Inelastic Collisions

Momentum conserved as before (if no external forces)

BUT K.E. not conserved : lost to deformation, heat, sound, ...

Special Case: Totally Inelastic ("sticky") collisions

$|\Delta KE| = |KE_f - KE_i|$ is max. when objects stick together after collision:

$$p_i = m_A v_{Ai} + m_B v_{Bi} \qquad = p_f = (m_A + m_B) v_f$$

$$KE_i = \tfrac{1}{2} m_A v_{Ai}^2 + \ldots \qquad KE_f = \tfrac{1}{2}(m_A + m_B) v_f^2 = \frac{p_f^2}{2(m_A+m_B)}$$

$$\frac{p_i^2}{2m_A}$$

Can show that $\Delta KE = KE_f - KE_i = \dfrac{-m_B}{(m_A+m_B)} KE_i$

i.e. greater fraction of KE_i is lost ($\Rightarrow$ more "damage")

when $m_B >> m_A$

also force of collision on A or B $F_{av} = \dfrac{(\Delta m v)_A}{\Delta t} \propto m_B$

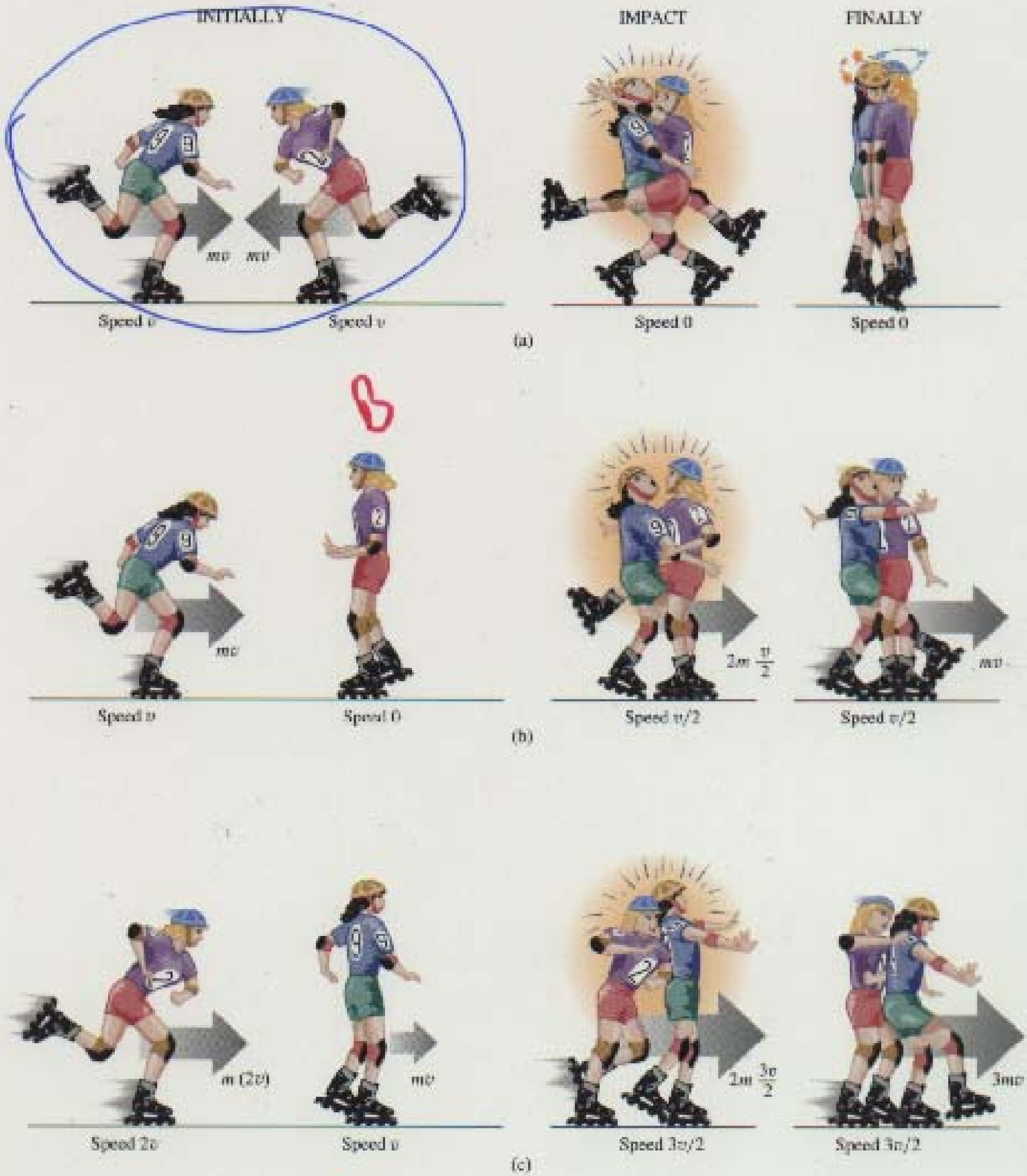
$p = mv$

cf. car hits parked truck $m_A << m_B$ $\quad \tfrac{1}{2} m v^2 = \dfrac{p^2}{2m}$

