Physics 1A QUIZ 4 . Closed Book. Write in blue or black ballpoint only.
2 Questions each worth 50 points:

> **Momentum** $p = mv$. **Kinetic energy** $KE = \tfrac{1}{2}mv^2 = \dfrac{p^2}{2m}$
>
> **Impulse** $\Delta(mv) = \int_0^t F dt$. **Work** $= \Delta(KE + PE) = \int_0^x F dx$.

1. Two gliders slide on a frictionless air track. The first (0.1kg) moves towards a stationary glider of mass 0.5 kg at a speed of +1 m/s. The gliders undergo an elastic collision.

a. Use the conservation laws to show that the final speeds of the smaller and larger masses must be -2/3 and +1/3 m/s respectively.
b. Therefore, what is (i) the momentum, and (ii) the kinetic energy of the *larger* mass only after the collision?

c. If instead the collision were totally inelastic, find (i) the momentum and (ii) the kinetic energy transferred to the larger mass only after the collision. (Hint: find the final common speed of both masses first).

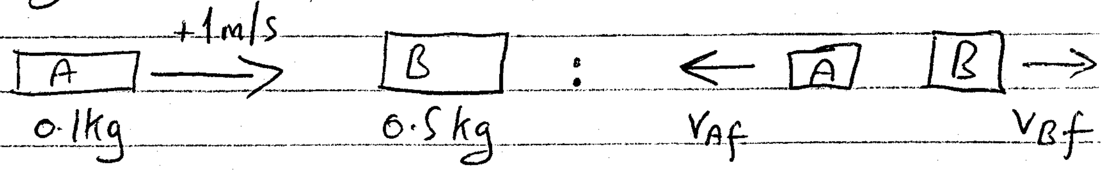
d. If the impulse (momentum transfer) of the collision lasted 0.2s in each case, what was the average force exerted on the masses for (i) the elastic and (ii) the inelastic collision.

2. Two skaters of masses m_A, m_B face each other on a frozen lake where the coefficient of friction is μ for both. They gently push each other apart and slide un-powered over the ice . Purely in terms of the skaters' masses m_A, m_B:
a. What is the ratio of (i) their initial speeds v_A , v_B, and (ii) their kinetic energies at the moment they separate? (Hint: $KE = p^2/2m$).
b. Therefore, what is the ratio of (i) the elapsed times and (ii) the distances moved by each skater before they each come to rest? (Remember the force of friction = μmg).

If m_A =40kg, m_B =60kg, μ =0.04, g=10 m/s^2, and skater A's initial speed is v_A =3 m/s:
c. What is the initial kinetic energy of both skaters combined?
d. How far does each skater move before coming to rest?
e. Briefly explain why the relative pushing strength of each skater does not affect these results.

Physics 1A Quiz 4 Solutions

1.



a) Momentum: Initial $P_i = m_A v_{Ai} = 0.1 \times 1\,m/s = 0.1\,Ns$

$=$ final $P_f = m_A v_{Af} + m_B v_{Bf}$

i.e. $0.1\, v_{Af} + 0.5\, v_{Bf} = 0.1 \qquad (1)$

Kinetic Energy: Elastic collision so

$(KE)_i = \frac{1}{2} m_A v_{Ai}^2 = \frac{1}{2} . 0.1 . 1^2 = (KE_f)$

$= \frac{1}{2} m_A v_{Af}^2 + \frac{1}{2} m_B v_{Bf}^2$

i.e. $\frac{1}{2} . 0.1\, v_{Af}^2 + \frac{1}{2} . 0.5\, v_{Bf}^2 = \frac{1}{2} . 0.1 . 1^2$

or $\quad v_{Af}^2 + 5 v_{Bf}^2 = 1 \qquad (2)$

Substitute from (1) $v_{Af} = 1 - 5 v_{Bf}$ into (2):

$(1 - 5 v_{Bf})^2 + 5 v_{Bf}^2 = 1$

i.e. $\quad 1 - 10 v_{Bf} + 25 v_{Bf}^2 + 5 v_{Bf}^2 = 1$

$\Rightarrow v_{Bf}(30 v_{Bf} - 10) = 0$

Solutions: $v_{Bf} = 0$ (A "misses" B entirely)

or $\quad v_{Bf} = \frac{10}{30} = \underline{+\frac{1}{3}\,m/s}$

Then $\quad v_{Af} = 1 - 5 v_{Bf} = 1 - \frac{5}{3} = \underline{-\frac{2}{3}\,m/s}$

(or take the given values and show they satisfy (1) and (2)).

