

**Physics 1A QUIZ 7 . Closed Book. Write in blue or black ballpoint only.**

Assume earth's gravity  $g = 10 \text{ m/s}^2$ . Density of water =  $1000 \text{ kg/m}^3$   
Atmospheric pressure  $P_0 = 10^5 \text{ Pa}$ . Hydrostatic equilibrium:  $\frac{dP}{dh} = \rho g$

1. An open tank holds a column of water 5.0m tall, covered by a layer of oil (density  $600 \text{ kg/m}^3$ ) which is 2.0m thick.

a. What is the hydrostatic pressure in Pascals at (i) the oil/water interface, and (ii) the bottom of the tank? (Hint: don't forget atmospheric pressure!).

b. Sketch a graph showing the pressure  $P$  as a function of depth  $h$  from the surface, paying close attention to the pressure gradient in the oil and the water.

2. A hollow spherical buoy is held below the surface of the ocean by a cable attached to the bottom, to warn scuba divers of an unstable wreck. The buoy's volume is  $0.008 \text{ m}^3$  and the cable's tension is 30N.

a. Show (i) that the buoyant force of the water on the sphere is 80N. Therefore, (ii) what is the mass of the sphere, in kg?

The cable is cut by a passing boat's propeller.

b. What is the instantaneous upward acceleration of the sphere as it begins to rise to the surface?

c. Once the sphere floats at equilibrium on the surface, what fraction of its total volume is visible above the surface?

3. A pinball machine uses a light horizontal spring to launch of ball of mass 0.08kg along a track. A player pulls back gently on the spring with a force of 9N, and the spring compresses by 0.07m. Therefore:

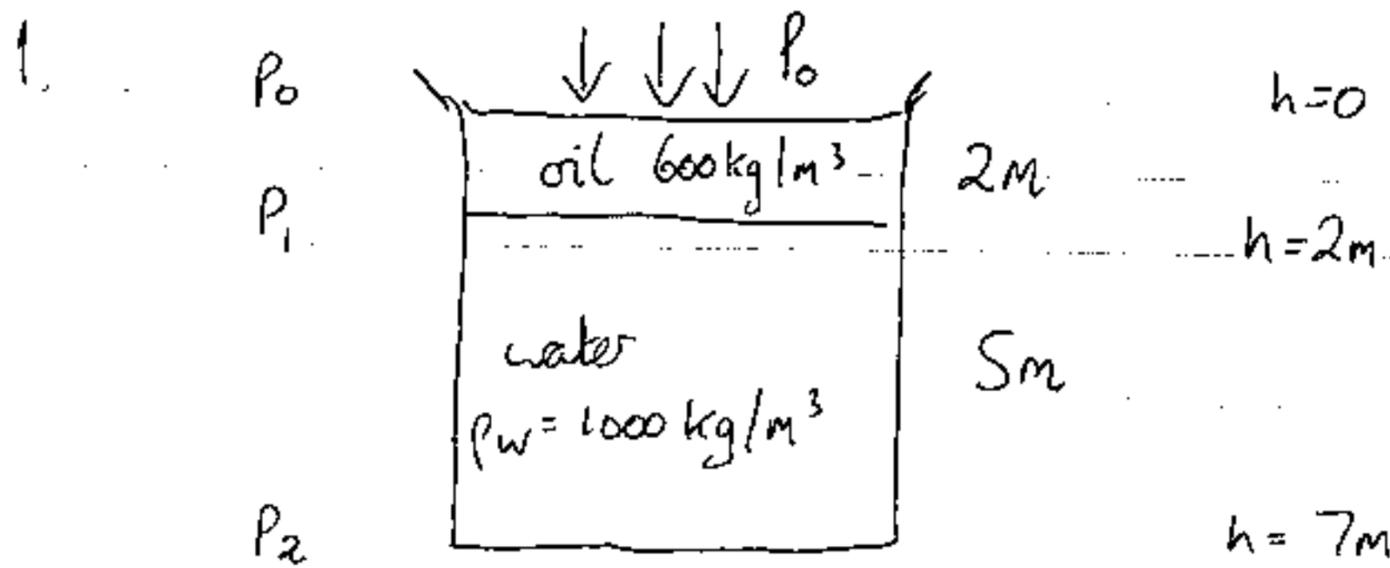
a. Show that the spring constant is about 129 N/m; give 5 significant figures.