truck hits parked car $m_A >> m_B$

16/5

Figure 7.10
Inelastic collisions



Totally Inelastic: Examples

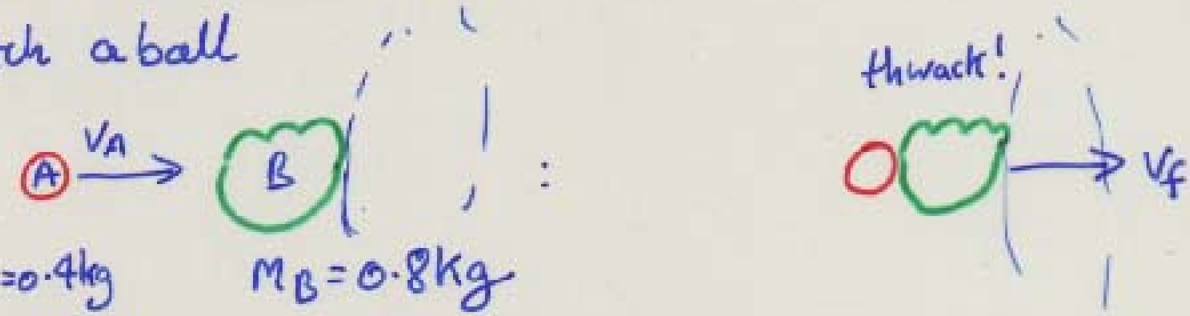
- Throw clay lump at door
- Car crash

$\Big\}$ ΔKE lost to work as force deforms objects $\Delta KE = F_{av}\,\Delta x$

("crumple zone" increases Δx so $F_{av} \downarrow$).

- Sack a quarterback (Ex. 7.9)
- Catch a ball

e.g



$m_A = 0.4\,kg$ $\quad m_B = 0.8\,Kg$

$v_A = 50\,m/s$

Just after collision (i.e. before catcher can apply an external braking force):

$$p_f = (m_A + m_B)\,v_f \qquad = p_i = m_A v_A = 0.4 \times 50 = 20\ kg\,m/s\ (Ns)$$

$$\text{So } v_f = \frac{m_A}{m_A + m_B}\,v_A = \frac{0.4}{0.8 + 0.4}\,v_A = \tfrac{1}{3}\,v_A = 16.67\,m/s$$

$$\text{Initial } KE_i = \frac{p_i^2}{2m_A} = \frac{20^2}{2 \times 0.4} = 500\,J.$$

