Physics 1A QUIZ 8 . Closed Book. Write in blue or black ballpoint only.

Assume earth's gravity $g = 10\ \text{m/s}^2$. Simple Harmonic Motion relations:

Displacement $x = A\cos(\omega t + \varphi)$; $\omega = \dfrac{2\pi}{T} = 2\pi f$. Kinetic energy $= \tfrac{1}{2}mv^2 = \tfrac{1}{2}m(\tfrac{dx}{dt})^2$

Spring: $\omega^2 = k/m$. Potential energy $= \tfrac{1}{2}kx^2$

Pendulum: $\omega^2 = g/L$. Potential energy $= mgh$.

1. A student attaches a 200g total mass to a vertical spring of length 100mm with force constant k=25 N/m.

a. What is the new equilibrium length of the spring, <u>in mm</u>?

The mass is now displaced a further 20mm, then released.

b. What is the frequency of the resulting oscillation, in Hz? 10.6a

c. Find the maximum acceleration of the mass on the spring. 10.6 d

d. Using energy arguments or otherwise, find the maximum speed of the mass on the spring. **(50 points)** 10.6c 10.7

2. A mass on a light wire of constant length L is designed to swing with a period T=1s on the Earth. This pendulum forms part of a clock, which is started by giving the mass at the bottom an initial "kick". The displacement x of the pendulum's mass from the center position can then be written as $x = A\sin\omega t$; $\omega^2 = g/L$.

a. Show that the required length of the wire (use g=10) is about 0.25m; give 3 decimal places.

b. An explorer takes this clock to Mars, and finds a new period of 2.46 s. Therefore, what is the acceleration due to gravity (g_M) on the Martian surface?

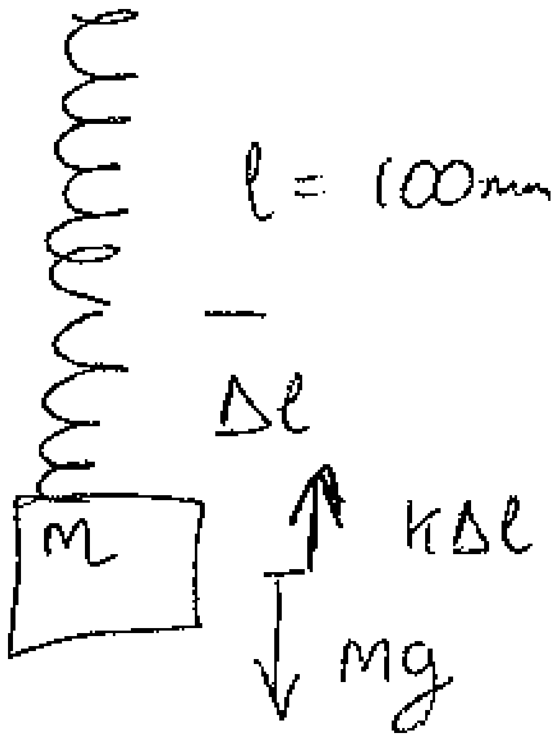
c. If the pendulum is started from x=0 with an initial speed $v(0)$=0.2 m/s, what is the amplitude A of the resulting motion on (i) the Earth and (ii) Mars? (Hint: Differentiate the equation of motion above).

d. Briefly explain why the pendulum has a larger displacement (amplitude) on Mars compared to the Earth, for the same initial kinetic energy. A diagram may help. **(50 points)**

Physics 1A Quiz 8 Solutions

1.

a)



At equilibrium

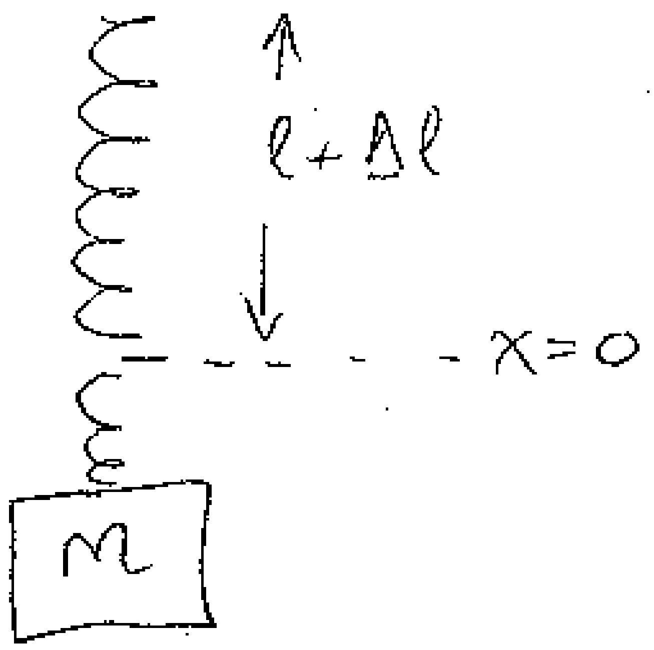
$F = k\Delta l = mg$

$\Rightarrow$ extension $\Delta l = mg/k$

$= \dfrac{0.2kg \times 10 m/s^2}{25 N/m} = 0.08m$

So new length $l + \Delta l = 100\,mm + 80\,mm = 180\,mm$.

