.8 – 19.11)



For given mass of gas

Initial state: V_i, P_i, T_i

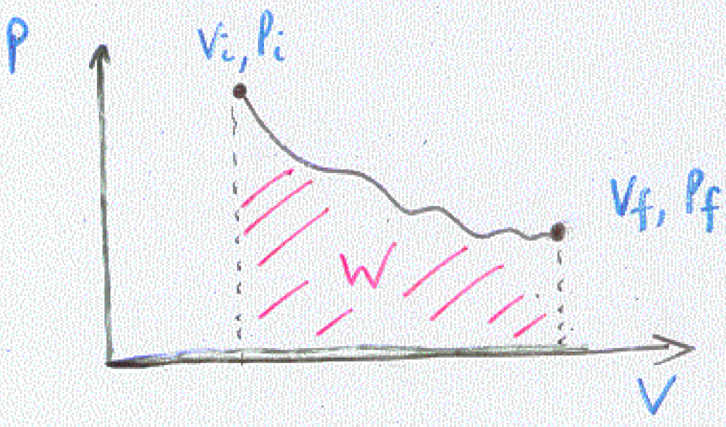
Final state: V_f, P_f, T_f

[If heat transfered to gas
  OR weight changed] $\Rightarrow$ piston can move up/down

If piston moves by dy, work $dW = \text{Force} \times dy$

But force $= P.A$ so $dW = P.A.dy = PdV$

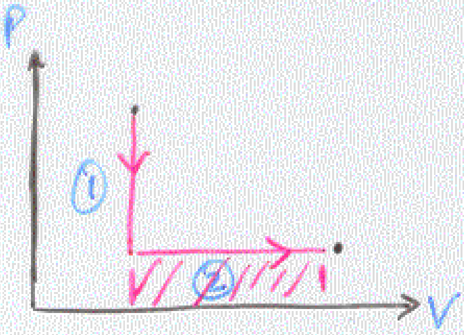
$\Rightarrow$ $\boxed{\text{total work done } \underline{\text{by}} \text{ gas } \; W = \int_{V_i}^{V_f} P\,dV}$



P-V diagrams

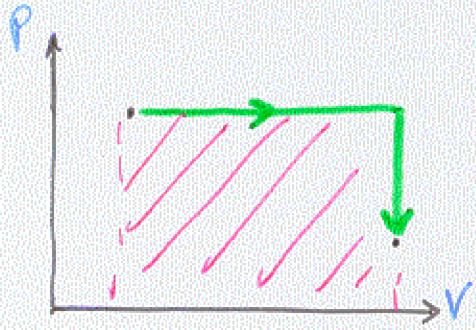
- Many paths from V_i, P_i to V_f, P_f
- depends on heat input Q and work done $W = \int P.dV$

e.g.



① Constant vol. (block piston)
Then
② Constant Pressure $\left(P = P_0 + \frac{mg}{A} \right)$

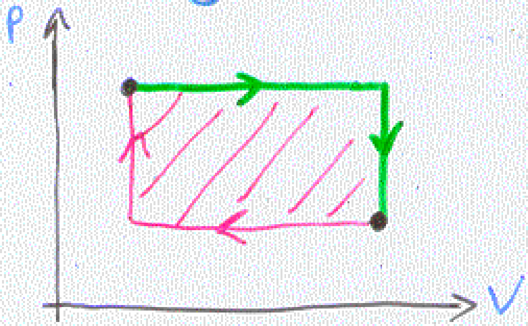
OR



① Constant pressure
Then
② Constant volume

Note: Work $= \int P.dV$ different for each path

So in a cycle:



Net Work $W > 0$

Work done by gas comes from Heat input Q

– an engine!

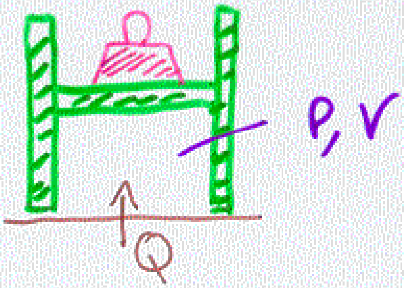
1st Law of Thermodynamics - Energy Conservation

For any path from V_i, P_i to V_f, P_f we find

$$\boxed{\begin{array}{ccccc} Q & - W & = & \Delta E_{int} & \text{is a constant} \\ \text{heat input} & \text{work output} & = & \text{change in internal energy} & \end{array}}$$

ΔE_{int} depends only on {initial, final} states

For small changes : $\boxed{dE_{int} = dQ - dW}$



Special cases:

e.g. remove weight

1. Adiabatic : $Q = 0$ (insulated vessel, or rapid change)
2. Constant vol : $W = 0$ since $\int P.dV = 0$ (lock piston)
3. Constant P : Add heat, lift weight $W = P(V_f - V_i)$

