**Physics 1A QUIZ 5 . Closed Book. 3 questions. Write in blue/black ink.**

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| Centripetal force $F_C = \dfrac{mv^2}{r}$. Gravitational force $F_G = \dfrac{(GM)m}{r^2}$ |
| Circumference $= 2\pi r$. Work $= \int_{r_1}^{r_2} F dr$. Earth's gravity $g = 10\text{m/s}^2$. |

1. A ferris wheel at the Del Mar Fair has a radius of 10m and rotates in a vertical circle at constant speed. The seats are suspended from the wheel's circumference so that riders remain upright throughout the rotation. The wheel rotates at 6 m/s.

20 a. What is the effective weight of a rider of mass 50kg (i) at the top, and (ii) at the bottom of the wheel's rotation?
10 b. What is the maximum speed of rotation possible to keep the passengers in their seats? **(30 points)**

2. A cork of mass 0.01 kg moves in a horizontal circle at the end of a light string 0.3m in length, which makes an angle of 30 degrees below the horizontal. Using a free-body diagram for the cork, and equating vertical and horizontal forces acting on it:

10 a. Show that the tension in the string is 0.2N. (Hint: vertical forces).
18 b. Find the (i) radius of the cork's circular path and hence (ii) its speed of rotation. (Hint: horizontal forces). Therefore find (iii) the rotation period of the cork.

The tension is provided by passing the other end of the string through a ring and hanging washers from this other end, as in the lab.
12 c. If the tension is doubled to 0.4N by adding washers, but the length of the rotating string is unchanged, find the resulting new equilibrium values of (i) the angle of the rotating part of the string, and (ii) the cork's rotation speed. **(40 points)**

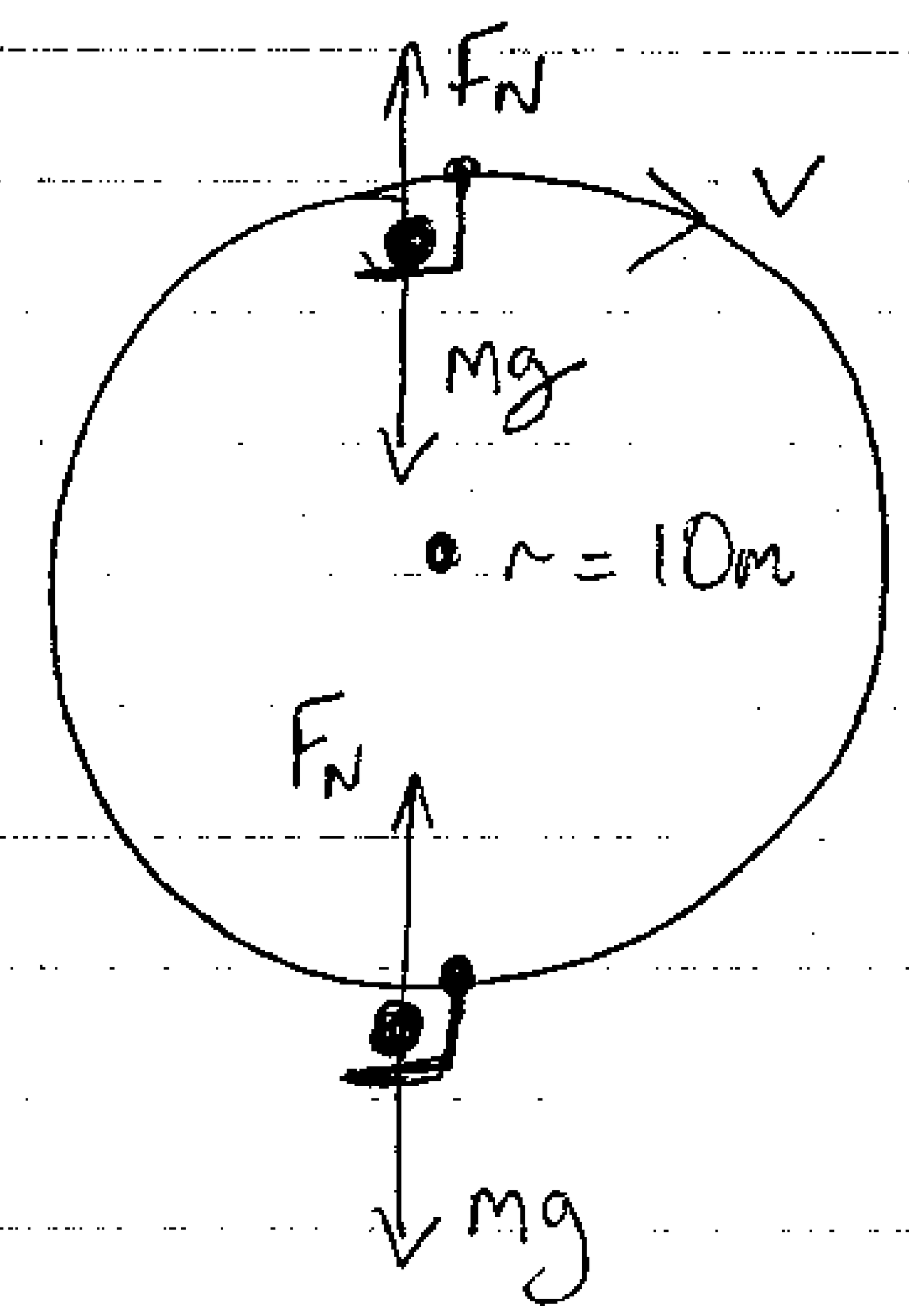
3. An astronaut stands very carefully on the surface of a small spherical planetoid with $GM = 6 \times 10^7 \ \frac{\text{Nm}^2}{\text{kg}}$, $R = 2 \times 10^4 \text{m}$.

