Electromagnetic (EM) Waves - ch. 34

Solution to Maxwell's equations in free space

$\Rightarrow$ Changing $\begin{matrix}\underset{\sim}{E}\\ \underset{\sim}{B}\end{matrix}$ field will induce a $\begin{matrix}\underset{\sim}{B}\\ \underset{\sim}{E}\end{matrix}$ field

We find solutions obey : $$\frac{\partial^2 E}{\partial t^2} = c^2 \frac{\partial^2 E}{\partial x^2} \qquad (1)$$

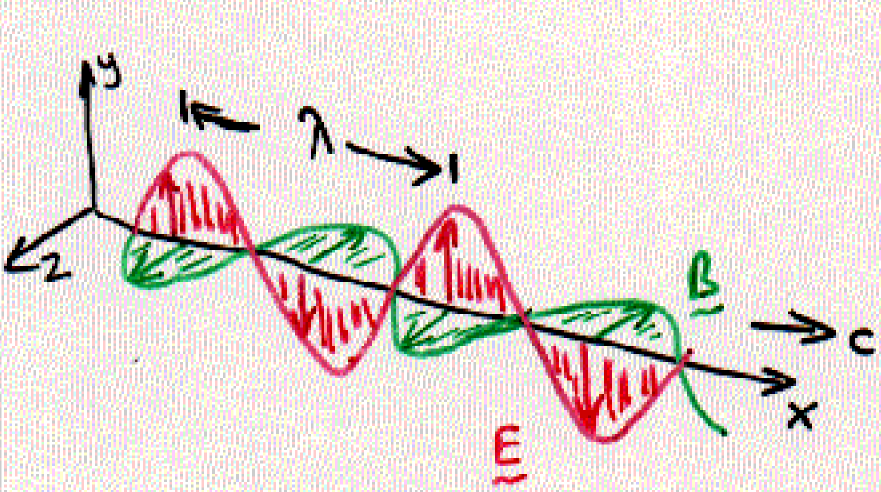
with corresponding $$B = E/c \qquad (2)$$

Where $c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$

Can verify that if $\underset{\sim}{E}$ field is a wave : $\underset{\sim}{E} = \underset{\sim}{E}_0 \sin(kx - \omega t)$,

(1) satisfied when speed of wave $\omega/k = c$: speed of light!

- $\underset{\sim}{E}, \underset{\sim}{B}$ are $\perp$ to each other, and direction of wave travel $= \underset{\sim}{E} \times \underset{\sim}{B}$ (transverse wave)
- $\underset{\sim}{E}, \underset{\sim}{B}$ are in phase with $B = E/c$

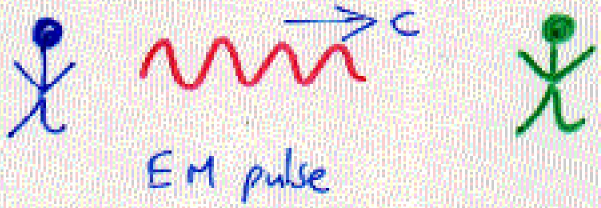


Linked $\underset{\sim}{E}, \underset{\sim}{B}$ fields propagate together

EM Waves Transport energy and momentum in fields

- true even in vacuum

mid-flight:
energy, momentum in particle



mid-flight:
energy, momentum in $\underset{\sim}{E}, \underset{\sim}{B}$

$\underset{\sim}{E}, \underset{\sim}{B}$ fields exert a force on charged particles

$$\underset{\sim}{F} = q\,(\underset{\sim}{E} + \underset{\sim}{v} \times \underset{\sim}{B})$$

$\Rightarrow$ transfer of energy and momentum
(easier to measure than $\underset{\sim}{E}, \underset{\sim}{B}$ directly for light)

As for sound, Intensity $[W/m^2] \propto (\text{amplitude})^2$

With $\underset{\sim}{E} = \underset{\sim}{E}_0 \sin(kx - \omega t)$ and $B = E/c$ we find:

$$I = \left(\frac{1}{\mu_0} E B\right)_{av} = \frac{1}{2} \frac{1}{c\mu_0} E_0^2 \quad [W/m^2]$$

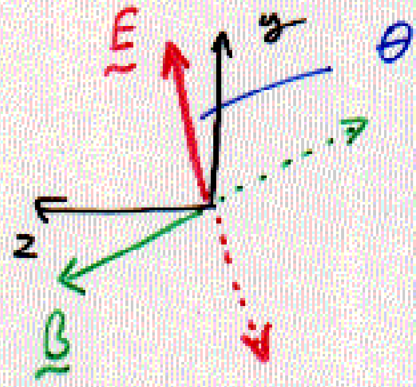
with electric field $\underset{\sim}{E}$ in Volt/meter: $\underset{\sim}{E} = \frac{\partial V}{\partial x}$