   BUT heat required $Q = \Delta E_{int} + W \geq W$

4. Cycle : $\Delta E_{int} = 0$ so $Q = W$ (heat $\Rightarrow$ work)

Also 5 : Isothermal : constant temp } later!
and 6 : Free expansion : work $W = 0$ }

Kinetic Theory of Gases (ch. 20)

- Relate microscopic (atomic) behavior to macroscopic

Counting atoms/molecules - Avagadro's number

$$N_A \equiv 6.02 \times 10^{23}$$

Why this number: 6.02×10^{23} atoms of ^{12}C has mass 12g
^{1}H 1g

of <u>moles</u> of substance $n = \frac{N}{N_A} = \frac{\text{total mass (g)}}{N_A \times \text{atomic mass}}$

Ideal Gas Law (Boyle's Law)

From expt: $\boxed{PV = nRT}$

↑ ideal gas constant 8.31 J/mol/K

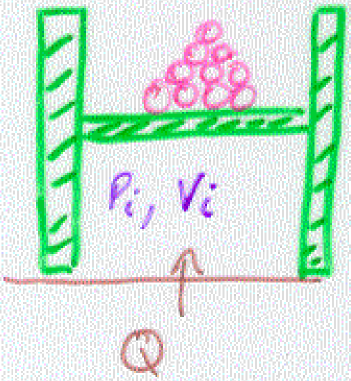
or using $N = (n N_A)$ = number of atoms

$$PV = n (N_A k_B) T = \underline{N k_B T}$$

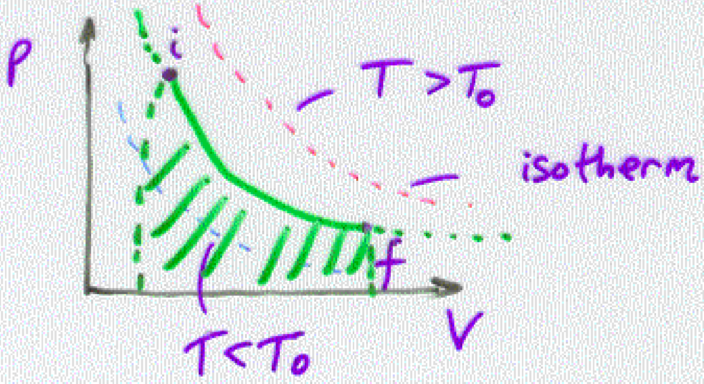
where $R = N_A \ k_B$

└ Boltzmann's constant

Isothermal Expansion and Work



Slowly reduce weight, while adding heat Q to keep constant temp. T_0

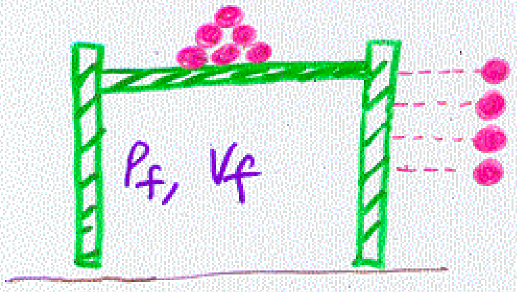


From $PV = nRT$, $\quad P = \dfrac{nRT_0}{V}$

$\therefore$ Work done by expanding gas $W = \int P.dV$

i.e. $$W = nRT_0 \int_{V_i}^{V_f} \frac{dV}{V} = nRT_0 \ln\left(\frac{V_f}{V_i}\right)$$

In this case, work $W = \sum m_i g h_i$ for the elevated weights:



Example: $0.5\,m^3$ of helium at $20°C$, $P = 100\,kPa$

is slowly compressed to $0.2\,m^3$ isothermally

a) Final Pressure P_f : $\quad P_f V_f = P_i V_i = nRT$

$$\Rightarrow \quad \frac{P_f}{P_i} = \frac{V_i}{V_f} \quad \Rightarrow \quad P_f = 100\,kPa \times \left(\frac{0.5}{0.2}\right) = \underline{250\,kPa}$$

b) Work required $= -$ (work done by gas, W)

$$W = \int P.dV = nRT \int \frac{dV}{V} = P_i V_i \ln\left(\frac{V_f}{V_i}\right)$$

$$= 100 \times 10^3 \times 0.5 \times \ln\left(\frac{0.2}{0.5}\right) = -45814\,J$$

$\therefore$ Force required on piston $\times$ distance $= \underline{45814\,J}$

c) Number of moles of gas $n = \dfrac{P_i V_i}{RT} = \dfrac{0.5 \times 100 \times 10^3}{8.31 \times 293\,K} = 20.5$