10.1

Use Eq. (10.1): $F = ks$. In our case $F = 50\,\text{N}$ and $s = 25\,\text{cm} - 20\,\text{cm} = 5\,\text{cm}$, so the spring constant is

$$k = \frac{F}{s} = \frac{50\,\text{N}}{0.05\,\text{m}} = 1.0\,\text{kN/m}\,.$$

10.5

The work W done on the spring is equal to the gain in its elastic potential energy:

$$W = \Delta\text{PE}_e = \frac{1}{2}ks^2 = \frac{1}{2}(2.00 \times 10^2\,\text{N/m})(0.100\,\text{m})^2 = 1.00\,\text{J}\,.$$

10.10

The force F that needs to be applied to a spring to stretch its length by an amount x is $F = kx$, where k is its spring constant. In our case a 40.0-N force results in a stretch of $14.0\,\text{cm} - 10.0\,\text{cm} = 4.0\,\text{cm} = 0.040\,\text{m}$, and so $k = 40.0\,\text{N}/0.040\,\text{m} = 1.0 \times 10^3\,\text{N/m}$. Thus the work W that must be done to stretch the spring from $x_i = 14.0\,\text{cm} - 10.0\,\text{cm} = 0.40\,\text{cm}$ to $x_f = 18.0\,\text{cm} - 10.0\,\text{cm} = 8.0\,\text{cm}$ is

$$\begin{aligned} W &= \int dW = \int_{x_i}^{x_f} F(x)\,dx = \int_{x_i}^{x_f} kx\,dx = \frac{1}{2}k\left(x_f^2 - x_i^2\right) \\ &= \frac{1}{2}(1.0 \times 10^3\,\text{N/m})\left[(0.080\,\text{m})^2 - (0.040\,\text{m})^2\right] \\ &= 2.4\,\text{J}\,. \end{aligned}$$

10.61

The motion of the ant as seen by the child is a one-dimensional SHM, which is the circular motion of the ant projected onto a line perpendicular to the line-of-sight of the child. The maximum displacement of the ant either to the left or the right measured from the center of the motion, which coincides with the center of the record, is R, the radius of the record. Since the record turns at 78 rpm, the period of the motion is $T = (1/78)\,\text{min}$, and the corresponding frequency is

$$f = \frac{1}{T} = \frac{78}{(1\text{min})(60\,\text{s/min})} = 1.3\,\text{Hz}\,.$$

According to Eq. (10.12) the angular frequency is $\omega = 2\pi f = (2\pi\,\text{rad})(1.3\,\text{s}^{-1}) = 8.2\,\text{rad/s}$.

10.63

The acceleration as a function of time of an object in SHM is given by Eq. (10.17): $a_x = -A\omega^2 \cos\omega t$. For maximum acceleration $a_x(\text{max})$, set $\cos\omega t = \pm 1$ to obtain $|a_x(\text{max})| = A\omega^2 = A(2\pi f)^2$. With $A = 0.50\,\text{cm}$ and $f = 50\,\text{Hz}$,

$$|a_x(\text{max})| = 4\pi^2 A f^2 = 4\pi^2(0.50 \times 10^{-2}\,\text{m})(50\,\text{Hz})^2 = 4.9 \times 10^2\,\text{m/s}^2\,.$$

10.67

Compare the expression $x = 5.0\cos(0.40t + 0.10)$ for the SHM of the body in question with Eq. (10.13), $x = A\cos\omega t = x_{max}\cos 2\pi ft$, which describes a standard SHM with an initial displacement of A.

(a) $A = 5.0\,\text{m}$, by direct comparison between the two equations.

(b) Set $2\pi f = 0.40\,\text{s}^{-1}$ to obtain $f = 0.40\,\text{s}^{-1}/2\pi = 0.064\,\text{Hz}$.

(c) Set $t = 0$ in the expression for the phase to obtain the initial phase: $\varepsilon = 0.40 \times 0 + 0.10 = 0.10\,\text{rad}$.

