Waves (chap 17)

First a review of Simple Harmonic Motion (ch. 16)

Restoring force $\propto$ displacement, y

$$m\frac{d^2y}{dt^2} = -ky \quad \text{(Newton II)}$$

$\Rightarrow$ Solution $y = y_m \sin(\underbrace{\omega t + \varphi}_{\text{radians!}})$ with $\omega^2 = k/m$

$$= \underbrace{y_m \cos\varphi}\,\underline{\sin\omega t} + \underbrace{y_m \sin\varphi}\,\underline{\cos\omega t}$$

y_m: Amplitude (oscillates between $\pm y_m$)

φ: initial phase (radian). 2π rad $= 360°$

ω: Angular frequency (rad/s)

SHM repeats when $\omega T = 2\pi$ $\Rightarrow$ period $T = \frac{2\pi}{\omega}$

# of cycles/s $f = \frac{1}{T} = \frac{\omega}{2\pi}$ [s^{-1} or Hz]

Energy $E = k.e. + p.e. = \frac{1}{2}m\left(\frac{dy}{dt}\right)^2 + \frac{1}{2}ky^2$

When $y = 0$, all energy kinetic:

$$\Rightarrow E = \frac{1}{2}m\omega^2 y_m^2$$

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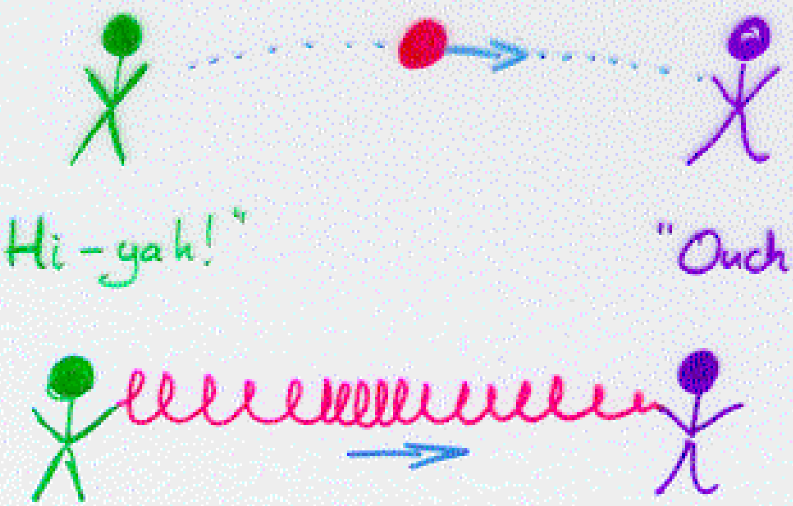
Wave Definition: a disturbance which propagates i.e. transfers energy (and info) without transfering matter

e.g.



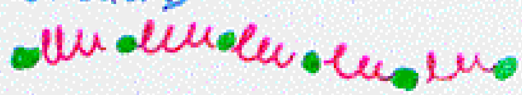
particle

wave

Waves travel quickly (e.g. 330 m/s in air)

" transfer energy efficiently (possibly ≈ 100%)

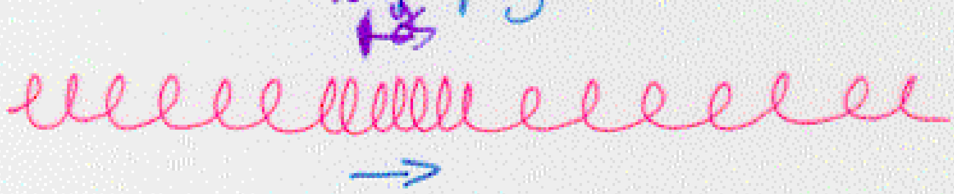
- **Mechanical**: coupling forces between atoms



- **Electromagnetic:** disturbance in $\underset{\sim}{E}$, $\underset{\sim}{B}$, works in vacuo
- **Quantum-Mechanical:** All matter has wave-like properties (Physics 2D)

