Physics 1A QUIZ 6 . Closed Book. Write in blue or black ballpoint only.
3 Questions:

Note: You must use $g = 10\ m/s^2$ for the earth's gravity.
Angular speed $\omega = v/r = 2\pi/T$. Angular acceleration $\alpha = \frac{d\omega}{dt} = \frac{1}{r}\frac{dv}{dt}$
Moment of Inertia $I = \sum_i m_i r_i^2$. Torque $\tau = r_\angle F = I\alpha$
Angular momentum $L = I\omega$; Rotational k.e. $= \frac{1}{2} I\omega^2 = L^2/2I$

1. For a scene in the movie *MI3*, Tom Cruise jumps backwards off the top of a building (height h=8m) and falls head-first towards the ground. To effect the stunt safely, the m=70kg actor wears a harness which is attached to a long cable. The cable is wound multiple times around a large drum of radius r=0.75m, which is free to rotate with moment of inertia I=900 kgm^2. The drum is positioned out of sight at the top of the building. Therefore, when Mr. Cruise jumps, his fall is slowed by the drum's rotation as the cable unwinds. (The action is later sped up and the visible cable removed from the shot by special-effects editors).

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a. Using energy arguments, find (i) the actor's speed and hence (ii) the rotation period of the drum after the fall of 8m. (Hint: Mr. Cruise's downward speed is related to the drum's angular speed by $v = r\omega$).

15 b. Therefore, what percentage of the total gravitational potential energy lost in the fall is "absorbed" by the drum's rotational energy? **(35 points)**

c. **(Extra credit: 5 points):** Find the tension in the cable during the fall. (Hint: Use free-body diagrams, and first find the actor's acceleration).

2. In a school playground, a see-saw consists of a wooden beam 4m long balanced on a pivot at its center. A toddler of mass 20 kg sits at one end.

10 a. (i) At what distance from the pivot should her parent (mass 80 kg) sit in order for the see-saw to be level in equilibrium (i.e. no net torque)? (ii) What is the resulting force of the pivot on the beam?

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Another child of mass 15 kg jumps onto the end of the see-saw, this time on the same side as the parent.

10 2b. Show that the center of gravity of the system is now displaced about ~~0.52m~~ from the pivot, on the parent's side. Give 4 significant figures. 0.26m

10 c. By summing the contributions from the three riders, find the moment of inertia of the system about the pivot (neglect the mass of the beam).
(30 points)

d. **(Extra credit: 5 points)** Therefore find the angular acceleration of the system as it begins to rotate about the pivot, in radian/s^2.

3. Elsewhere in the playground, a child of mass 20 kg rides on the outside edge of a merry go-round with radius r=1.5m, moment of inertia I=180 kgm^2. (The merry-go-round is basically a thick horizontal disk). The merry-go-round spins with a period of 2.0s.

12 a. What is (i) the total moment of inertia and so (ii) the total angular momentum of the {child+merry-go-round} system about the central axis? (Give correct units for each).

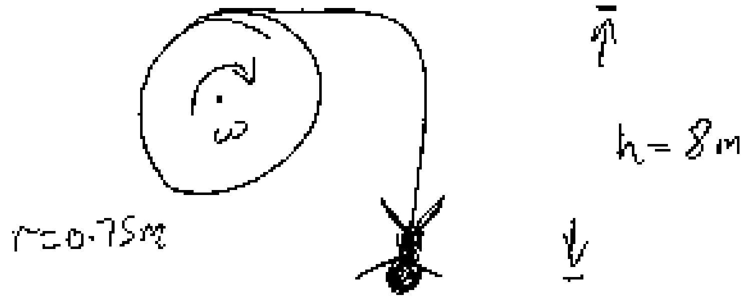
11 b. The child crawls directly to the center of the merry-go-round as it spins, and sits at the center. Using conservation of angular momemtum, show that the new period of rotation is now 1.6s.

12 c. Therefore, calculate the rotational kinetic energy of the system (i) before and (ii) after the child moves to the center.
(35 points)

d. **(Extra credit: 5 points):** Briefly explain why energy is not conserved even though angular momentum is conserved in this maneuver.