(d) Plug $t = 2.0\,\text{s}$ into the expression for x: $x = 5.0\cos\left[(0.40\,\text{s}^{-1})(2.0\,\text{s}) + 0.10\right] = 3.1\,\text{m}$.

10.80

Obviously, the most likely moment when the rock might lose contact with the platform is when the platform reaches the highest point in its vertical SHM and is just about to move downward. At this point, the downward acceleration of the SHM of amplitude A and frequency f is at its maximum value of $a_0 = \omega^2 A = (2\pi f)^2 A = 4\pi^2 f^2 A$. Now, suppose the rock is just about to lose contact with the platform at this point, then the contact force F_N between the rock and the platform is zero, and so the only remaining force exerted on the rock is $F_W = mg$, the weight of the rock of mass m. Since the rock has *not* quite lost contact with the platform yet and is still undergoing the same SHM as the platform, its acceleration is still a_0. Thus for the rock

$$+\!\downarrow \sum F = F_W = mg = ma_0 = 4\pi^2 f^2 Am\,,$$

which gives the critical frequency $f = f_C$, above which the rock will start to clatter:

$$f_C = \frac{1}{2\pi}\sqrt{\frac{g}{A}} = \frac{1}{2\pi}\sqrt{\frac{9.81\,\text{m/s}^2}{0.10\,\text{m}}} = 1.6\,\text{Hz}\,.$$

10.83

Apply Eq. (10.20), $f_0 = (1/2\pi)\sqrt{k/m}$. Here $f_0 = 18\,\text{Hz}$ and $m = 0.20\,\text{g}$, so

$$k = 4\pi^2 f_0^2 m = 4\pi^2(18\,\text{Hz})^2(0.2 \times 10^{-3}\,\text{kg}) = 2.6\,\text{N/m}\,.$$

10.85

The weight of the potatoes of mass m is $F_W = mg$, which is the force F applied on the scale. The resulting displacement of the scale is $x = 2.50\,\text{cm} = 0.0250\,\text{m}$; hence from $F_W = mg = F = kx$ we get

$$k = \frac{mg}{x} = \frac{(2.00\,\text{kg})(9.81\,\text{m/s}^2)}{0.0250\,\text{m}} = 785\,\text{N/m}\,.$$

The frequency f_0 of the SHM is found from Eq. (10.20):

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{785\,\text{N/m}}{2.00\,\text{kg}}} = 3.15\,\text{Hz}\,.$$

10.105

The frequency $f_\oplus$ of a simple pendulum of length L in small-amplitude oscillation on the surface of the Earth is given by $f_\oplus = (1/2\pi)\sqrt{g_\oplus/L} \propto \sqrt{g_\oplus}$, where $g_\oplus$ is the acceleration of gravity on the surface of the Earth. Similarly, if the pendulum is moved to the surface of the Moon, its new frequency will be $f_{\text{☾}} = (1/2\pi)\sqrt{g_{\text{☾}}/L} \propto \sqrt{g_{\text{☾}}}$. Thus

$$f_{\text{☾}} = f_\oplus \sqrt{\frac{g_{\text{☾}}}{g_\oplus}} = f_\oplus \sqrt{\frac{g_\oplus/6}{g_\oplus}} = \frac{f_\oplus}{\sqrt{6}} \approx 0.41 f_\oplus\,.$$

10.112

Before it is raised, the pendulum bob is located a distance L below the pivot point of the pendulum. When raised through an angle θ it is located a distance $L\cos\theta$ below the same pivot point. Thus the bob is raised through a vertical distance of $\Delta h = L - L\cos\theta = L(1-\cos\theta)$. The corresponding change in *gravitational*-PE of the bob-Earth system is $\Delta PE_G = mg\Delta h = mgL(1-\cos\theta)$. Using the expansion $\cos\theta = 1 - \frac{1}{2}\theta^2 + \cdots \approx 1 - \frac{1}{2}\theta^2$, valid for $|\theta| \ll 1$, this becomes

$$\Delta PE_G = mgL\,(1-\cos\theta) \approx mgL\left[1 - \left(1 - \frac{1}{2}\theta^2\right)\right] = \frac{1}{2}mgL\theta^2\,.$$