Polarization of EM Waves

$\mathbf{E}$, $\mathbf{B}$ fields are $\perp$ to direction of travel

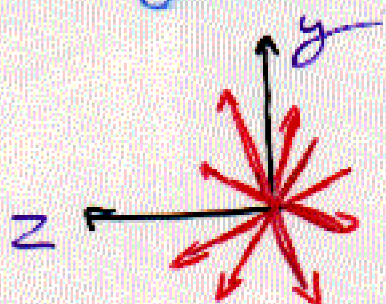
Plane containing $\mathbf{E}$ vector is plane of polarization

Viewed end-on :



Some sources produce ~ 100% polarized EM radiation (e.g. TV and radio transmitters)

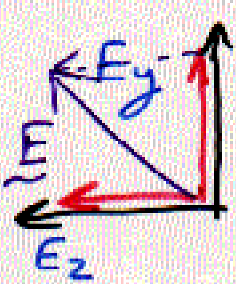
Most sources produce a jumble of EM waves, random phases



un-polarized light

Can represent $\mathbf{E}$ as two $\perp$ polarized waves with equal intensity, random phase :

$$\mathbf{E} = \mathbf{E}_y + \mathbf{E}_z$$

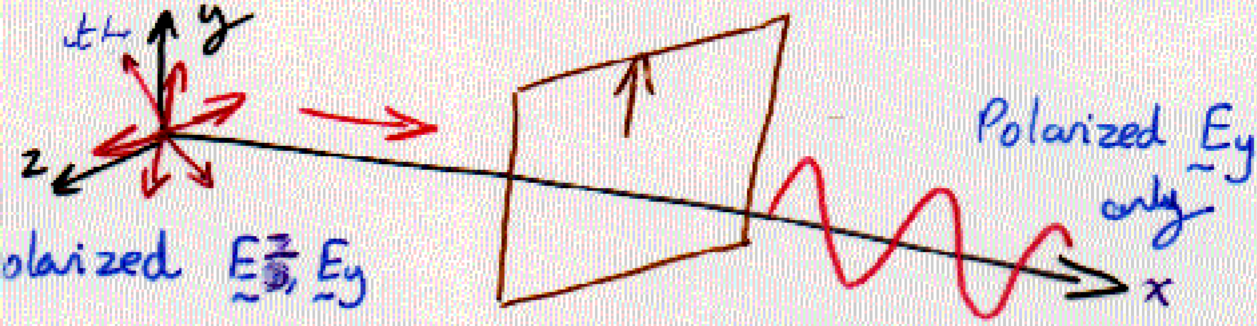


$$I_y = I_z = \tfrac{1}{2} I_0$$

↑ total intensity

Polarizing Filters ...

Some materials (e.g. polaroid) only allow one plane of $\underset{\sim}{E}$, $\underset{\sim}{B}$ to be transmitted (+ absorb the rest)



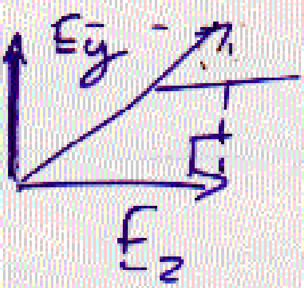
$I_0 = \frac{1}{2}\frac{1}{\mu_0 c} E_0^2$ $\qquad$ $I_1 = \frac{1}{2}\frac{1}{\mu_0 c} E_y^2$

$I_0 = \frac{1}{2}\frac{1}{\mu_0 c}(E_z^2 + E_y^2)$ with, on average $E_{0z} = E_y$

$\Rightarrow$ $\boxed{\text{transmitted intensity } I_1 = \tfrac{1}{2} I_0}$

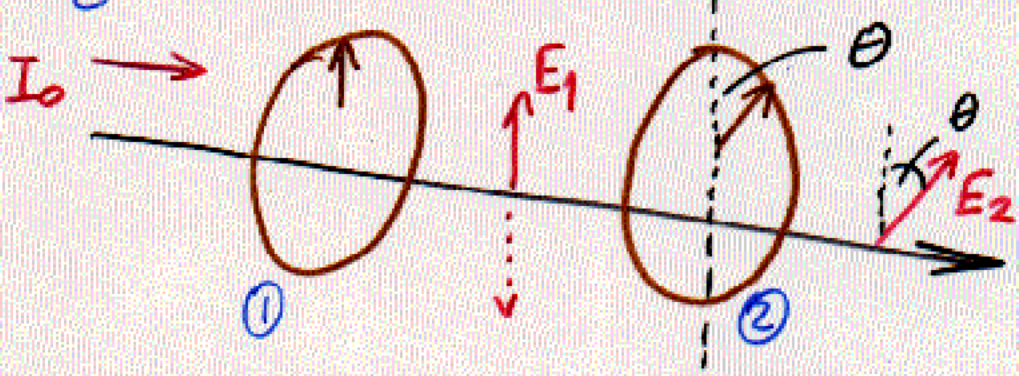
i.e. single polarizer cuts unpolarized intensity by a factor of

