Ch. 17

Answers to Questions

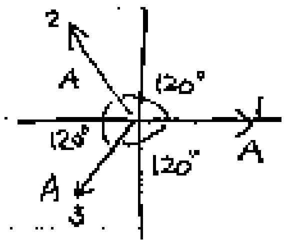
①

3. $v = \sqrt{\frac{\tau}{\mu}}$

$A = B > C > D$

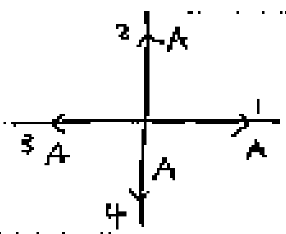
4. (a) 4 (b) 4 (c) 3

8. (a) 120°, 240°

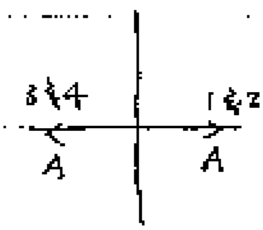


(b) 90°, 180°, 270°

or 0°, 180°, 180°



or



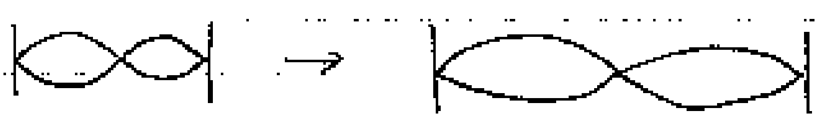
13. $v = \lambda f$

$\Rightarrow \sqrt{\frac{\tau}{\mu}} = \lambda f$

$\therefore \mu = \frac{\tau}{\lambda^2 f^2}$

A particular frequency f happens to produce the standing wave.

Since string A has shorter wavelength, μ_A is greater than μ_B. (Of course, tension τ for both strings is the same.)

14. (a)  2nd harmonic

λ increases

$v = \sqrt{\frac{\tau}{\mu}}$ hasn't changed. $\Rightarrow$ $\underset{\text{fixed}}{v} = \overset{\text{increased}}{\lambda} f$ $\Rightarrow$ f decreases.

(b) $\underset{\text{increased}}{v} = \lambda \underset{\text{fixed}}{f}$ $\therefore$ λ same, f increases.

(c) λ same, f decreases

Exercises & Problems

4E. (a) $T = 4 \cdot (0.17 s)$
$= 0.68\ s$

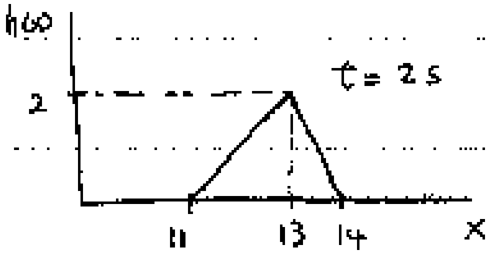
(b) $f = 1/T = 1/0.68 s = 1.47\ Hz$

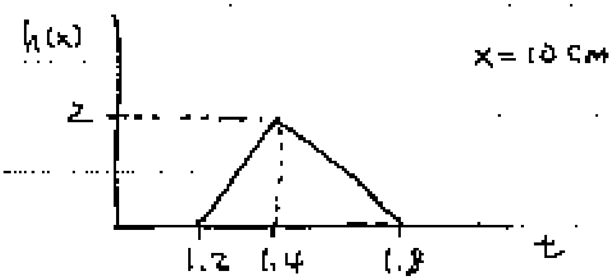
(c) $v = \lambda f = (1.4 m)\cdot(1.47 Hz) = 2.06\ m/s$

8E. (a) A fixed point on the wave satisfies
$x - 5t =$ constant.
$\Rightarrow \frac{dx}{dt} = 5$ (cm/s)

(b) positive x direction

(c)





13P. (a) expression for wave : $y(x,t) = y_m \sin(kx - \omega t)$ for travelling in +x direction

