Acoustic ("Sound") Waves - Ch. 18

Solids: can transmit either:

acoustic (longitudinal, compression) waves

OR transverse waves

Fluids: No shear forces $\Rightarrow$ acoustic waves only.

Speed of Sound in Materials:

In general $\boxed{v = \sqrt{\frac{\text{elastic}}{\text{inertial}}} = \sqrt{\frac{B}{\rho}}}$ (sec. 18.3)

where Bulk Modulus [Pascals] $B \equiv \frac{\text{Pressure change}}{\text{frac. vol. change}} = \frac{dP}{\left(\frac{dV}{V}\right)} = -V\frac{dP}{dV}$

For ideal gas, $PV^{\gamma} = \text{constant}$ (fast $\Rightarrow$ adiabatic)

$$\Rightarrow B = -V\frac{dP}{dV} = V \cdot -\frac{\text{const.}\,\gamma}{V^{\gamma+1}} = \gamma P.$$

$\therefore$ Ideal gas has $v = \sqrt{\frac{\gamma P}{\rho}}$, $\propto \sqrt{T}$ since $PV = nRT$

e.g. air at 300K has $v \approx 330$ m/s

c.f. helium ≈ 1000 m/s   rock ~ 6000 m/s

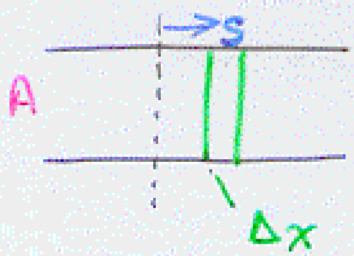
water 1400 m/s   (earthquakes)

Harmonic Sound Wave in a Gas

- a series of (adiabatic) compressions and rarefactions
- each element does work on neighbor $\Rightarrow$ transmits energy as it oscillates

Displacement s of air element along x direction

$s = s_m \cos(kx - \omega t)$



Cross section = A

element thickness = Δx

Vol. of element $V = A\,\Delta x$

When element is displaced, vol. changes by $\Delta V = A(s + \Delta s) - A.s$

i.e. $\Delta V = A\,\Delta s$

$\Rightarrow$ Change in Pressure $\Delta P = -B\dfrac{\Delta V}{V} = -B\dfrac{\Delta s}{\Delta x} \rightarrow -B\dfrac{\partial s}{\partial x}$

$\therefore \boxed{\Delta P = -B\dfrac{\partial s}{\partial x}} = +Bks_m \sin(kx - \omega t)$ from above

e.g. Sample Problem 18.2 :

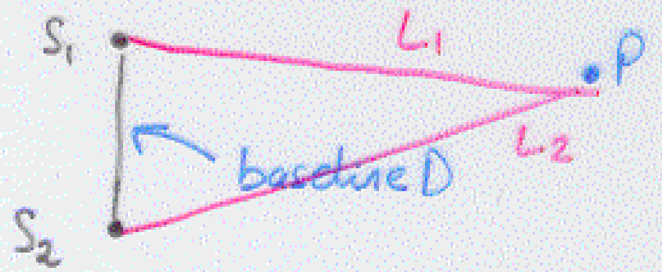
"loud" $\Delta P = 28\,\text{Pa} \Rightarrow s_m \sim 11\,\mu\text{m}$ (use $B = \rho v^2$)

"quiet" $\Delta P \sim 10^{-5}\,\text{Pa} \Rightarrow s_m \sim 10^{-11}\,\text{m}$ < atomic diameter !!!

(c.f. $P \sim 10^5\,\text{Pa}$)

Path Length and Interference of Sound Waves

2 sources S_1, S_2 in phase.



Path lengths L_1, L_2 to point P

Using $y_1 = y_m \sin(kL_1 - \omega t)$ etc.
$y_2 = y_m \sin(kL_2 - \omega t)$

phase diff. between S_1, S_2 measured at P is

$$\Delta\phi = k(L_1 - L_2) = \frac{2\pi \Delta L}{\lambda}$$

Constructive if $\Delta\phi = 0, 2\pi, 4\pi, \ldots\ 2m\pi$ i.e. $\Delta L = m\lambda$

Destructive if $\Delta\phi = \cancel{0}\ \pi, 3\pi, \ldots\ (m+\frac{1}{2})2\pi$. i.e. $\Delta L = (m+\frac{1}{2})\lambda$

– depends on position P, baseline D and wavelength λ.

Interference also caused by reflections

→ "dead spots" in concert halls.