When the spring is released, it pushes against the ball, which is accelerated rapidly without rolling.

b. Ignoring friction, find (i) the maximum acceleration and (ii) the final speed of the pinball when the spring is released.

c. If the ball's sliding coefficient of friction on the track is 0.6, now what is its speed at the instant the spring returns to its original length? (**Hint:** use work/energy arguments for this, even if you did not do so for part b)

# Physics 1A Quiz 7 Solutions



a) From  $\frac{dP}{dh} = \rho g$        $P = P_0 + \rho g h$

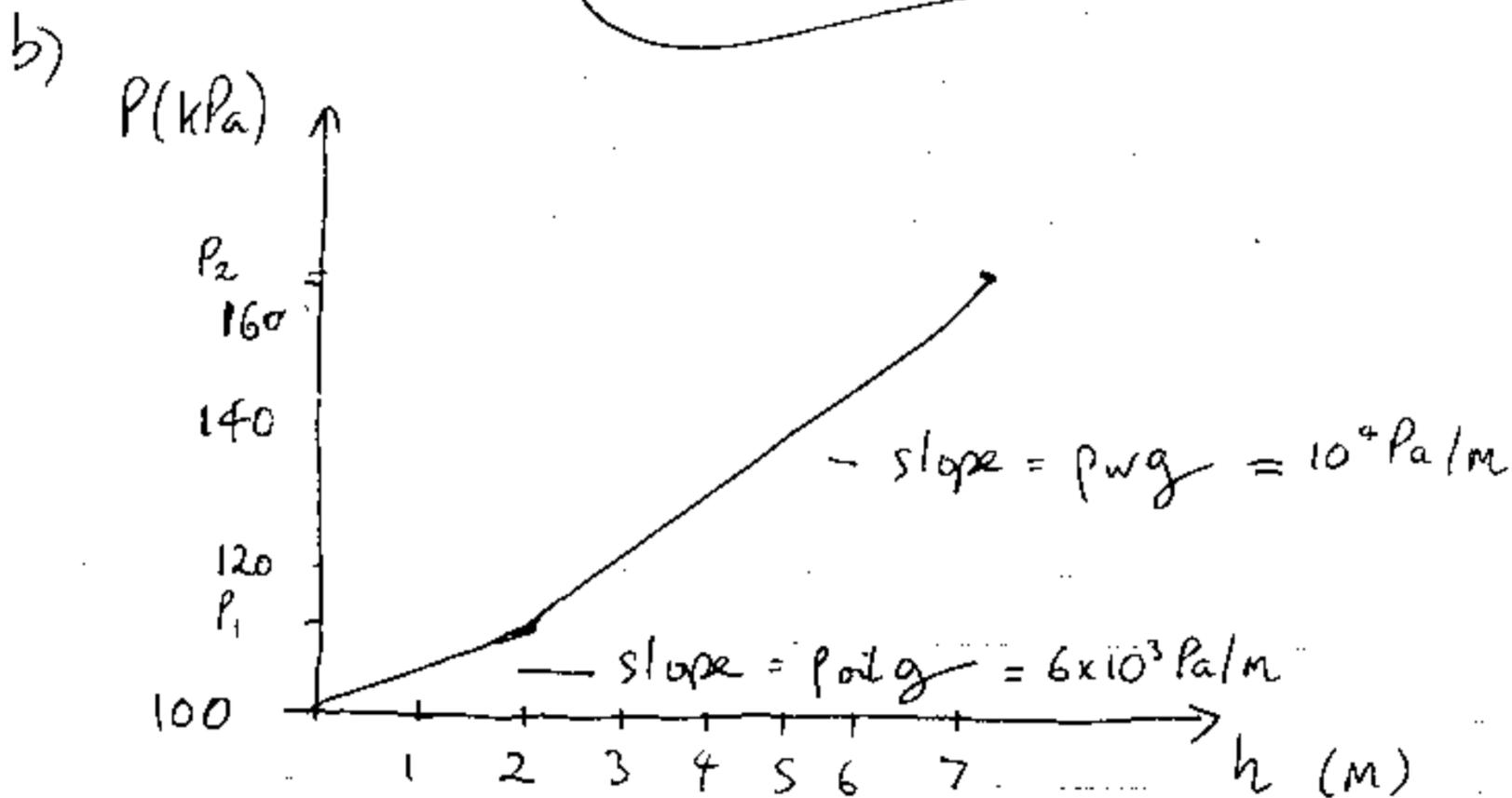
i) For oil,  $P = P_0 + \rho_{\text{oil}} g h$  with  $h = 2\text{m}$  at interface