$k = \frac{2\pi}{\lambda} = 62.8\ m^{-1}$

$\omega = 2\pi f = 2\pi \cdot (400 Hz) = 2510\ rad/s$

$y_m = 0.02\ m$

$\Rightarrow y(x,t) = 0.02 \sin(62.8x - 2510t)$ [x, y in meters; t in seconds]

(b) speed of a point on the cord is
$\frac{\partial y}{\partial t} = -\omega y_m \cos(kx - \omega t)$

$\Rightarrow$ max. speed $= \omega y_m = (2510\ rad/s)\cdot(0.02 m)$
$= 50\ m/s$

(c) wave speed $= v = \lambda f = (0.1 m)\cdot(400\ s^{-1})$
$= 40\ m/s$

$(Hz = s^{-1})$

23E. (a) $v = \lambda f = \left(\frac{2\pi}{k}\right)\cdot\left(\frac{\omega}{2\pi}\right) = \frac{\omega}{k}$

$\therefore\ v = \frac{30\ s^{-1}}{2\ m^{-1}} = 15\ m/s$

(b) $v = \sqrt{\frac{\tau}{\mu}}$

$\Rightarrow\ \tau = \mu v^2 = (1.6\times10^{-4}\ kg/m)\cdot(15\ m/s)^2$

$= 0.036\ \underbrace{kg\,m/s^2}_{=N} = 0.036\ N$

27P. (a) Reading the amplitude from the graph, it's about 5 cm.

(b) From the graph, about 40 cm.

(c) $v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{3.6N}{25\times10^{-3}\ kg/m}} = 12\ m/s$

(d) $T = \frac{1}{f} = \frac{\lambda}{v} = \frac{0.4\ m}{12\ m/s} = 0.033\ s$

(e) $y(x,t) = y_m \sin(kx+\omega t)$

$\left(\begin{array}{c}\text{speed of a particle}\\ \text{in the string}\end{array}\right) = \frac{\partial y}{\partial t} = +\omega y_m \cos(kx+\omega t)$

$\therefore$ max. speed $= \omega y_m = (2\pi f)\cdot y_m$

$= \left(\frac{2\pi}{0.033s}\right)\cdot(0.05m) = 9.4\ m/s$

(f) $y(x,t) = y_m \sin(kx+\omega t)$

← travelling in $-x$ direction.

$y_m = 0.05\ m$

$\omega = 2\pi f = 2\pi\cdot(30\ Hz) = 190\ rad/s$

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.4m} = 16\ m^{-1}$

But when $x=0, t=0$, $y(0,0) \cong 3.6\times10^{-2}$ according to the graph.

$\Rightarrow$ Wave has a phase.

$\Rightarrow\ y(x,t) = 0.05 \sin(16x + 190t + \phi)$

$$y(0,0) = 0.05 \sin\phi \cong 3.6\times10^{-2}$$

$$\therefore \ \phi = 0.8 \text{ rad}$$

Therefore,

$$y(x,t) = 0.05 \sin(16x + 190t + 0.8)$$

35P. (a) max. value of transverse speed.

$$\left(\frac{\partial y}{\partial t}\right)_{max} = \omega y_m = (2\pi f)\cdot y_m$$

$$= 2\pi\cdot(120\ s^{-1})\cdot(0.005\ m)$$

half of 1cm

$$= 3.77 \text{ m/s}$$

(b)

$$y(x,t) = y_m \sin(kx-\omega t)$$

T is along the string and its transverse component is $T\sin\theta$.

Now, θ is given by

$$\text{slope} = \tan\theta = \frac{\partial y}{\partial x} = k y_m \cos(kx-\omega t)$$

At maximum angle θ_m,

$$\tan\theta_m = k y_m = \frac{2\pi}{\lambda}\cdot y_m = \frac{2\pi f}{v}\cdot y_m \qquad (\because \lambda = \frac{v}{f})$$

$$= 2\pi f y_m \sqrt{\frac{\mu}{T}} \qquad (\because v = \sqrt{\frac{T}{\mu}})$$

$$= 2\pi\ (120\ s^{-1})\cdot(0.005\ m)\cdot\sqrt{\frac{0.12\ kg/m}{90 N}}$$

$$= 0.138$$

$$\therefore \ \theta_m = \tan^{-1}(0.138) = 7.86^\circ$$

Therefore, the maximum value of the transverse component of the tension $T\sin\theta_m$ is

$$(90N)\cdot\sin(7.83^\circ) = 12.3\ N$$

(c) From (a)

$$\frac{\partial y}{\partial t} = -\omega y_m \cos(kx-\omega t) \qquad \text{——} \quad (1)$$