Wave Characteristics

- **Longitudinal** – displacement ∥ direction of propagation



a.k.a. compression waves e.g. sound in liquids and gases

– can still measure displacement y from eqm. position x_0: $y = (x - x_0)$

- **Transverse** – displacement ⊥ direction of travel

e.g. waves in rope or string, waves on ocean surface

Waveform described by $y = y(x)$

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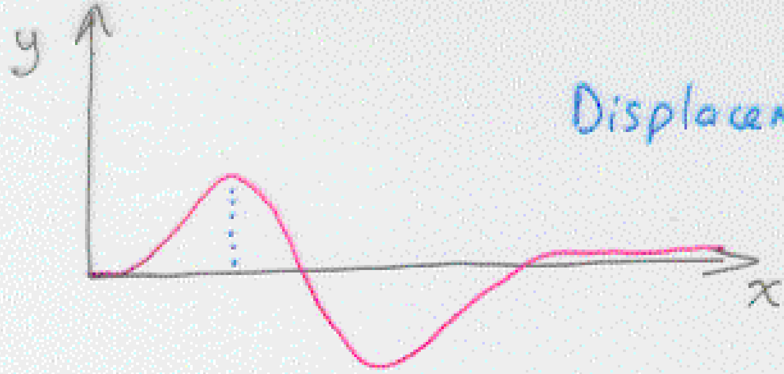
Waveforms

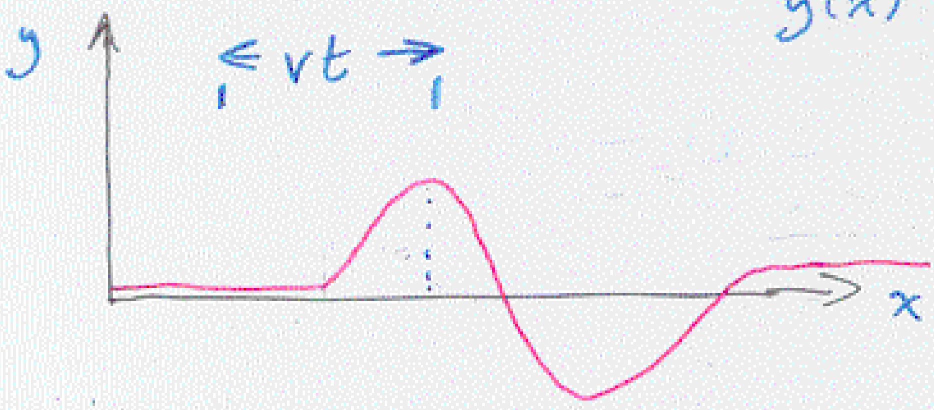
Take "snapshot" of wave pulse at $t=0$:



Displacement $y(x,0)$

Wave moves right at speed v, so at time t

$$y(x) \rightarrow y(x-vt)$$



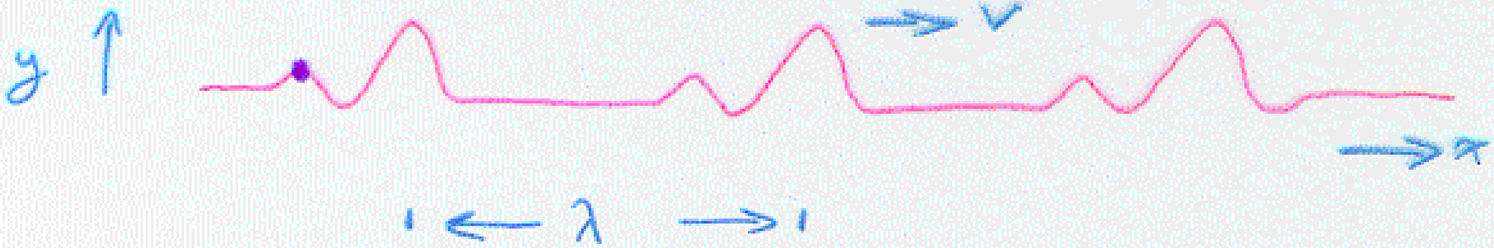
$\therefore$ Wave traveling right has form $y(x,t) = h(x-vt)$

Similarly $y(x,t) = h(x+vt)$ travels left

[Note: $h(x)$ must be single-valued in x]

Periodic Waves : Waveform repeats in space

Again "freeze" wave at $t = 0$:



Pattern repeats each Wavelength λ

Each point on wave vibrates at frequency f [Hz]

In 1s : wave moves distance v

$\Rightarrow \frac{v}{\lambda}$ wavelengths pass through point

$\therefore$ each point oscillates $f = \frac{v}{\lambda}$ times in 1s

$$\Rightarrow \quad \boxed{v = f\lambda}$$

Note: Vibration period $T = \frac{1}{f}$ (s)