(Adding a 2nd polarizer with // transmission axis should make no difference: E_y transmitted 100%.)



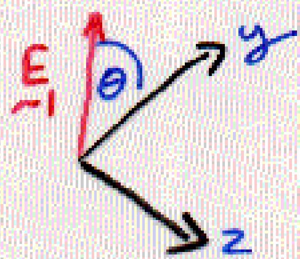
$E_0^2 = E_y^2 + E_z^2$

Combining two polarizers: Malus' Law



Take 2 polarizers, axes mis-aligned by θ

Unpolarized light incident on ① $\Rightarrow$ $I_1 = \frac{1}{2} I_0$, with plane polarized beam. Take transmission axis of ② as the y-axis:



$\underline{E}_1 = \underline{E}_z + \underline{E}_y$

y-component of $\underline{E}_1$ is $E_1 \cos\theta$, with z-component absorbed

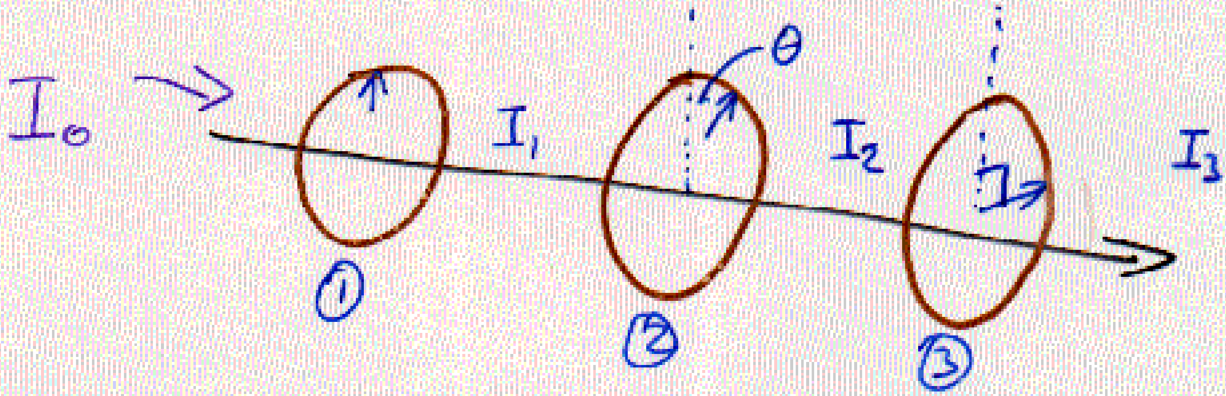
$\Rightarrow$ transmitted $|\underline{E}_2| = |\underline{E}_y| = E_1 \cos\theta$

So since intensity $\propto E^2$ $\boxed{I_2 = I_1 \cos^2\theta}$

— convenient means to vary intensity of a light source (from 0 to $\frac{1}{2} I_0$).

Application: Liquid Crystal Displays (LCDs)

Example: 3 Polarizers : first + last at 90°



We can rotate polarized light by 90° using 3 polaroid sheets

$I_2 = I_1 \cos^2\theta$ as before, transmitted at θ to the vertical.

After polarizer 3 : $I_3 = I_2 \cos^2(90° - \theta) \neq 0$ if $\theta \neq 0$

e.g. for unpolarized incident $I_0 = 800\ W/m^2$ and $\theta = 30°$

After 1 : $I_1 = \frac{1}{2} I_0 = \underline{400\ W/m^2}$

2 : $I_2 = I_1 \cos^2\theta = I_1 \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4} I_1 = \frac{3}{8} I_0 = \underline{300\ W/m^2}$

3 $I_3 = I_2 \cos^2 60° = I_2 \left(\frac{1}{2}\right)^2 = \frac{1}{4} I_2 = \frac{3}{16} I_0 = \underline{75\ W/m^2}$

Can show that I_3 is maximum when $\theta = 45°$

(Then $I_3 = \frac{1}{4} I_0$)