

Physics 1A QUIZ 3 . Closed Book. Write in blue or black ballpoint only.
2 Questions each worth 50 points:

Note: You must use $g = 10 \text{ m/s}^2$ for the earth's gravity.

You may assume the constant - acceleration equations:

$$v = v_0 + at \quad x = x_0 + v_0 t + \frac{1}{2} at^2 \quad v^2 = v_0^2 + 2ax$$

1. A roller coaster car of mass 600kg runs on a track with coefficient of friction $\mu=0.1$. The car is first driven at constant speed along a horizontal section of track, before rising un-powered up a 45-degree slope which is 40m long (and therefore has a height gain of $40 \sin 45^\circ = 28.284\text{m}$). Using work/energy arguments, or otherwise:

- 10pt a. Show that the coaster must be moving at about 25m/s at the base of the slope in order to reach the top. Give 4 significant figures.
- 10pt b. What is the minimum power output, in kW, of the electric motor required to maintain this speed against friction along the horizontal portion of the track?

The coaster reaches the top of the slope with no remaining speed, then rolls back down under gravity alone.

- 10pt c. Show that the new speed as it returns to the bottom of the slope is about 22.6 m/s. (Give 4 significant figures).

- 10pt d. If the brakes are then applied with a constant force of 5000N (in addition to the existing friction) (i) how long does it take the coaster to stop, (ii) what is the total stopping distance?

- 10pt e. At night, the coaster is parked at the end of the track with the brakes off. What is the largest angle of slope this portion of the track can have, in degrees, if the coaster is to remain stationary? (Hint: use a free-body diagram).

2. A janitor pushes a 20kg vacuum cleaner along a train car's carpet ($\mu=0.6$) with a horizontal force of 125N, accelerating from rest relative to the train. The length of the carpet is 15m.

15m ✓
a. What is (i) the total work done on the vacuum cleaner, (ii) the work done against friction, and (iii) the vacuum cleaner's final speed?

10m ✓
b. Show that it takes about 11 s to reach the end of the train. Use 4 significant figures.

An observer standing by the track sees the train pass by with the janitor pushing her vacuum cleaner inside. The train travels at 20 m/s. (So, for example, in 11s the vacuum cleaner appears to be pushed over a distance of $220\text{m}+15\text{m}=235\text{m}$).

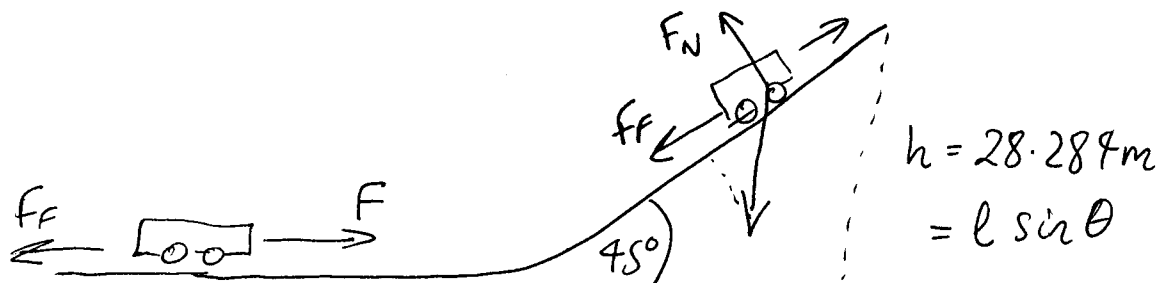
Now taking all quantities with respect to this trackside observer:

15m ✓
10m ✓
c. What is the (i) total work done on the vacuum cleaner, (ii) the work done against friction, and (iii) the vacuum's final speed? (Hints: The initial speed is just the speed of the train, 20 m/s, and the distance over which friction acts is now 235m, not 15m).

d. Do your answers in (c) make sense, given your answers in part (a)?

Physics 1A Quiz 3

1.



Normal force $F_N = mg \cos \theta \Rightarrow$ friction $F_f = \mu F_N = \mu mg \cos \theta$
opposing motion

- a) Initial k.e. at bottom = work done against gravity +
work done against friction +
remaining k.e.

$$\frac{1}{2} M V_0^2 = mgh + F_f l + 0$$
$$= mgl \sin \theta + \mu mgl \cos \theta$$

$$\Rightarrow V_0^2 = 2gl (\sin \theta + \mu \cos \theta) \text{ with } \theta = 45^\circ$$
$$= 2 \times 10 \times 40 (\sin 45^\circ + 0.1 \cos 45^\circ) = 622 \text{ m}^2/\text{s}^2$$

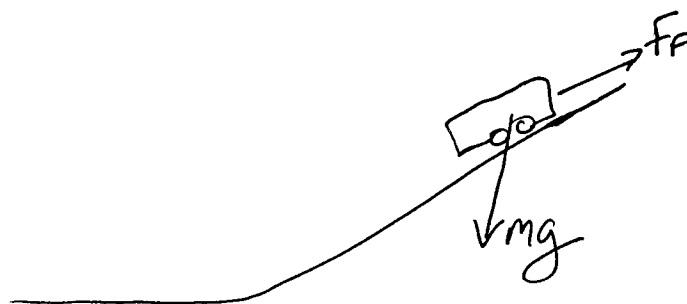
$$\Rightarrow V_0 = 24.95 \text{ m/s}$$

- b) Power = $\frac{\text{Work}}{\text{time}} = \text{Force} \times \frac{\text{distance}}{\text{time}} = \text{Force} \times \text{speed}$

with force of motor = $F_f = \mu mg$ (horizontal)

$$\therefore \text{Power} = \mu mg V_0 = 0.1 \times 600 \times 10 \times 24.95 = 14.97 \text{ kW}$$

c)