10 a. What is the acceleration due to gravity at the surface?

The astronaut throws a small rock horizontally just fast enough for it to complete a circular orbit close to the surface.
10 b. What is (i) the speed and (ii) the orbital period of the rock?
10 c. How fast would the rock have to be launched to escape the planetoid's gravity completely? **(30 points)**

# Physics 1A Quiz 5 Solutions

1.




a) For rider mass $m = 50\,kg$, $mg = 500N$.

i) At top of circle, inward centripetal force

$$f_c = \frac{mv^2}{r} = mg - F_N$$

$\Rightarrow$ effective weight $F_N = m\left(g - \frac{v^2}{r}\right)$

with $v = 6\,m/s$ $\quad F_N = 50(10 - 6^2/10) = 50 \times 6.4\,m/s^2$

$$= 320\,N$$

ii) At bottom of circle, $F_c = \frac{mv^2}{r} = F_N - mg$ (inwards)

$\Rightarrow \quad F_N = m\left(g + \frac{v^2}{r}\right) = 50\left(10 + \frac{6^2}{10}\right)$

$= 50 \times 13.6\,m/s^2$

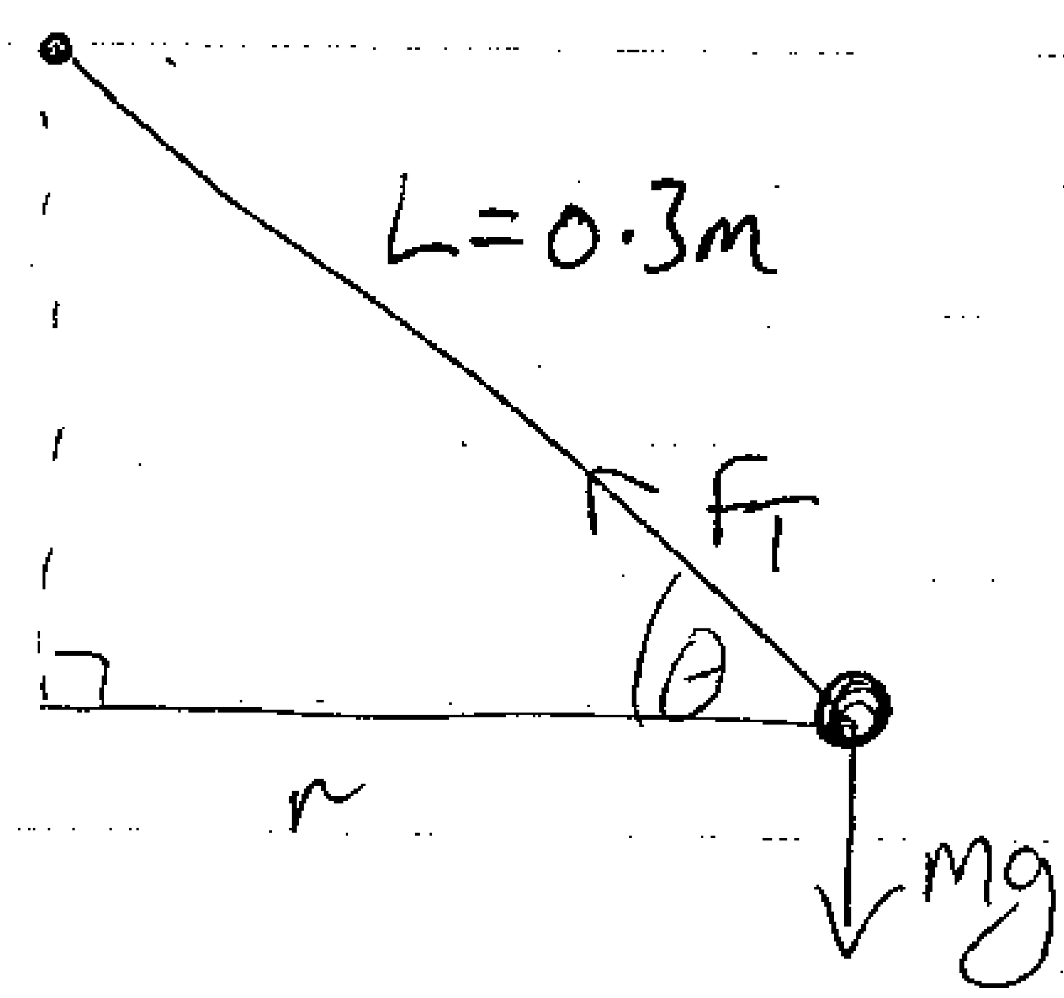
$\Rightarrow \quad F_N = 680\,N$ (so weight varies by $\pm 36\%$.)