i.e.  $P_1 = 10^5 \text{ Pa} + 600 \times 10 \times 2 = \underline{112 \text{ kPa}}$

ii) For water, at depth  $h$ ,  $P = P_1 + \rho_w g (h - 2\text{m})$

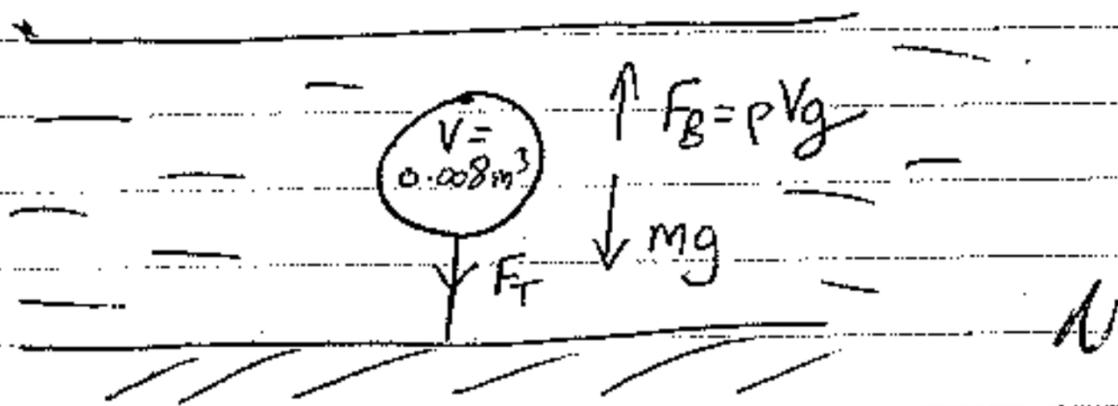
So where  $h = 7\text{m}$  at bottom of tank

$P_2 = \underline{112 \text{ kPa}} + 1000 \times 10 \times (7 - 2)$   
 $= \underline{162 \text{ kPa}}$



2.

3d

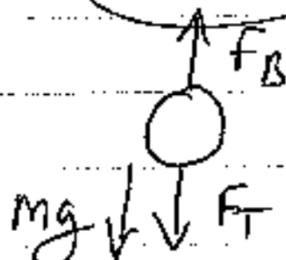


a) i) Buoyant force = weight of water displaced (Archimedes)

$$F_B = \rho V g = 1000 \times 0.008 \times 10 = 80 \text{ N}$$

ii) Equating forces up/down:

$$mg + F_T = F_B$$



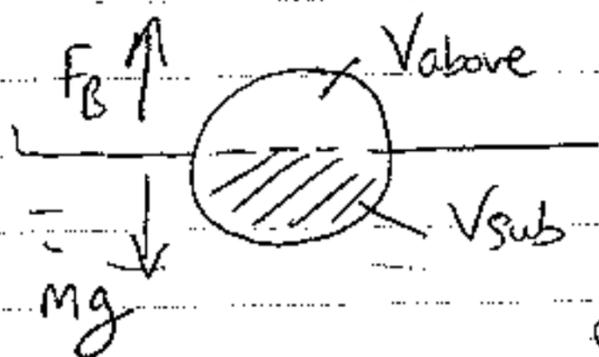
Given  $F_T = 30 \text{ N}$ :  $mg = F_B - F_T \Rightarrow m = \frac{80 - 30}{g} = 5 \text{ kg}$