Physics 1A Quiz 6 Solutions

1.



a) After falling distance h, P.E. lost = $(KE)_{actor} + (KE)_{drum}$

i.e. $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$, with $\omega = v/r$

So $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\, v^2/r^2 = \frac{1}{2}v^2(m + I/r^2)$ 10

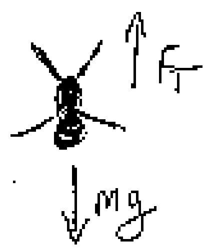
$\Rightarrow v^2 = \frac{2mgh}{(m + I/r^2)} = \frac{2 \times 70kg \times 10m/s^2 \times 8m}{70kg + 900/0.75^2} = 6.706$ 10

(i) $\Rightarrow v = 2.59 m/s$ so rotation period $T = \frac{2\pi}{\omega} = \frac{2\pi r}{v} = \underline{1.9s}$
(ii)

b) Total grav. PE lost = $mgh = 70 \times 10 \times 8 = 5600 J$ 15

Final KE of drum $= \frac{1}{2}I\omega^2 = \frac{1}{2}I\left(\frac{v^2}{r^2}\right) = 5364.8 J$

c)



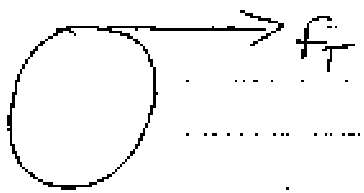
For Mr. Cruise, $ma = mg - F_T$

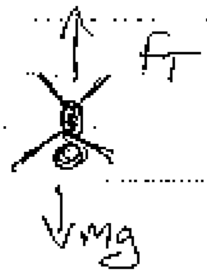
So if we find accel. a, $F_T = m(g - a)$

Method (1) Use $v^2 = v_0^2 + 2ah$ with $v_0 = 0\ m/s$, $h = 8m$

$\Rightarrow a = 0.419 m/s^2 \Rightarrow F_T = 70kg\,(10 - 0.419) = \underline{670.7N}$

1c Method II





For Mr. Cruise, $ma = mg - f_T$ as before (1)

For drum, torque $\tau = f_T . r = I \times$ angular accel.

$$f_T r = I\frac{d\omega}{dt} = \frac{I}{r}\frac{dv}{dt} = \frac{Ia}{r} \Rightarrow f_T = \frac{Ia}{r^2} \quad (2)$$

From (1), $a = (mg - f_T)/m$, substitute into (2)

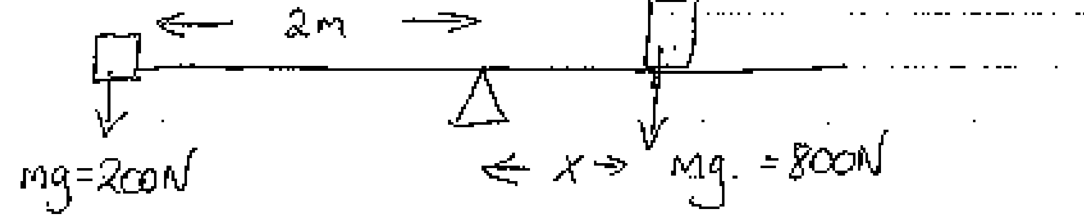
$$\Rightarrow f_T = \frac{I}{r^2}\frac{(mg - f_T)}{m}$$

$$\text{or } \left(I/mr^2 + 1\right).\, f_T = \frac{I\cancel{m}g}{\cancel{m}r^2}$$

$$\Rightarrow f_T = \frac{Ig/r^2}{\left(1 + I/mr^2\right)} = \frac{900 \times 10/0.75^2}{\left(1 + \frac{900}{70 \times 0.75^2}\right)} = 670.7\,N$$

and from (1), $a = \frac{mg - f_T}{m} = 0.419\ m/s^2$ as before.

2.



a) At equilibrium, total torque $\sum \tau = 0$

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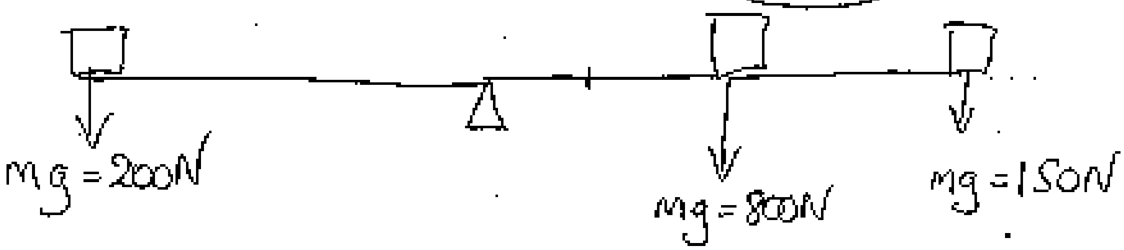
i.e. $200N \times 2m = 800N \times x$

i) $\Rightarrow x = 0.5m$ from pivot

ii) Force of pivot on beam is a reaction force $F = \sum mg$
$= 200N + 800N = 1000N.$

b)



