Ch. 21

Answer to Questions

1. $\Delta S = -\frac{Q}{T_{sun}} + \frac{Q}{T_{earth}} > 0 \quad (\because T_{sun} > T_{earth})$

increase

4. $b > a > c > d$

$(\because \Delta S = nR \ln \frac{V_f}{V_i} + nC_v \ln \frac{T_f}{T_i})$

6. $c > a > b$

$(\because W = \varepsilon |Q_H|, \ \varepsilon = 1 - \frac{T_C}{T_H}$, & $|Q_H|$ are same for a, b, c.)

10. increase

Exercise & Problems

6E. (a) $W = \int_{V_1}^{2V_1} p\,dV = nRT \int_{V_1}^{2V_1} \frac{dV}{V} = nRT \ln \frac{2V_1}{V_1}$

$\therefore W = 9.22 \times 10^3 J$

(b) Isothermal process : $\Delta T = 0 \Rightarrow \Delta E = nC_v \Delta T = 0$

$\therefore Q = W$

$\Rightarrow \Delta S = \int \frac{dQ}{T} = \frac{Q}{T} = \frac{W}{T} = 23.1 \ J/K$

(c) $\Delta S = 0$ for all reversible adiabatic processes.

7E. (a) $Q = mc\Delta T = (386 \ J/kg\cdot K)\cdot(2 \ kg)\cdot(100°C - 25°C)$

$= 5.8 \times 10^4 \ J$

(b) $\Delta S = mc \ln \frac{T_f}{T_i}$ (according to sample problem 21-1)

$= (2kg)\cdot(386 J/kg\cdot K)\cdot \ln\left(\frac{100+273}{25+273}\right)$

$= 1.7 \times 10^2 \ J/K$

9E. Constan volume $\Rightarrow$ $W = 0$.

$\therefore \Delta E = Q$

$\Rightarrow dQ = dE = \frac{3}{2} n R dT$

$$\Delta S = \int \frac{dQ}{T} = \int_{T_i}^{T_f} \frac{(3/2) n R dT}{T} = \frac{3}{2} n R \ln \frac{T_f}{T_i}$$

$$= 3.59 \text{ J/K}$$

10E. Now $dQ = \Delta E = n C_p dT = n(C_v + R) dT = n\left(\frac{3}{2}R + R\right) dT$

$$= \frac{5}{2} n R dT$$

Replace the factor 3/2 in the last problem by 5/2.

$$\Delta S = \frac{5}{2} n R \ln \frac{T_f}{T_i} = 5.98 \text{ J/K}$$

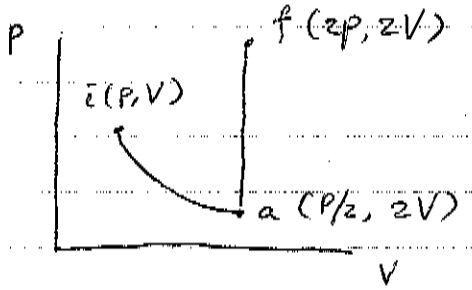
11E. (a) $\Delta S = \frac{Q}{T} = \frac{m L_F}{T} = \frac{(12 \times 10^{-3} \text{ kg}) \cdot (333 \text{ kJ/kg})}{273 \text{ K}}$

$$= 14.6 \text{ J/K}$$

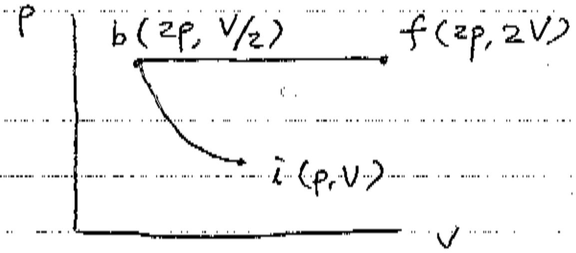
(b) $\Delta S = \frac{Q}{T} = \frac{m L_V}{T} = \frac{(5 \times 10^{-3} \text{ kg}) \cdot (2260 \text{ kJ/kg})}{373 \text{ K}}$

$$= 30.3 \text{ J/K}$$

23P.