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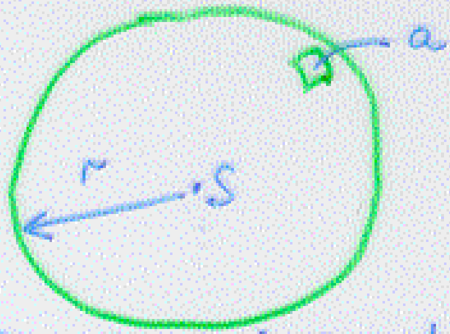
Sound Intensity - Inverse Square Law

For point source producing wave power P [Watt]



At distance r, power is spread over sphere, area $4\pi r^2$

If source is uniform, Intensity $\equiv$ Power / unit area

$$\boxed{I = \frac{P}{A} = \frac{P}{4\pi r^2}}$$: "inverse square law"

$\Rightarrow$ Detector (microphone) of area "a" receives power

$$P_{mic} = \frac{P_{source}}{4\pi r^2} \cdot a$$: larger "ears" $\Rightarrow$ more power received.

Sound intensity given by

$$\boxed{I = \tfrac{1}{2} \rho v \omega^2 s_m^2}$$ (p. 433)

frequency (ω)   amplitude (s_m)

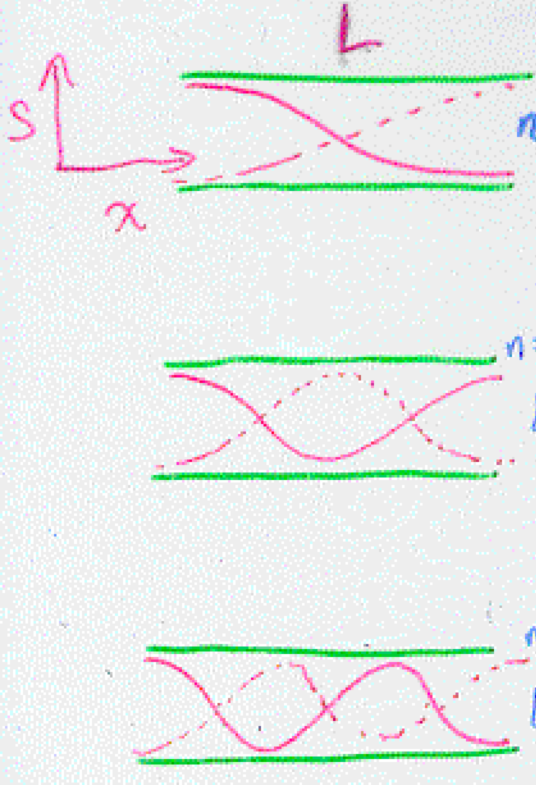
$\Rightarrow$ for "quiet" sound, $I \sim 10^{-12}\, W/m^2$

Music : Resonance in Pipes

Closed end : air cannot move : <u>node</u>

Open end : no resistance to air column : antinode.

e.g. **Open pipe** | **Closed-end pipe**



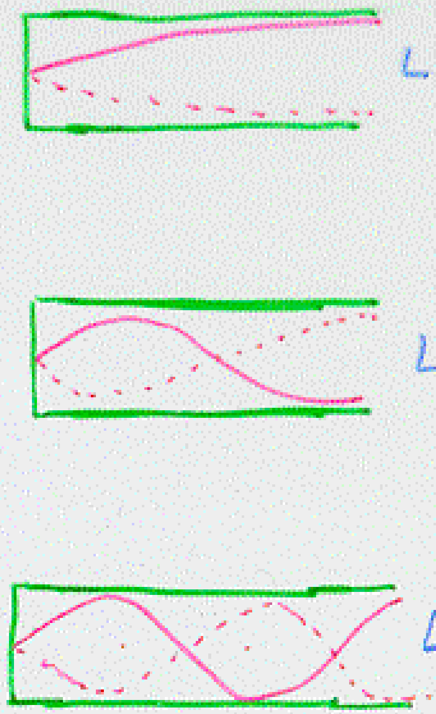
$n=1$ $L = \lambda/2$

$n=2$ $L = \frac{2\lambda}{2}$

$n=3$ $L = \frac{3\lambda}{2}$

$$\lambda = \frac{2L}{n} \quad \text{(c.f. string)}$$

$n = 1, 2, 3 \ldots.$



$L = \lambda/4$

$L = \frac{3\lambda}{4}$

$L = \frac{5\lambda}{4}$

$$\lambda = \frac{4L}{n}$$

$n = 1, 3, 5, \ldots..$

and $f = v/\lambda \propto \frac{1}{L}$ for both cases

$\Rightarrow$ sets size of musical instrument.