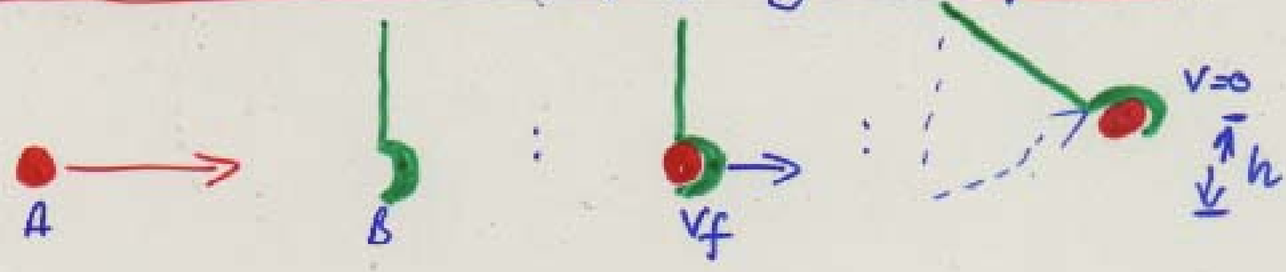
$$\text{Final } KE_f = \frac{p_f^2}{2(m_A + m_B)} \text{ with } p_f = p_i \Rightarrow KE_f = \frac{20^2}{2 \times (0.4 + 0.8)} = 166.7\,J$$

$\Rightarrow$ K.E. lost $= 333.3\,J$ or $2/3$ of the original

$\hookrightarrow$ heat, sound etc.

So catcher only needs to do work $W = F_{av}\,\Delta x = 1/3$ original KE to bring ball to rest.

Demo: Ballistic Pendulum for finding bullet speed v_A



Gun fires into mass B, bullet A and mass B together gain height (converting KE_f into PE). Measure $m_A, m_B, h \rightarrow v_A = ?$

Momentum before collision $p_i = m_A v_A$

= Momentum after $p_f = (m_A + m_B) v_f$

So KE *just* after collision $KE_f = \frac{p_f^2}{2(m_A+m_B)} = \frac{m_A^2 v_A^2}{2(m_A+m_B)}$

As combined mass swings up to height h

Final P.E. $= (m_A+m_B) g h = KE_f$

$$\Rightarrow \quad v_A^2 = \frac{2(m_A+m_B)^2 g h}{m_A^2} \quad \underline{\text{or}} \quad v_A = \sqrt{2gh}\left(1+\frac{m_B}{m_A}\right)$$

Note: "Stopping Power" of bullet refers to momentum, not energy

"Penetration" of bullet refers to energy $\frac{1}{2}mv^2 = F_{av}\,\Delta x$

Inelastic Collisions in time-reverse (un-collisions?)

- when objects fly apart

e.g. rocket ejects fuel, astronaut throws ball, bomb explodes into fragments (3D), recoil of gun.

If initial $\vec{p}_i = 0$, final $\vec{p}_f = m_A \vec{v}_A + m_B \vec{v}_B + \dots = 0$

e.g. for 2 masses, $m_A v_A = m_B v_B$ after separation $(*)$

If initial $KE_i = 0$, final KE_f = energy input

(e.g. from spring, muscles, chemical, ...)

$KE_f = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$ with constraint of $(*)$

$\Rightarrow \quad KE_f = \frac{1}{2} m_A v_A^2 \left(1 + \frac{m_A}{m_B}\right)$

So actual speeds v_A (and $v_B = \frac{m_A}{m_B} v_A$) are set by energy input only.

Since $F_{AB} = -F_{BA}$, doesn't matter if A or B pushes harder against other.

Example: 2 skaters on ice ($\mu = 0.1$) push apart

- how can they tell who is heavier?