(I)



(II)

Coordinates of points a & b are calculate by ideal gas law ($\frac{PV}{T} = \frac{P_a \cdot 2V}{T}$ & $\frac{PV}{T} = \frac{2P \cdot V_b}{T}$)

(a) Heat absorbed by the gas in process ia (isothermal):

$$Q_{ia} = W_{ia} = nRT \ln\left(\frac{2V}{V}\right) = pV \ln 2$$

Heat absorbed in process af ($\Delta V = 0 \Rightarrow W = 0$) :

$$Q_{af} = \Delta E_{af} = \frac{3}{2} nR \Delta T_{af} = \frac{3}{2} nR \Delta (pV)_{af}$$

$$= \frac{3}{2}\left[(2p)(2V) - pV\right] = 4.5\, pV$$

Similarly, heat absorbed in process ib :

$$Q_{ib} = W_{ib} = -pV \ln 2$$

$$Q_{bf} = \Delta E_{bf} + W_{bf} = \frac{3}{2}\left[(2p)(2V) - pV\right] + (2p)\cdot\left(\frac{3V}{2}\right)$$

$$= \frac{15}{2} pV$$

(b) Work done in process ia :

$$W_{ia} = Q_{ia} = pV \ln 2$$

$$W_{if} = 0 \quad (\because \Delta V = 0)$$

$$W_{ib} = Q_{ib} = -pV \ln 2$$

$$W_{bf} = 2p\left(2V - \frac{V}{2}\right) = 3pV$$

(c) $$\Delta E_{if} = E_f - E_i = \frac{3}{2} nR (T_f - T_i)$$

$$= \frac{3}{2}(P_f V_f - P_i V_i) = \frac{3}{2}\left[(2p)(2V) - pV\right] = 4.5\, pV.$$

(Same for both paths I & II.)

(d) ΔS_{if} is the same for both paths. (State function.)

$$\Delta S_{if} = \Delta S_{ia} + \Delta S_{af} = nR \ln \frac{V_a}{V_i} + \int_a^f \frac{dQ}{T}$$

$$= nR \ln\left(\frac{2V}{V}\right) + nC_V \int_{T_a}^{T_f} \frac{dT}{T}$$

$$= nR \ln 2 + \frac{3}{2} nR \ln\left(\frac{T_f}{T_a}\right)$$

$$= nR \ln 2 + \frac{3}{2} nR \ln\left[\frac{(2p)(2V)/nR}{pV/nR}\right] = 4nR \ln 2$$

$$= 4\cdot(1\,mol)\cdot(8.31\,J/mol\cdot K)\cdot \ln 2 = 23\,J/K$$

24P. (a) Denote the copper block as block 1 & the lead block as block 2.
The equilibrium temperature T_f satisfies

$$m_1 c_1 (T_f - T_{i,1}) + m_2 c_2 (T_f - T_{i,2}) = 0$$

$$\therefore\ T_f = \frac{m_1 c_1 T_{i,1} + m_2 c_2 T_{i,2}}{m_1 c_1 + m_2 c_2} \qquad \left(\begin{array}{l} c_1 = 386\,J/kg\cdot C^\circ \\ c_2 = 128\,J/kg\cdot C^\circ \end{array} \right)$$

$$= 320\,K$$

(b) Since the two-block system is in thermally insulated from the environment, the change in internal energy is zero.

(c) $$\Delta S = \Delta S_1 + \Delta S_2 = m_1 c_1 \ln\left(\frac{T_f}{T_{i,1}}\right) + m_2 c_2 \ln\left(\frac{T_f}{T_{i,2}}\right)$$

$$= 1.72\,J/K$$

29P. (a) $W = p_0(4V_0 - V_0) = 3p_0V_0$

(b) From b to c, $\Delta V = 0 \Rightarrow W = 0 \Rightarrow Q = \Delta E = nC_v\Delta T$