e.g. tuning fork vibrates at $f = 440$ Hz

sound speed $v = 330$ m/s

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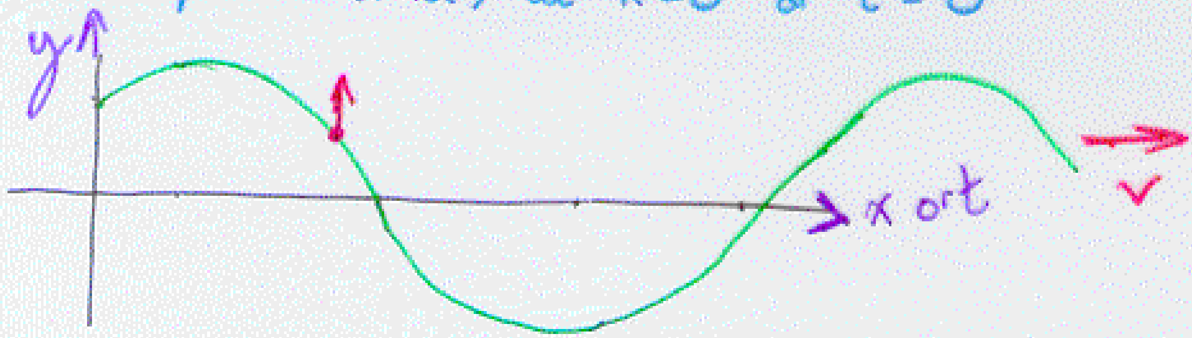
General form of Harmonic wave function:

$$y = y_m \sin(kx - \omega t + \phi) \quad \text{(moves} \Rightarrow \text{right)}$$

OR $$y = y_m \sin 2\pi\left(\frac{x}{\lambda} - \frac{t}{T} + \frac{\phi}{2\pi}\right) : \text{ need 4 numbers to specify}$$

ϕ = initial phase (rad) at $x = 0$ or $t = 0$



Note: If $\phi = 90°$ or $\pi/2$, $\Rightarrow$ $y = y_m \cos(kx - \omega t)$

Speed of wave $v = f\lambda = \dfrac{\lambda}{T} = \dfrac{\omega}{k}$ (direction $\pm$?)

Speed of point <u>on</u> wave $\dfrac{\partial y}{\partial t} = -\omega y_m \cos(kx - \omega t + \phi)$

Acceleration of point (= force / mass) $\dfrac{\partial^2 y}{\partial t^2} = -\omega^2 y_m \sin(\quad) = -\omega^2 y$

— i.e. SHM

Example: Displacement on string $y = 0.1 \sin \pi(4x - 6t + 1/4)$

Give: wavelength, amplitude, wave speed,

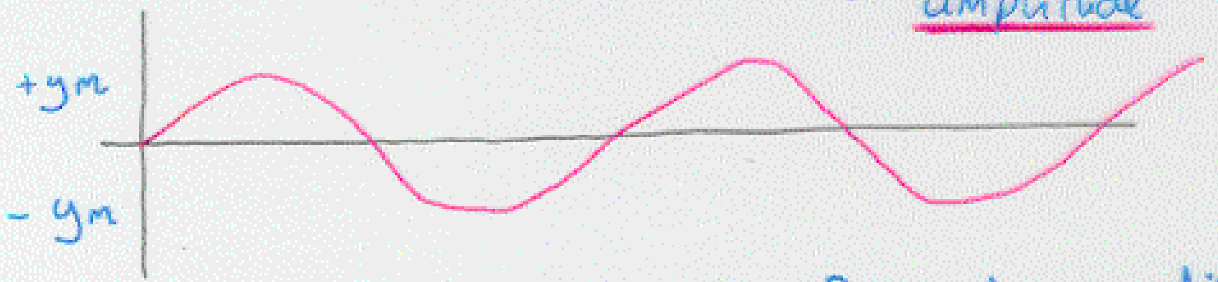
motion of bead at $x = 3.5$ m

Harmonic Waves – each point executes SHM

(chapter 16)

h

$\Rightarrow$ Profile $y(x,t) = \underbrace{y_m}_{\text{"amplitude"}} \sin(kx - \omega t)$



Freeze at $t=0$: 1 cycle $= 2\pi$ rad. over distance $x = \lambda$

i.e. $k\lambda = 2\pi$ or $\boxed{k = \frac{2\pi}{\lambda}}$: wave number [m^{-1}]

Now watch point on wave at $x = 0$:

executes SHM: $y(0,t) = y_m \sin(-\omega t)$

One cycle $= 2\pi$ rad in time period $t = T$

i.e. $\omega T = 2\pi$ or $\boxed{\omega = \frac{2\pi}{T} = 2\pi f}$

ω : angular frequency [rad s^{-1}]

Note: $\boxed{\text{wave speed } v = f\lambda = \frac{\omega\, 2\pi}{k\, 2\pi}}$ [$m s^{-1}$]