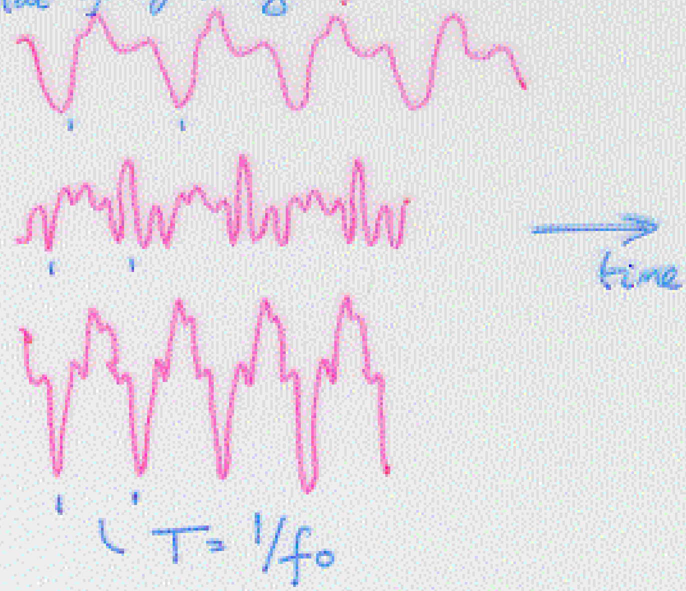
Characterising Musical Notes

Pitch = fundamental frequency f_0

flute :

oboe :

sax :



Note: 1 octave = factor of 2 change in f_0.

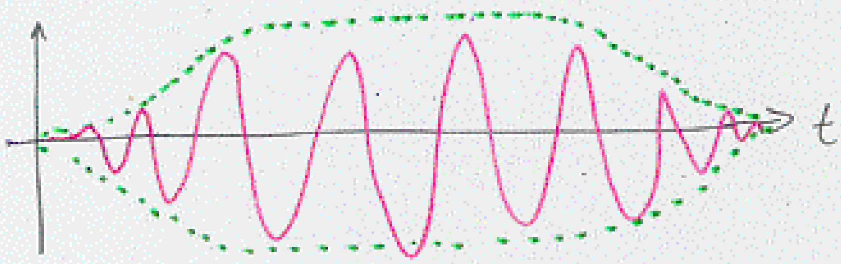
"Timbre" : waveform of instrument (depends on Fourier harmonics present)

e.g. flute has few, low power harmonics ~ sine wave

sax has 'sharp-edged' wave-form ⇒ high frequencies present.

"Loudness": Intensity $\propto (\text{amplitude})^2$

"Envelope" : Defines, start, duration, end of note.



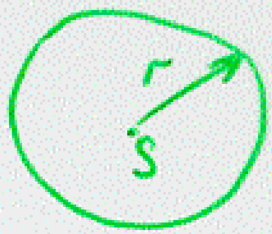
Example: A speaker produces 10W of sound power, emitting isotropically. If the ear can detect $\Delta P = 10^{-2}\,Pa$, how far can the sound be heard?

Use: Intensity $I = \frac{1}{2}\rho v \omega^2 s_m^2$

and $\Delta P_m = -B\left(\frac{\partial s}{\partial x}\right)_m = v\rho\omega s_m$

$$\Rightarrow I = \frac{1}{2}\frac{\rho v \cancel{\omega^2} \Delta P_m^2}{\rho^2 v^2 \cancel{\omega^2}} = \frac{1}{2}\frac{\Delta P_m^2}{\rho v}$$: independent of frequency!

For isotropic source



$$I = \frac{Power}{4\pi r^2}$$

So $I = \frac{Pow}{4\pi r^2} = \frac{1}{2}\frac{\Delta P_m^2}{\rho v}$

$\Rightarrow$ range of audible sound $r^2 = \frac{Pow}{2\pi}\,\frac{\rho v}{\Delta P_m^2}$

or $r = \sqrt{\frac{Pow \times \rho v}{2\pi}} \cdot \frac{1}{\Delta P_m}$

$\rho = 1.2\ kg/m^3$, $v = 330\ m/s$, $Pow = 10W$, $\Delta P_m = 10^{-2}\,Pa$

$$\Rightarrow r = 251\ m$$