$C_v = \frac{3}{2}R$ for monatomic gas

Using ideal gas law,

$$T_b = \frac{P_b V_b}{nR} = \frac{4P_0V_0}{nR}, \quad T_c = \frac{P_c V_c}{nR} = \frac{(2P_0)(4V_0)}{nR} = \frac{8P_0V_0}{nR}$$

$$\Rightarrow \underset{\Delta E}{\underset{\parallel}{Q}} = \frac{3}{2}nR\left(\frac{8P_0V_0}{nR} - \frac{4P_0V_0}{nR}\right) = 6p_0V_0$$

$$= 6RT_0 \quad (\because n = 1\,mol)$$

Since the process bc is at constant volume,

$$\Delta S = \int \frac{dQ}{T} = \underset{\underset{1\,mol}{\uparrow}}{n}C_v \int_{T_b}^{T_c} \frac{dT}{T} = nC_v \ln\frac{T_c}{T_b}$$

$$= \frac{3}{2}R\ln 2$$

(e) For a complete cycle, $\Delta E = 0$ & $\Delta S = 0$.

36E. (a) $|Q_H| = \frac{|W|}{\varepsilon} = \frac{8.2\,kJ}{0.25} = 33\,kJ$

$|Q_c| = |Q_H| - |W| = 33\,kJ - 8.2\,kJ = 25\,kJ$

(b) $|Q_H| = \frac{|W|}{\varepsilon} = \frac{8.2\,kJ}{0.31} = 26\,kJ$

$|Q_c| = |Q_H| - |W| = \cancel{26\,k}\ 26\,kJ - 8.2\,kJ = \cancel{18}\ 18\,kJ$

45P. (a) $W = 2p_0(2V_0 - V_0) + p_0(V_0 - 2V_0)$

$= p_0 V_0 = 2.27 \times 10^3\ J$

(b) $Q_{abc} = \Delta E_{abc} + W_{abc}$

$\Delta E_{abc} = \frac{3}{2} nR\Delta T = \frac{3}{2}\Delta(pV) = \frac{3}{2}\left[(2p_0)(2V_0) - p_0 V_0\right]$

$= 4.5\, p_0 V_0$

$W_{abc} = 2p_0(2V_0 + V_0) = 2p_0 V_0$

$\therefore Q_{abc} = (4.5 + 2)p_0 V_0 = 6.5\, p_0 V_0$

$= 1.48 \times 10^4\ J$

(c) $\epsilon = \frac{W}{Q_H} = \frac{W}{Q_{abc}} = \frac{p_0 V_0}{6.5 p_0 V_0} = 15.4\%$

(d) $\epsilon_{ideal} = 1 - \frac{T_a}{T_c} = 1 - \frac{p_0 V_0 / nR}{4p_0 V_0 / nR} = 1 - \frac{1}{4} = \frac{3}{4}$

$= 75\%$

which is considerably higher than 15.4%.

61P. The work done by the motor in 10 min. is

$$W = \underset{\substack{\uparrow \\ \text{power}}}{Pt} = (200\,W)\cdot(10\,min)(60\,s/min) = 1.2\times10^5\,J$$

Then $|Q_c| = K|W| = \frac{T_C}{T_H - T_C}\cdot|W| = \frac{(270K)}{300K-270K}\cdot(1.2\times10^5 J)$

$$= 1.08\times10^6\,J$$

65E. There are two possible states for each molecule. And they are all independent each other.

So $N_{total} = \underbrace{2\times2\times\cdots\times2}_{N\text{ times}} = 2^N$

68P. (a) $W = \frac{50!}{(25!)(50-25)!} = 1.26\times10^{14}$

(b) $N_{total} = 2^{50} = 1.13\times10^{15}$

(c) The percentage of time in question is equal to the probability for the system to be in the central configuration.

$$p = \frac{W}{2^{50}} = \frac{1.26\times10^{14}}{1.13\times10^{15}} = 11.1\,\%$$

(d) $N = 100$,

$$W = \frac{100!}{50!\cdot50!} = 1.01\times10^{29}$$

$$N_{total} = 2^{100} = 1.27\times10^{30}$$

$$p = \frac{W}{N_{total}} = \frac{1.01\times10^{29}}{1.27\times10^{30}} = 8\,\%$$

(e) As N increases the number of available microstates increase as 2^N, so there are more states to be occupied, leaving the probability less for the system to remain in its central configuration.