Physics 1A QUIZ 9 . Closed Book. Write in blue or black ballpoint only.

Wave equation : $y = A\sin\frac{2\pi}{\lambda}(x - vt)$.
Beat frequency $f = f_1 - f_2$. Area of sphere $= 4\pi r^2$.

1. A transverse harmonic wave on the ocean surface has a vertical displacement in meters given by $y = 0.7\sin\pi(0.125x - 0.625t)$.

a. Show that (i) the wavelength is 16m, and (ii) find the amplitude, speed, and frequency of this wave.

A surfer sits on her board at x=10m, waiting patiently for a larger wave to come by.

b. Find her vertical speed and acceleration at time t=0.

c. Find the *maximum* values of her speed and acceleration as this wave moves across the ocean surface.

d. At what position (between x=0 and x=10m) would another surfer have to be located, such that their vertical speed *relative* to the first surfer can have its *maximum possible* value at some time in their motion? (A diagram may help; no calculation necessary).
(50 points)

2. A twin-engine helicopter hovers at an altitude of 6000m. An observer stands in a field at 8000m from the point on the ground directly below the aircraft. (Speed of sound in air v=340 m/s).

a. Show that it takes about 29s for the sound of the helicopter to reach the observer. Give 2 decimal places. (Hint: Pythagoras).

b. The observer uses a sound meter which measures the average power received from a sound source. This meter is attached to a dish microphone of collecting area 0.25m^2. If the meter reads 145 μW, what is the average acoustic power emitted by the helicopter's engines? (Assume that they emit sound uniformly over a sphere).

c. The observer now hears beats in the volume of the engine noise with a beat period of 0.6667 s. If the average tone produced by the engines combined is measured to be 200Hz, what are the fundamental frequencies of the tones produced by each engine?
(50 points)

Physics 1A Quiz 9 Solutions

1. $y = 0.7\sin\pi(0.125x - 0.625t) = A\sin\frac{2\pi}{\lambda}(x - vt)$

So (i) $\frac{2\pi}{\lambda} = 0.125\pi \Rightarrow \lambda = \frac{2}{0.125} = 16\text{m}$

(ii) Amplitude $A = 0.7\text{m}$, speed $v = \frac{0.625}{0.125} = 5\text{m/s}$
So freq. $f = v/\lambda = 5/16 = 0.3125\text{Hz}$

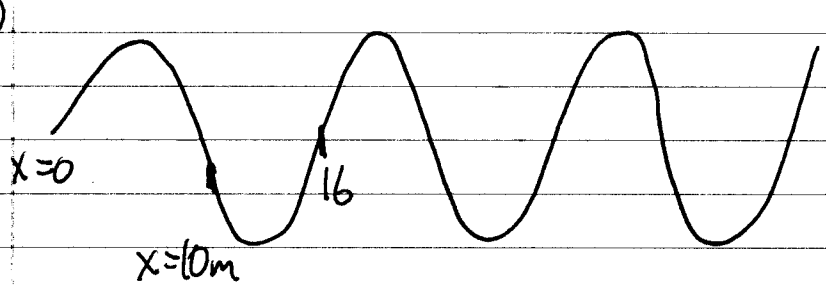
b) At $x = 10\text{m}$, $y = 0.7\sin\pi(0.125\times10 - 0.625t)$
$= 0.7\sin(1.25\pi - 0.625\pi t)$
So speed of surfer $\frac{dy}{dt} = \underline{-0.7\times0.625\pi}\cos(1.25\pi - 0.625\pi t)$
$= 1.374\text{m/s}$

And accel. $a = \frac{d^2y}{dt^2} = -\omega^2 y = \underline{-4\pi^2 f^2 \times 0.7}\sin(1.25\pi - 0.625t)$
$= 2.70\text{m/s}^2$

For $t = 0$,
$\frac{dy}{dt} = -1.374\cos(1.25\pi) = +0.972\text{m/s}$
$d^2y/dt^2 = -2.70\sin(1.25\pi) = +1.91\text{m/s}^2$

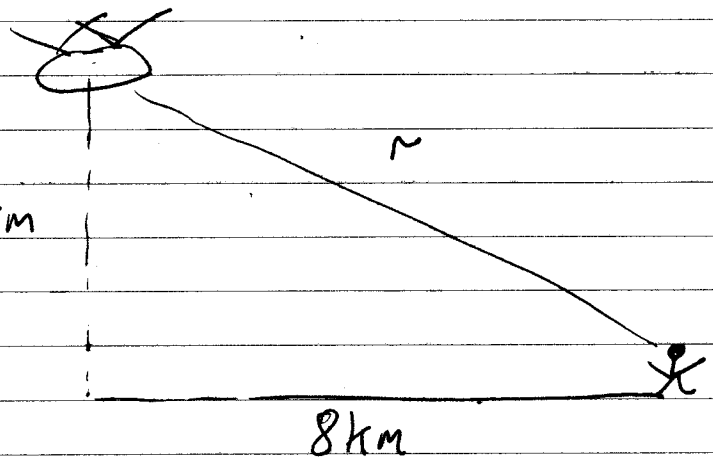
c) At max. values, $\sin(\,) = 1$ and $\cos(\,) = 1$
$\Rightarrow \left.\frac{dy}{dt}\right|_{max} = (-)1.374\text{m/s}$, $\left.\frac{d^2y}{dt^2}\right|_{max} = (-)2.70\text{m/s}^2$

d)



Max. relative speed occurs for $x_2 = x_1 \pm \lambda/2 = 10 \pm 16/2$
$\Rightarrow x_2 = 2\text{m}.$

2.



a) Distance $r = \sqrt{6^2+8^2} = 10\,km$

So time for sound to travel $t = \frac{r}{v} = \frac{10^4 m}{340 m/s} = 29.41 s$

b) Power P is spread over sphere area $4\pi r^2$

$\Rightarrow$ Intensity $I = \frac{P}{4\pi r^2}$. For detector area $a = 0.25 m^2$

Power received $Ia = \frac{aP}{4\pi r^2} = 145 \times 10^{-6} W$

$$\Rightarrow P = 145 \times 10^{-6} \times \frac{4\pi (10^4 m)^2}{0.25 m^2} = 728.8 kW$$

c) Beat period $T_{beat} = \frac{1}{f_1 - f_2} = 0.667 s \Rightarrow f_1 - f_2 = 1.5 Hz$ (1)

Average freq. $\frac{f_1 + f_2}{2} = 200 Hz$ (2)

Solve (1) and (2) $\Rightarrow$ $f_1 = 200 + 0.75 Hz = 200.75 Hz$

$f_2 = 200 - 0.75 Hz = 199.25 Hz$.