The transverse Component of the tension due to the string to the left is $-T\sin\theta$.

$$-\tan\theta = -\frac{\partial y}{\partial x} = -k y_m \cos(kx-\omega t) \qquad \text{——} \quad (2)$$

Two quantities ((1) & (2)) reach their max. at $\cos(kx-\omega t) = -1$.

At this moment, $\sin(kx-\omega t) = 0$ so $y(x,t) = 0$.

(d) $P = \frac{dW}{dt} = \frac{T_{trans} \cdot dy}{dt}$

($dW = T_{trans} \cdot dy$ because only transverse component of the tension does work.)

P has its maximum when the transverse component T_{trans} of the tension and the string speed $\frac{dy}{dt}$ have their maximum value.

So $P_m = (12.3N) \cdot (3.77\ m/s) = 46.4\ W$

(e) As shown in (c), $y = 0$.

(f) P becomes 0 when T_{trans} & $\frac{\partial y}{\partial t}$ become zero.

(g) $P = 0$ when $\cos(kx-\omega t) = 0$ and when it does $\sin(kx-\omega t) = \pm 1$. The string displacement is $\pm y_m = \pm 0.5$ cm

36E. $y(x,t) = y_m \sin(kx-\omega t) + y_m \sin(kx-\omega t + \phi)$

$$= 2y_m \cos\frac{\phi}{2} \cdot \sin\left(kx-\omega t + \frac{\phi}{2}\right)$$

when $\phi = \frac{\pi}{2}$,

$$\text{amplitude} = 2y_m \cos\frac{\pi}{4} = \sqrt{2}\, y_m$$

46E. (a) $v = \sqrt{\frac{\tau}{\mu}} = 1.4 \times 10^2 \ m/s$

(b) From the figure,

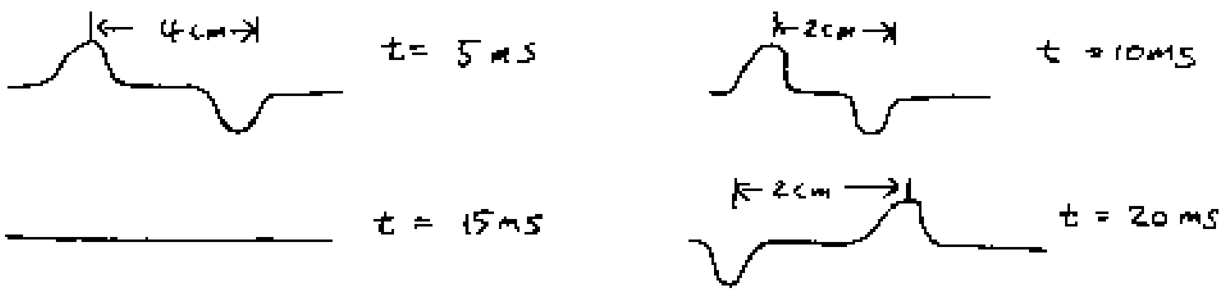
$\lambda = \frac{2}{3} \cdot (90 cm) = 60 \ cm$

(c) $f = \frac{v}{\lambda} = \frac{1.4 \times 10^2 \ m/s}{0.6 \ m} = 2.4 \times 10^2 \ Hz$

47E. $T = 2 \cdot (0.5 s) = 1 s$

$\lambda = \frac{v}{f} = vT = 10 \ cm$

60P. (a)



(b) The energy is not lost at t = 15ms. Rather, it takes the form of the kinetic energy of the particles in the string moving in the direction perpendicular to the string.