Start at rest, $p_i = 0$. Just after separation (before friction has had time to change momenta)

$$p_f = M_A V_A + M_B V_B = 0 \quad \text{, i.e. speed ratio } \frac{V_A}{V_B} = \frac{M_B}{M_A} \quad (*)$$

(with actual speed set by total energy input $= (KE_A + KE_B)$ doesn't matter which skater pushes harder)

Skater A is brought to rest by work done by friction over dist. Δx

$$W = F_F \Delta x_A = (KE)_A \quad \text{with } F_F = \mu_k M_A g$$

$$\text{i.e. } \mu M_A g \Delta x_A = \tfrac{1}{2} M_A V_A^2 \quad \text{or} \quad \Delta x_A = \frac{V_A^2}{2 \underbrace{\mu g}_{= \text{accel. } a}}$$

Similarly for skater B: stopping distance $\Delta x_B = \dfrac{V_B^2}{2\mu g}$

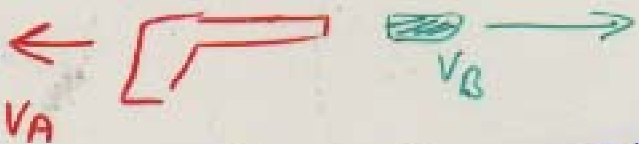
So ratio $\dfrac{\Delta x_A}{\Delta x_B} = \dfrac{V_A^2}{V_B^2} = \dfrac{M_B^2}{M_A^2}$ from $(*)$

e.g. if initial KE = 810 J, $M_A = 30\,\text{kg}$, $\underline{M_B = 60\,\text{kg}}$, $\mu = 0.1$, $g = 10\,\text{m/s}^2$

$(*) \Rightarrow V_A = 2V_B = 6\,\text{m/s}$, $V_B = 3\,\text{m/s}$

$\Rightarrow \Delta x_A = 18\,\text{m}$, $\Delta x_B = \tfrac{1}{4} \Delta x_A = 4.5\,\text{m}$.

Example cf. Hecht ex 7.8 : recoil



$m_A = 0.9\,kg$ $\quad m_B = 0.008\,kg$. If gun recoils at $v_A = -3.1\,m/s$,

gun momentum $p_A = m_A v_A = -2.79\,Ns = -p_B = -m_B v_B$

So bullet speed $v_B = \dfrac{-p_A}{m_B} = +352\,m/s$ with $p_B = +2.79\,Ns$

Total explosion kinetic energy $= KE_A + KE_B = \dfrac{p_A^2}{2m_A} + \dfrac{p_B^2}{2m_B}$

$$= 4.32\,J + 486.5\,J = 491\,J$$

If barrel length $\Delta x = 15\,cm$, av. force on bullet given by

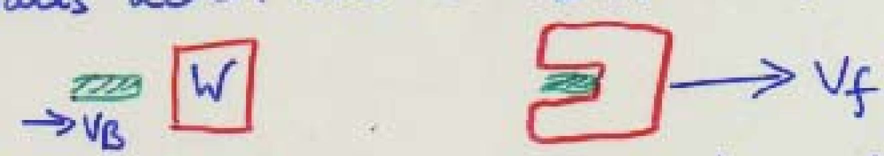
work $= \underset{\int F.dx}{F_{av}\,\Delta x} = \tfrac{1}{2} m_B v_B^2 = \dfrac{p_B^2}{2m_B} \rightarrow F_{av} = \dfrac{486.5\,J}{0.15\,m} = 3.24\,kN$

From Newton III, F_{av} on gun also $= 3.24\,kN$ acting over

time $\Delta t = \dfrac{\Delta(m_A v_A)}{F_{av}} \approx 860\,\mu s$.

If bullet hits wooden block, mass also $0.9\,kg$, and penetrates $20\,cm$ into it. ($\Delta x_w = 0.2\,m$)

Now:



Common $v_f = \dfrac{m_B v_B}{m_B + m_w} = +3.1\,m/s$ as before

$\Rightarrow$ Can now find $\Delta KE = \dfrac{p_B^2}{2m_B} - \dfrac{p_B^2}{2(m_B+m_w)} = F_{wood}\,\Delta x_w$.