[A note in trigonometry: as an alternative to the expansion for $\cos\theta$ above, consider the trigonometric identity $\frac{1}{2}(1-\cos\theta) = \sin^2\left(\frac{1}{2}\theta\right)$. Note that $\sin^2\left(\frac{1}{2}\theta\right) \approx \left(\frac{1}{2}\theta\right)^2 = \frac{1}{4}\theta^2$ for $|\theta| \ll 1$, so $\frac{1}{2}(1-\cos\theta) \approx \frac{1}{4}\theta^2$, and $1-\cos\theta \approx \frac{1}{2}\theta^2$ for small θ.]

10.114

According to Problem (10.112), a pendulum bob of mass m which was released from its initial position, in which the string makes a small angle θ_0 with the vertical, will lose gravitational potential energy in the amount of $\frac{1}{2}mgL\theta_0^2$ by the time it swings to its lowest position. By conservation of energy, all the gravitational energy lost is turned into the kinetic energy of the bob, which is now moving at its maximum linear speed, v_{max}. Hence $\frac{1}{2}mv_{max}^2 = \frac{1}{2}mgL\theta_0^2$, or $v_{max} = \theta_0\sqrt{gL}$. The corresponding angular speed is then

$$\omega_0 = \frac{v_{max}}{L} = \frac{\theta_0\sqrt{gL}}{L} = \theta_0\sqrt{\frac{g}{L}}\,.$$

Note that, since ω_0 is the angular speed at the bottom of the swing, it is also the maximum angular speed of the swing, since this is where the *gravitational*-PE of the pendulum is the lowest and its kinetic energy the highest.

10.122

Consider the bob of mass m undergoing a uniform circular motion with a speed v in a horizontal plane. The radius of the circular track is $R = L\sin\theta$. Apply Newton's Second Law to the bob. Vertically it is not moving so

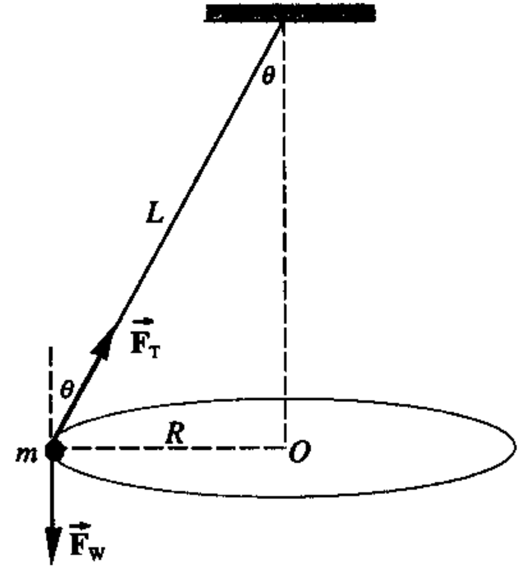
$$+\uparrow\sum F_y = F_T\cos\theta - F_W = 0\,;$$

and horizontally

$$\overset{+}{\rightarrow}\sum F_x = F_T\sin\theta = ma_c = m\frac{v^2}{R}\,.$$

Rewrite the equation for F_y as $F_T\cos\theta = mg$ and use this to divide the one for F_x. The result is $\sin\theta/\cos\theta = v^2/gR$. Solve for v:

$$v = \sqrt{\frac{gR\sin\theta}{\cos\theta}}\,.$$



The period T of the pendulum then follows from $v = 2\pi R/T$ as

$$T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{gR\sin\theta/\cos\theta}} = 2\pi\sqrt{\frac{R\cos\theta}{g\sin\theta}} = 2\pi\sqrt{\frac{(L\sin\theta)\cos\theta}{g\sin\theta}} = 2\pi\sqrt{\frac{L\cos\theta}{g}}\,.$$