b)



Take $x=0$ at the new length

$\Rightarrow$ Net restoring force

$F = m\dfrac{d^2x}{dt^2} = -kx$

$\Rightarrow$ SHM with $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{25}{0.2}} = 11.2$ rad/s

So frequency $f = \dfrac{\omega}{2\pi} = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}} = 1.78\,Hz$

c) Accel. $a = -\omega^2 x$, maximum at $x = \pm A = 20\,mm$

i.e. $a_{max} = (-)\,\omega^2 A = \dfrac{k}{m}A = \dfrac{25}{0.2} \times 20\times10^{-3}m = 2.5\,m/s^2$

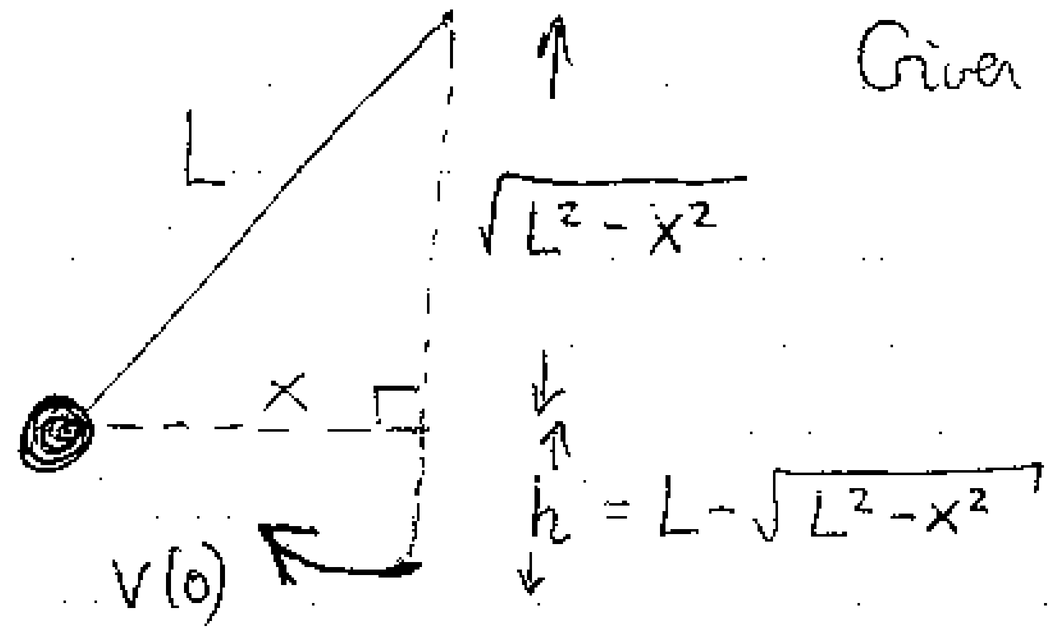
d) Initially, $KE = 0$ and total $E = P.E. = \frac{1}{2}kA^2$

K.E. is maximum when $PE = 0$, so total $E = \frac{1}{2}mV_{max}^2 = \frac{1}{2}kA^2$

~~$\Rightarrow$ total E/a~~ $\Rightarrow V_{max} = A\sqrt{\dfrac{k}{m}} = A\omega = 0.223\,m/s$

[OR: Take $x = 20\times10^{-3}m \cos 11.2t$ and find $\dfrac{dx}{dt}$]

2.



Given $x = A \sin \omega t$

$\omega^2 = g/L$

a) Since $T = 2\pi/\omega$, required length $L = \frac{g}{\omega^2} = \frac{gT^2}{4\pi^2}$

So with $T = 1s$, $L = \frac{10 m/s^2 \times 1^2 s^2}{4\pi^2} = \underline{0.253 m}$

b) Now with $T_M = 2.46s$, $g_M = L\omega_M^2 = \frac{4\pi^2 L}{T_M^2}$

$\Rightarrow g_M = \frac{4\pi^2 \times .253 m}{(2.46 s)^2} = \underline{1.65 m/s^2}$

c) $x = A \sin \omega t$ so speed $V = \frac{dx}{dt} = \omega A \cos \omega t$

At $t = 0$, $V = V(0) = A\omega$ so amplitude $A = \frac{V(0)}{\omega} = \frac{V(0)T}{2\pi}$

i) Earth $T = 1s$, $V(0) = 0.2 m/s$ $\Rightarrow A = \frac{0.2 m/s \times 1s}{2\pi} = \underline{0.0318 m}$

ii) Mars $T = 2.4s$ $\qquad A_M = \underline{0.0783 m}$

d) For same initial KE $\frac{1}{2} m V(0)^2$, pendulum reaches a max. height h above starting point ($x = 0$)

where $PE = mgh = \frac{1}{2} m V(0)^2$ [See diagram].

So as $g \downarrow$ on Mars, $h \uparrow$ and amplitude A given by

$h = L - \sqrt{L^2 - A^2}$ must also $\uparrow$ ($A = h\sqrt{2L - h}$ in fact)