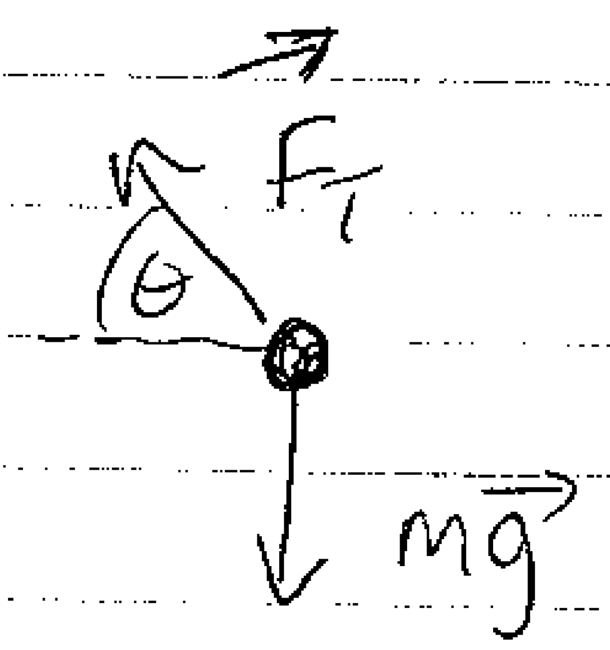
b) At max. speed $V_{max}$, $F_N \geq 0$ at top of circle

So $F_N = m(g - V_{max}^2/r) \geq 0$

$\Rightarrow V_{max} \leq \sqrt{rg} = \sqrt{10 \times 10} = 10\,m/s$

2.

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a) Equate vertical forces : $F_T \sin 30° = mg$

10 $\Rightarrow F_T = \frac{mg}{\sin 30°} = \frac{0.01 \times 10}{1/2} = 0.2\,N$

b) i) Rotation radius $r = L\cos 30° = 0.3 \cos 30° = 0.260\,m$

ii) Rotation speed $v$ given by : $F_c = \frac{mv^2}{r} = F_T \cos\theta$

i.e. $v^2 = \frac{F_T r \cos 30°}{m} = \frac{mg}{\sin 30°} \cdot \frac{L \cos 30° \cdot \cos 30°}{m}$

8

$v^2 = gL\cos^2 30° / \sin 30° = 10 \times 0.3 \times (3/4) / \tfrac{1}{2}$

$\Rightarrow v = \sqrt{9/2} = 2.12\,m/s$ So (iii) period $T = \frac{2\pi r}{v} = 0.77s$

c) If $F_T$ now $= 0.4N$, vertical forces $\Rightarrow 0.4 \sin\theta = mg$

$\Rightarrow \sin\theta = \frac{10 \times 0.01}{0.4} = 0.25 \Rightarrow \theta = 14.48°$

