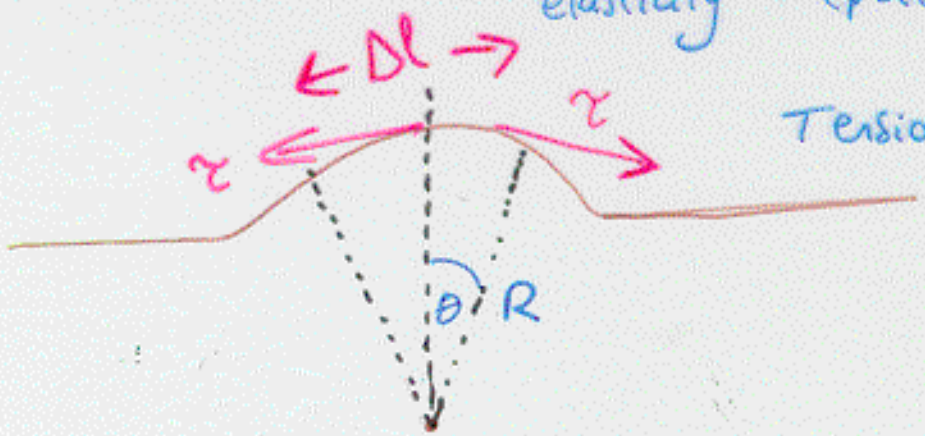
(Transverse) Wave Speed on a String

Wave medium must have inertia (kinetic energy)
elasticity (potential ")



Force on segment Δl of string $F = 2\tau \sin\theta$

where $\sin\theta \approx \frac{1}{2}\Delta l/R \Rightarrow F \approx \tau \Delta l / R$

Centripetal force $= \frac{M v^2}{R}$ where $M = \mu \Delta l$

μ: linear mass density [$kg\,m^{-1}$]

Equate $\Rightarrow \frac{\mu \Delta l v^2}{R} = \frac{\tau \Delta l}{R}$

$\Rightarrow v^2 = \frac{\tau}{\mu} = \frac{\text{tension}}{\text{linear density}}$

- does not depend on amplitude or frequency

Energy and Power of Wave on String

$y = y_m \sin(kx - \omega t)$

Consider string as many coupled oscillators

For small string element dx, mass $= \mu dx$

$$\text{k.e. } K = \tfrac{1}{2} m \left(\frac{dy}{dt}\right)^2 = \tfrac{1}{2} \mu dx \cdot \omega^2 y_m^2 \cos^2(kx - \omega t)$$

Rate of change of k.e = rate of energy transport

$$\text{i.e. } \frac{dK}{dt} = \tfrac{1}{2} \mu \underbrace{\frac{dx}{dt}}_{=v} \cdot \omega^2 y_m^2 \cos^2(kx - \omega t)$$

- Average $\cos^2(kx - \omega t)$ over time $\Rightarrow \frac{1}{2}$.
- On average, potential = kinetic energy

$\Rightarrow$ Energy transport rate $\overline{P} = \tfrac{1}{2} \mu v \omega^2 y_m^2$
≡ average wave power

Note: power $\propto$ (freq.)^2, also $\propto$ (amplitude)^2

(see Sample Problem 17.4)

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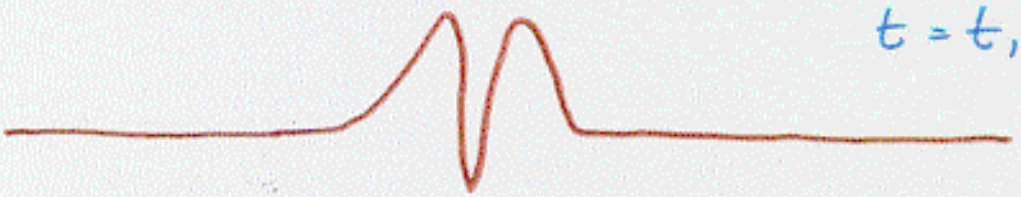
Superposition of Waves

If $y_1(x,t)$ and $y_2(x,t)$ are waves, the resulting displacement where they overlap is:

$$y = y_1 + y_2$$: just add 'em up!

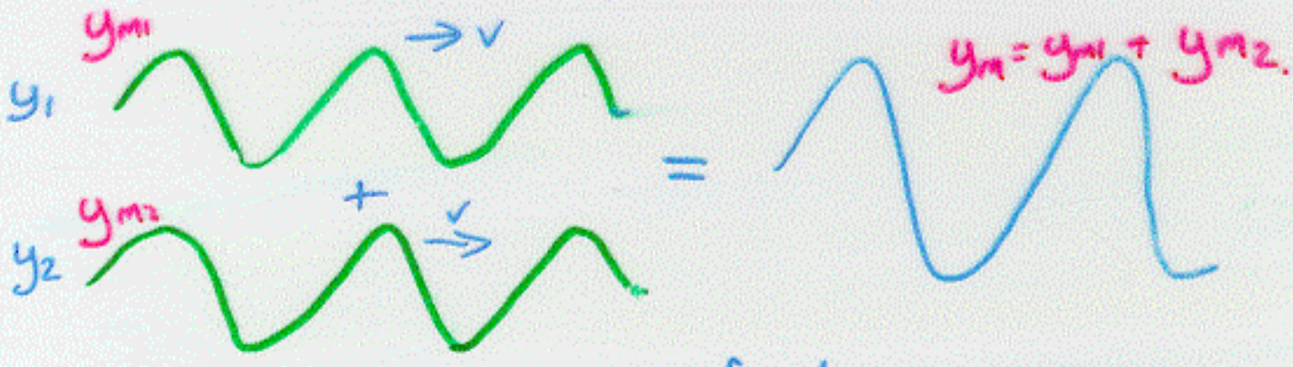
⇒ wave pulses pass "through" each other

e.g.



Interference

When two waves combine in phase ($\phi = 0$)



constructive interference

c.f. combining out of phase ($\phi = \pi = 180°$)

destructive interference

Purely a wave phenomenon:

Since wave power $\propto$ (amplitude)^2, if $y_{m1} = y_{m2} = y_m$:

Constructive: amplitude $= 2y_m \Rightarrow$ power $\propto 4y_m^2$

Destructive: " $= y_{m1} - y_{m2} = 0 \Rightarrow$ power $= 0$