b) Larger mass B [0.5] ⟶ $+\frac{2}{3}$ m/s

Momentum now $P_B = M_B V_{Bf} = 0.5 \times \frac{2}{3} = \frac{1}{3}$ Ns

$\therefore KE = \frac{P_B^2}{2M_B} = \frac{(\frac{1}{3})^2}{2 \times 0.5} = \frac{1}{9}$ J (0.111 J)

c) Inelastic case: [0.1][0.5] ⟶ V_f

Momentum: $M_A V_{Ai} = (M_A + M_B) V_f$

$\Rightarrow V_f = \frac{M_A V_{Ai}}{(M_A + M_B)} = \frac{0.1 \times 1}{0.1 + 0.5}$

$= \frac{1}{6}$ m/s (0.1667 m/s)

(i) For larger mass only, P_B now $= 0.5 \times \frac{1}{6} = \frac{1}{12}$ Ns (0.083)

(ii) K.E. now $= \frac{P_B^2}{2M_B} = \frac{(\frac{1}{12})^2}{2 \times 0.5} = \frac{1}{144}$ Ns (0.00694)

d) Impulse $= \Delta(mv) = F_{av} \Delta t$

Just look at mass A since $(F_{av})_{AB} = (F_{av})_{BA}$ from Newton III

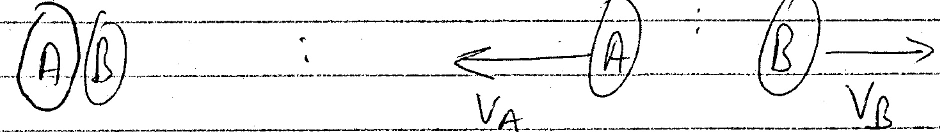
Elastic: $F_{av} \Delta t = M_A (V_{Af} - V_{Ai}) = 0.1\left(1 - (-\frac{2}{3})\right) = \frac{1}{6}$ N

$\Rightarrow F_{av} = \frac{1}{6} \div 0.2s = \frac{5}{6}$ N or 0.833 N.

Inelastic: $F_{av} \Delta t = M_A (V_f - V_{Ai}) = 0.1\left(1 - \frac{1}{6}\right) = \frac{1}{12}$ Ns

$\Rightarrow F_{av} = \frac{1}{12} \div 0.2 = \frac{5}{12}$ N or 0.417 N

- half of the elastic collision force.



a) Initial momentum $P_i = 0$ = final momentum P_f.

$$\therefore \quad M_A V_A + M_B V_B = 0 \qquad (1)$$

i) Initial speeds $\dfrac{V_A}{V_B} = -\dfrac{M_B}{M_A}$ →

ii) $\dfrac{\text{Kinetic energy of } A}{\text{" \quad " \quad " } B} = \dfrac{\frac{1}{2} M_A V_A^2}{\frac{1}{2} M_B V_B^2} = \dfrac{M_A}{M_B} \cdot \left(\dfrac{-M_B}{M_A}\right)^2$

$$= M_B / M_A$$

b) Once moving, each skater is slowed by friction
$F_A = \mu M_A g \qquad F_B = \mu M_B g$

$\therefore$ to change momentum to zero
Impulse $F_A t_A = M_A V_A \Rightarrow \mu \cancel{M_A} g t_A = \cancel{M_A} V_A$

or $t_A = \dfrac{V_A}{\mu g}$ so $t_A \propto V_A$, $t_B \propto V_B$

$\Rightarrow t_A / t_B = M_B / M_A$ from eqn. 1

To change KE to zero $\overbrace{F_A X_A}^{\text{WORK}} = \frac{1}{2} M_A V_A^2$

$\mu \cancel{M_A} g X_A = \frac{1}{2} \cancel{M_A} V_A^2$

$\Rightarrow X_A \propto V_A^2$ so $\dfrac{X_A}{X_B} = \dfrac{V_A^2}{V_B^2} = \left(\dfrac{M_B}{M_A}\right)^2$

2 c) Use $m_A = 40kg$ $m_B = 60kg$, $\mu = 0.04$, $g = 10m/s^2$

$V_A = 3m/s$

Then from (1) $V_B = \frac{m_A}{m_B} V_A = \frac{40}{60} \times 3 = 2m/s$

$\therefore$ total K.E. $= \frac{1}{2} m_A V_A^2 + \frac{1}{2} m_B V_B^2$

$= \frac{p_A^2}{2m_A} + \frac{p_B^2}{2m_B}$ with $p_A = p_B = 40 \times 3 = 120 Ns$

total K.E. $= \frac{(120)^2}{2}\left(\frac{1}{40} + \frac{1}{60}\right) = 300 J$

(or 0.07 Calories expended)

d) Skater A: $\mu g x_A = \frac{1}{2} V_A^2 \Rightarrow x_A = \frac{\frac{1}{2}(3)^2}{0.04 \times 10} = 11.25m$

Skater B: $x_B = x_A \left(\frac{m_B}{m_A}\right)^2 = 25.3125 m$

e) No matter who pushes harder, Newton's 3rd law tells us $F_{AB} = -F_{BA}$, so the total momentum $= 0$ and the skaters' speeds are set by the total energy converted to kinetic energy, regardless of who did the most work.