12 Horizontal forces: $\frac{mv^2}{r} = F_T \cos\theta$ with $r = L\cos\theta$

$\Rightarrow mv^2 = F_T L \cos^2\theta$ or $v = \cos\theta \sqrt{\frac{F_T L}{m}}$

$= \cos 14.48° \sqrt{\frac{0.4 \times 0.3}{0.01}} = 3.35\,m/s$

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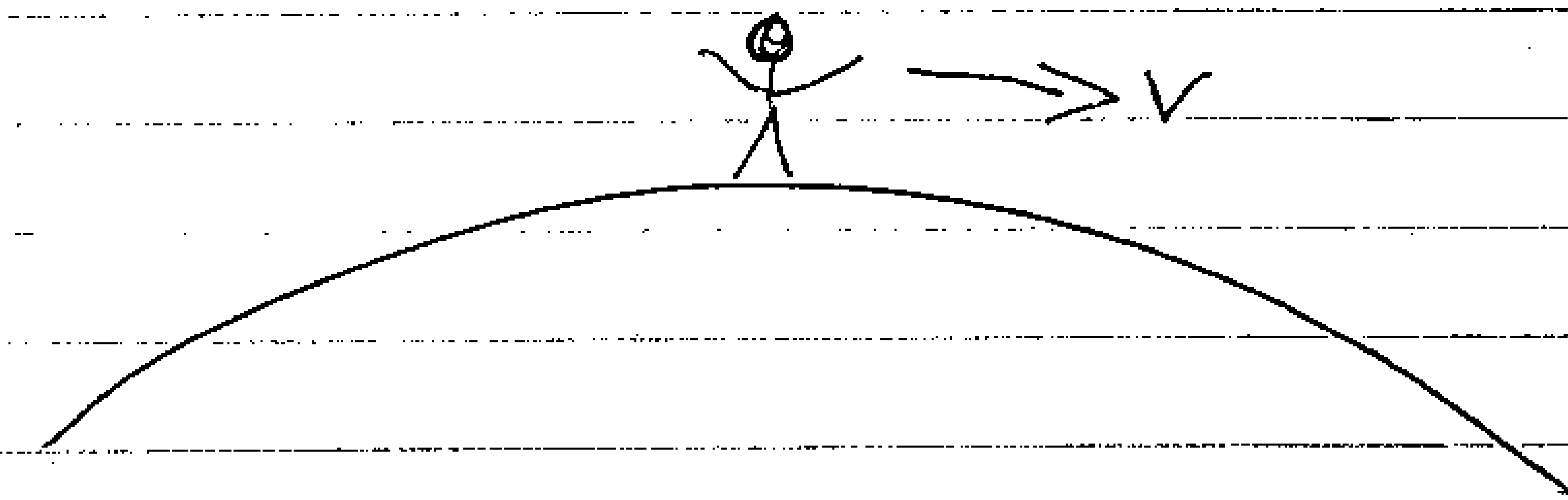
3.




a) $F_G = \frac{GMm}{R^2} = mg_p \Rightarrow g_p = \frac{GM}{R^2} = \frac{6\times10^7}{(2\times10^4)^2} = 0.15\,m/s^2$

b) i) For a circular orbit at $r = R$: $F_c = \frac{mv^2}{R} = \frac{GMm}{R^2}$

$$\Rightarrow v = \sqrt{\frac{GM}{R}} = \sqrt{\frac{6\times10^7}{2\times10^4}} = \sqrt{3000} = 54.77\,m/s$$

ii) $T = \frac{2\pi R}{v}$

c) To escape planetoid's gravity

Initial K.E. $\frac{1}{2}mv_0^2$ > Work done against gravity

i.e. $\frac{1}{2}mv_0^2 > \int_R^\infty \frac{GMm}{r^2}\,dr = GMm\int_R^\infty \frac{1}{r^2}\,dr$

$$\frac{1}{2}v_0^2 > GM\left[\frac{-1}{R}\right]_R^\infty = \frac{GM}{R} = g_p.R$$

$$\therefore v_0 > \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

$$v_0 > 77.45\,m/s.$$