As coaster rolls downhill from top: initial $KE = 0$
 " $PE = mgh$

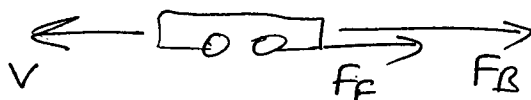
$\therefore \Delta KE = \Delta PE - (\text{work against friction})$

$$\frac{1}{2}mv^2 - 0 = mgh - F_f l = mgh - \mu mg l \cos \theta$$

$$\Rightarrow v^2 = 2gl (\sin \theta - \mu \cos \theta) \text{ with } \theta = 45^\circ, \mu = 0.1.$$

$$\Rightarrow v = \boxed{22.56 \text{ m/s}}$$

d) Back on level ground, apply brakes



$$\text{Total force} = F_B + F_f = 5000 \text{ N} + \mu mg = 5000 + 0.1 \times 600 \times 10 = 5600 \text{ N}.$$

i) If coaster stops in time Δt , momentum change $\frac{\Delta mv}{\Delta t} = F_B + F_f$

$$\therefore \text{time taken } \Delta t = \frac{600 \text{ kg} \times (v - 0)}{F_B + F_f} = \frac{600 \times 22.56}{5600}$$

$$= \boxed{2.417 \text{ s}}$$

OR: Find the accel. $a = \frac{F_B + F_f}{m} = \frac{5600 \text{ N}}{600 \text{ kg}} = -9.33 \text{ m/s}^2$

then use time $\Delta t = \frac{-(v - 0)}{a} = \boxed{2.417 \text{ s}}$

d) Work done in stopping coaster = $(f_B + f_F) x$

(10)

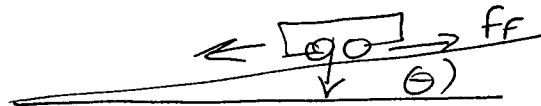
$$= KE = \frac{1}{2} m v^2$$

$$\Rightarrow x = \frac{\frac{1}{2} m v^2}{f_B + f_F} = \frac{\frac{1}{2} m (22.56)^2}{5600 \text{ N}} = \frac{27.27 \text{ m}}{}$$

OR Use $v^2 - 0^2 = 2ax$ with $a = -9.33 \text{ m/s}^2$ from (c)

OR use $x = vt + \frac{1}{2} a t^2$ with $t = \Delta t$ from part (c)

e)



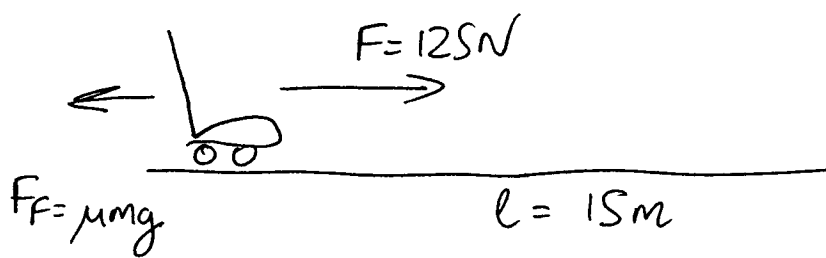
Coaster parked, no motion means $F_f \leq \mu F_N = mg \sin \theta$

ie. $\mu mg \cos \theta \geq mg \sin \theta$

or $\tan \theta \leq \mu = 0.4$

$\Rightarrow \theta = 37^\circ$

2.



- a) $F_{\text{net}} = 125\text{ N}$ so work = $125\text{ N} \times 15\text{ m} = 1875\text{ J}$
 Work against friction $W_f = F_f l = \mu mg l = 0.6 \times 20 \times 10 \times 15$
 $= 1800\text{ J}$

$\therefore \text{final K.E.} = W_{\text{TOT}} - W_f = 1875 - 1800 = 75\text{ J}$
 $\frac{1}{2} m v^2 = 75\text{ J} \Rightarrow v = \sqrt{\frac{2 \times 75}{20}} = 2.74\text{ m/s}$

- b) Net accel. of vacuum $a = \frac{F - F_f}{m} = \frac{125}{20\text{ kg}} - \mu g = 0.25\text{ m/s}^2$

$\therefore \text{time to cover } 15\text{ m is } t = \frac{v - 0}{a} = \frac{2.74 - 0}{0.25} = 10.95\text{ s}$



- i) total work $F_{\text{TOT}} = 125\text{ N} \times \text{distance}$
 $\times (15\text{ m} + 10\text{ m/s} \times 10.95\text{ s}) = 29250\text{ J}$

- ii) Against friction $W_f = F_f (l + v_{\text{train}} t) = 28080\text{ J}$

- iii) Change in K.E. $\frac{1}{2} m (v^2 - v_{\text{train}}^2) = 1170\text{ J}$

$\Rightarrow \text{final speed wrt ground } v^2 = v_{\text{train}}^2 + \frac{2 \times 1170}{20\text{ kg}} = 162.25\text{ m}^2/\text{s}^2$
 $\Rightarrow v = \sqrt{162.25} = 12.74\text{ m/s}$
 517 m/s^2

- d) Adding the (vacuum) speed of 2.74 m/s to the train's speed of $20\text{ m/s} \Rightarrow 22.74\text{ m/s}$ relative to ground, so answer makes sense.

$20 + 2.74 = 22.74$ makes sense