b) If cable is cut,  $F_T = 0$  and net force upwards =  $F_B - mg$

So from Newton II:  $ma = \overbrace{F_B - mg}^{30 \text{ N}} \Rightarrow a = \frac{F_B}{m} - g$

i.e.  $a = \frac{80}{5} - 10 = 6 \text{ m/s}^2$

c)



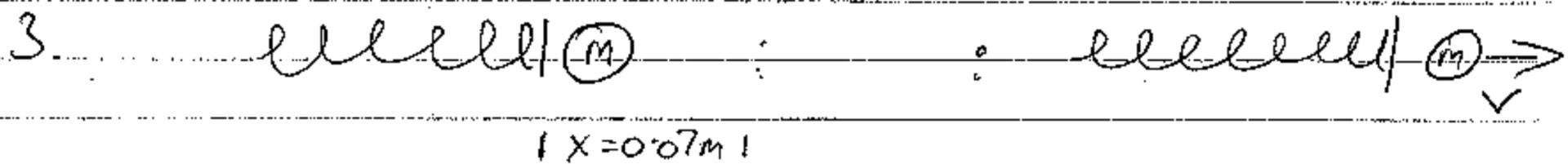
Once afloat,  $mg = F_B = \rho V_{\text{sub}} g$

So  $V_{\text{sub}} = \frac{mg}{\rho g} = \frac{m}{\rho} = \frac{5 \text{ kg}}{1000 \text{ kg/m}^3}$

$\Rightarrow V_{\text{sub}} = 0.005 \text{ m}^3$

$\therefore V_{\text{above}} = V - V_{\text{sub}} = 0.008 - 0.005 = 0.003 \text{ m}^3$   
 i.e. a fraction  $\frac{0.003 \text{ m}^3}{0.008 \text{ m}^3} = \frac{3}{8}$  of total (0.375)

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a) Hooke's law:  $F = kx$  with  $x = 0.07 \text{ m}$ ,  $F = 9 \text{ N}$

$\Rightarrow$  Spring constant  $k = 128.57 \text{ N/m}$  ( $= 9/0.07$  or  $\frac{900}{7} \text{ N/m}$ )

b) i) Accel  $a = \frac{F}{m}$  is a max. when  $F = kx$  is a maximum

So  $a_{\text{max}} = \frac{kx_{\text{max}}}{m} = \frac{9 \text{ N}}{0.08 \text{ kg}} = 112.5 \text{ m/s}^2$  ( $> 10g!$ )

ii) Final K.E. of ball  $\frac{1}{2}mv^2 = \text{P.E. of spring} - \frac{1}{2}kx^2$

$\Rightarrow v = \sqrt{\frac{k}{m} x^2}$ ,  $x = \sqrt{\frac{128.57 \times 0.07}{0.08}} = 2.81 \text{ m/s}$

(OR: Solve  $m \frac{d^2x}{dt^2} = -kx$  to find  $\frac{dx}{dt}$  at  $x=0$ : next week!)

c) With friction present

(Stored P.E. of Spring) = (Work done against friction) + (K.E.)

i.e.  $\frac{1}{2}kx^2 = F_f \cdot x + \frac{1}{2}mv^2$   
 $= (\mu mg) x$

$\Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}kx^2 - \mu mgx$

or  $v^2 = \left(\frac{k}{m}\right)x^2 - 2\mu gx$

$= 128.57 (0.07)^2 - 2 \times 0.6 \times 10 \times 0.7$

$\Rightarrow v_{\text{now}} = 2.65 \text{ m/s}$  (less than before)