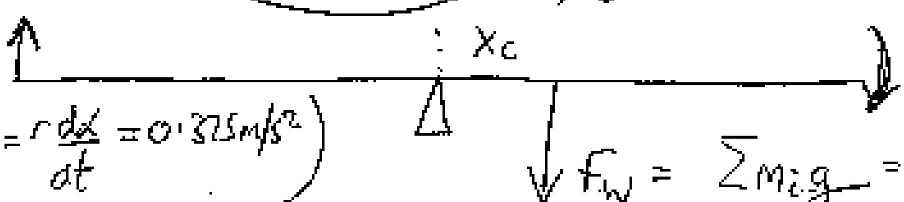
Let c.g. be at position x_c from pivot. Then

$$x_c = \frac{\sum m_i g x_i}{\sum mg} = \frac{\overbrace{200\times(-2)+800\times0.5}^{=0}+150\times2}{800+200+150N}$$

$= 0.261m$ 10

c) Moment of inertia ~~about pivot~~ $I = \sum m_i x_i^2 = (200\times2^2+800\times0.5^2+150\times2^2)$
i.e. $I = 1600\ kg\ m^2$ 10

d)



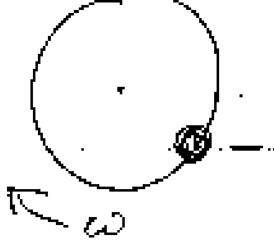
$\alpha = \frac{d\omega}{dt}$

$\left(\frac{dv}{dt} = r\frac{d\alpha}{dt} = 0.375 m/s^2\right)$

$f_W = \sum m_i g = 1150N$

Net torque $\tau = F_W x_c = I\alpha \Rightarrow \alpha = \frac{f_W x_c}{I} = \frac{1150\times0.261}{1600} = 0.1875\ rad/s^2$

3\. Top view



$r = 1.5$ m, $I_0 = 180$ kg m^2

a) i) Moment of inertia $I_{tot} = I_0 + mr^2 = 180 + (20 \times 1.5^2) = 225$ kg m^2

/12 ii) So ang. momentum $L = I\omega = I\dfrac{2\pi}{T} = \dfrac{225 \times 2\pi}{2} = 225\pi$ kg m^2/s (706.86 kg m^2/s)

b) Now with child at center, $(r = 0)$, $I = I_0 = 180$ kg m^2

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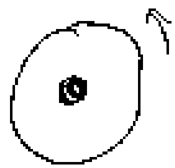

By conservation, $L = I_0\omega = 225\pi =$ constant

$\Rightarrow \omega = 2\pi/T = \dfrac{225\pi}{180} = 1.25\pi$ rad/s

or $T = \dfrac{2\pi}{\omega} = 1.60$ s

c) Before: $KE = \frac{1}{2}I\omega^2 = L^2/2I = \dfrac{(225\pi)^2}{2 \times 225} = 1110.3$ J

After: $KE = \dfrac{L^2}{2I_0} = \dfrac{(225\pi)^2}{2 \times 180} = 1387.9$ J

So KE increases by $\Delta KE = 277.61$ J

d) As child crawls in wards, no net torque $\Rightarrow L =$ constant $= I\omega$ at all times

But child does work in decreasing their radius to $r = 0$.
Work done by child against centripetal force reaction $W = \int F_c \, dr$

Advanced!

i.e. $W = \int \dfrac{mv^2}{r} \, dr$ with rotation speed $v = r\omega = \dfrac{rL}{I(r)}$

and $I = I_0 + mr^2$

$\Rightarrow W = \displaystyle\int \frac{m r^2 L^2}{r\,(I_0 + mr^2)^2} = \int \frac{m r L^2}{(I_0 + mr^2)^2} = \frac{1}{2}\left[\frac{L^2}{(I_0 + mr^2)}\right]_{r=r}^{r=0} = \Delta KE$